20. Calculate correct to six places of decimals.

$(1.01)^{3/2}-(0.99)^{3/2}$.

21. Sum the series to infinity :

$\frac{5}{1!}+\frac{7}{3!}+\frac{9}{5!}+.....$

22. Sum to n terms of the series

3.5.7+5.7.9+7.9.11+......

4186/M11

MAY 2010

Paper I — CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. If $y=\frac{1}{(x+1)(2x-1)}$, find y_n.

2. At which point is the tangent to the curve $x^2+y^2=5$ parallel to the line $2x-y+6=0$.

3. Find the radius of the curvature of the curve $x^4+y^4=2$ at the point $(1,1)$.

4. Show that $\int_0^{\pi/4}\log(1+\tan\theta)d\theta=\frac{\pi}{8}\log 2$.

5. Evaluate $\int(\log x)^2 dx$.

6. Expand $f(x)=x$ as a fourier series in the interval $(-\pi,\pi)$.

7. Show that any convergent sequence is a bounded sequence.

8. Discuss the convergence of the series

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\ldots.$$

9. Show that the series $\Sigma\left|\frac{x^{n-1}}{(n-1)!}\right|$ converges absolutely for all values of n.

10. Sum to infinity, the series :

$$1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots..$$

11. Show that :

$$\log\left(\frac{n+1}{n-1}\right)=\frac{2n}{n^2+1}+\frac{1}{3}\left(\frac{2n}{n^2+1}\right)^3+\frac{1}{5}\left(\frac{2n}{n^2+1}\right)^5+\ldots..$$

12. Sum the series :

$$\frac{3}{1^2.2^2}+\frac{5}{3^2.4^2}+\ldots.+\frac{2n+1}{n^2(n+1)^2}.$$

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Find the maxima (or) minimum values of $2(x^2-y^2)-x^4+y^4$.

14. For the curves $x^2=4y$ and $y^2=4x$, find the angle of intersection.

15. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.

16. Show that $\int_0^\pi \theta\sin^3\theta d\theta=\frac{2\pi}{3}$.

17. If $f(x)=\begin{cases}-x & \text{in } -\pi<x<0 \\ x & \text{in } 0<x<\pi,\end{cases}$

expand $f(x)$ as a fourier series in $(-\pi,\pi)$.

18. State and prove Raabe's test.

19. Prove that in an absolutely convergent series, the series formed by its positive terms alone is convergent and the series formed by its negative terms alone is convergent and conversely.

18. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=8$ and touch the plane $4x+3y=25$.

19. Find the equation of the right circular cylinder which passes through the circle $x^2+y^2+z^2=9$, $x+y+z=3$.

20. Prove that :

$div\left(\overline{F}\times\overline{G}\right)=\overline{G}.curl\ \overline{F}-\overline{F}.curl\ \overline{G}$.

21. Evaluate $\iint_S \overline{F}\cdot\overline{n}\ dS$, where $\overline{F}=4xz\overline{i}-y^2\overline{j}+yz\overline{k}$ and S is the surface of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$.

22. Verify Stoke's theorem for

$\overline{A}=(2x-y)\overline{i}-yz^2\overline{j}-y^2z\overline{k}$

where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

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4187/M12 MAY 2010

Paper II — TRIGONOMETRY, ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Prove that $\left(\dfrac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4=\cos 8\theta+i\sin 8\theta$.

2. If $\dfrac{\sin\theta}{\theta}=\dfrac{2165}{2166}$, show that θ is nearly the circular measure of 3°.

3. Find the sum to n terms of the series $\sin\alpha+\sin 2\alpha+\sin 3\alpha+......+\sin n\alpha$.

4. Find the equation of the plane passing through (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

5. Find the perpendicular distance from $P(3,9,-1)$ to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$.

6. Prove that the lines $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of their point of intersection.

7. Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0,\ 2x-y+2z=5$ for a great circle.

8. Find the equation to the right circular cone whose vertex is at the origin, whose axis in the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which has a semi vertical angle of 30°.

9. Find a unit vector normal to the surface given by $f(x,y,z)=x^2y^2+y+1-z$ at the point (0, 0, 1).

10. If the second order partial derivatives of $\overline{F}$ are continuous, prove that $div(curl\,\overline{F})=0$.

11. Evaluate $\int_C \overline{F}$ where $\overline{F}=yz\overline{i}+2y\overline{j}-x^2\overline{k}$ and C is the curve $x=t,\ y=t^2,\ z=t^3,\ 0\le t\le 1$.

12. Show by applying Green's theorem that the area bounded by a simple closed curve C is $\frac{1}{2}\int_C (x\,dy-y\,dx)$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Prove that

$$\frac{\sin 7\theta}{\sin\theta}=7-56\sin^2\theta+112\sin^4\theta-64\sin^6\theta.$$

14. If $\cosh u=\sec\theta$ show that $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

15. Find the sum to infinity of the series

$$C\sin\alpha+\frac{C^2}{2!}\sin 2\alpha+\frac{C^3}{3!}\sin 3\alpha+.....$$

16. Find the equation of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

17. Find the magnitude and the equations of the line of the shortest distance between the lines $\frac{x-5}{3}=\frac{y-7}{-16}=\frac{z-3}{7}$ and $\frac{x-9}{3}=\frac{y-13}{8}=\frac{z-15}{-5}$.

21. Solve

(a) $q^2 = z^2 p^2 (1 - p^2)$

(b) $p^2 + q^2 = npq$

22. Solve the following differential equation using Laplace Transform.

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$, given that $y = 0$, $\frac{dy}{dt} = 2$ when $t = 0$.

Paper III — MODERN ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Show that $f : R \to R$ defined by $f(x) = 2x - 2$ is a bijection and find its inverse.

2. If H_1, H_2 are two subgroups of a group G, then prove that $H_1 \cap H_2$ is also a subgroup of G.

3. State and prove Euler's theorem.

4. Prove that a subgroup N of G is normal if and only if the product of two right cosets of N is again a right coset of N.

5. If f is a homomorphism of a group G into a group G', then prove that

(a) $f(e) = e'$ where e is the identity of G and e' is the identity of G'

(b) $f(a^{-1}) = (f(a))^{-1}$, $\forall a \in G$.

6. Prove that every field is an integral domain.

7. Solve $xp^2 - 2yp + x = 0$.

8. Solve $(D^2 - 3D + 2)y = \sin 3x$.

9. Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2$.

10. Solve $xzp + yzq = xy$.

11. Evaluate $L(te^{-t} \sin t)$.

12. Evaluate $L^{-1}\left[\frac{s}{s^2 + 2s + 5}\right]$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Let $f : A \to B$ be a function. Let A_1 and A_2 be subsets of A and B_1, B_2 be subsets of B. Prove the following :

(a) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

(b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

(c) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

14. Prove that every subgroup of cyclic group is cyclic.

15. Let G be a group and H be a subgroup of G. Then prove that

(a) $a \in H \Leftrightarrow aH = H$

(b) $aH = bH \Leftrightarrow a^{-1}b \in H$

(c) $a \in bH \Leftrightarrow a^{-1} \in Hb^{-1}$.

16. State and prove fundamental theorem on homomorphism of groups.

17. If R is a ring, then prove that for all $a, b, c \in R$.

(a) $a \cdot 0 = 0 \cdot a = 0$

(b) $a(-b) = -(ab) = (-a)b$

(c) $(-a)(-b) = ab$

(d) $a(b - c) = ab - ac$.

18. Solve $(D^2 - 2D + 2)y = e^x \cos x + x$.

19. Solve $\frac{dx}{dt} + 2x - 3y = t, \frac{dy}{dt} - 3x + 2y = e^{2t}$.

20. Solve $\frac{d^2y}{dx^2} + y = \sec x$, using variation of parameters.

6. Prove that every field is an integral domain.

7. Solve $xp^2 - 2yp + x = 0$.

8. Solve $(D^2 - 3D + 2)y = \sin 3x$.

9. Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2$.

10. Solve $xzp + yzq = xy$.

11. Evaluate $L(te^{-t} \sin t)$.

12. Evaluate $L^{-1}\left[\frac{s}{s^2 + 2s + 5}\right]$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Let $f : A \to B$ be a function. Let A_1 and A_2 be subsets of A and B_1, B_2 be subsets of B.

Prove the following :

(a) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

(b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

(c) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

14. Prove that every subgroup of cyclic group is cyclic.

15. Let G be a group and H be a subgroup of G. Then prove that

(a) $a \in H \Leftrightarrow aH = H$

(b) $aH = bH \Leftrightarrow a^{-1}b \in H$

(c) $a \in bH \Leftrightarrow a^{-1} \in Hb^{-1}$.

16. State and prove fundamental theorem on homomorphism of groups.

17. If R is a ring, then prove that for all $a, b, c \in R$.

(a) $a \cdot 0 = 0 \cdot a = 0$

(b) $a(-b) = -(ab) = (-a)b$

(c) $(-a)(-b) = ab$

(d) $a(b - c) = ab - ac$.

18. Solve $(D^2 - 2D + 2)y = e^x \cos x + x$.

19. Solve $\frac{dx}{dt} + 2x - 3y = t, \frac{dy}{dt} - 3x + 2y = e^{2t}$.

20. Solve $\frac{d^2y}{dx^2} + y = \sec x$, using variation of parameters.

(7 pages)

4189/M22

MAY 2010

Paper IV — STATISTICS

(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Calculate the median for the following distribution :

Marks	No. of students
5-10	7
10-15	15
15-20	24
20-25	31
25-30	42
30-35	30
35-40	26
40-45	15
45-50	10

2. Calculate the mean deviation from the following series :

x:	10	11	12	13	14
f:	3	12	18	12	3

3. Find the first three moments about $A=80$ of the following data :

58, 62, 48, 44, 60, 50, 55, 45, 42, 56.

4. Two judges in a beauty contest rank the 10 entries as below :

x:	1	6	5	10	3	2	4	9	7	8
y:	6	4	9	8	1	2	3	10	5	7

What degree of agreement is there between the judgement of the two judges?

5. Find $\Delta^4 U_0$ and $\Delta^2 U_2$, given $U_0=6, U_1=7, U_2=16, U_3=39, U_4=82$.

6. If $(AB)=256$, $(\alpha B)=768$, $(A\beta)=48$, $(\alpha\beta)=144$, test whether the attributes A and B are independent.

22. Analyse the variance in the following Latin square :

A 8	C 18	B 9
C 9	B 18	A 16
B 11	A 10	C 20

14. Fit a parabola of second degree to the following data :

x :	0	1	2	3	4
$y=f(x)$:	1	1.8	1.3	2.5	6.3

15. Determine the two equations of the lines of regression :

x :	10	12	13	12	16	15
y :	40	38	43	45	37	43

Find y, when $x=20$.

16. Given $N=200$, $(A)=100$, $(B)=120$, $(C)=160$, $(AB)=70$, $(AC)=90$, $(BC)=84$, find the least and greatest values of (ABC).

17. Calculate index numbers by :

(a) Laspeyre's method

(b) Paasche's method

(c) Marshall-Edgeworth method

(d) Fisher's method

from the following data :

Commodity	1980		1981	
	Price	Quantity	Price	Quantity
A	4	50	10	40
B	3	10	9	2
C	2	5	4	2

18. State and prove addition theorem and product theorem of probability.

19. Find the mean, standard deviation and mode of normal distribution.

20. (a) Explain the test of significance for difference of sample means.

(b) Intelligence test on two groups of boys and girls gave the following results :

	Mean	S.D.	N
Girls	75	15	150
Boys	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls?

21. (a) Describe the test for the difference between the means of two samples of different size.

(b) Two random samples drawn from two normal populations are given below. Test whether the two populations have the same variance.

Sample 1 :	20	16	26	27	23	22
Sample 2 :	17	23	32	25	22	24
Sample 1 :	18	24	25	19		
Sample 2 :	28	6	31	33	30	27

7. A random variable X has the following probability distribution :

x :	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Determine :

(a) The value of a

(b) $p(X<3)$

(c) $p(X\geq 3)$

(d) $p(0<X<5)$.

8. Prove :

(a) $E(aX+b)=aE(X)+b$

(b) $Var(X+c)=Var(X)$

a and b are constants.

9. Find the standard deviation of the Poisson distribution.

10. In a sample of 1000, the mean is 17.5 and the S.D. 2.5. In another sample of 800, the mean is 18 and S.D. 2.7. Assuming that the samples are independent, discuss whether the two samples have come from a population which have the same S.D.

11. In a sample of 8 observations, the sum of the squares of the deviations of the sample values from the sample mean was 84.4 and in another sample of 10 observations, it was 102.6. Test whether this difference is significant at 5% level, using F-test (Given : 5% point of F is 3.29 for 7, 9 degrees of freedom).

12. The following table gives the number of train accidents in a country that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week.

Days :	Sun	Mon	Tue	Wed	Thur	Fri	Sat
No. of accidents :	20	18	13	23	26	11	15

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Calculate mean and S.D. of the following frequency distribution of marks :

Marks :	0-10	10-20	20-30	30-40
No. of students :	5	12	30	45
Marks :	40-50	50-60	60-70	
No. of students :	50	37	21	

(6 pages)

4190/M23 **MAY 2010**

Paper V – MECHANICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. State and prove Lami's theorem.

2. Find the magnitude and direction of the resultant of three coplanar forces $P, 2P, 3P$ acting at a point and inclined mutually at an angle of 120°.

3. $ABCD$ and $A'B'C'D'$ are parallelograms. Prove that the forces $\overline{AA'}$, $\overline{B'B}$, $\overline{CC'}$ and $\overline{D'D}$ acting at a point will keep it at rest.

4. Two like parallel forces P and Q act on a rigid body at A and B respectively. If P and Q be interchanged in position, show that the point of application of the resultant will be displaced along AB through a distance $\frac{P-Q}{P+Q}AB$.

5. If D is any point on the base BC of triangle ABC such that $\frac{BD}{DC} = \frac{m}{n}$ and $\angle ADC = \theta$, $\angle BAD = \alpha$, $\angle DAC = \beta$, then prove that

 (a) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

 (b) $(m+n)\cot\theta = n\cot B - m\cot C$.

6. A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane and its upper end against an equally rough vertical wall. If θ be the inclination of the ladder to the vertical, prove that $\tan\theta = \frac{2\mu}{1-\mu^2}$, where μ is the coefficient of friction.

7. A stone is thrown with a velocity of $39.2 m/\sec$ at 30° to the horizontal. Find at what times it will be at height of 14.7 m ($g = 9.8 m/\sec^2$).

8. If h and h_1 be the greatest heights in the two paths of a projectile with a given velocity for a given velocity for a given range R, prove that $R = 4\sqrt{hh_1}$.

18. A uniform rod rests in limiting equilibrium within a rough hollow sphere. If the rod subtends an angle 2α at the center of the sphere and if λ be the angle of friction, show that the inclination of the rod to the horizontal is

$$\tan^{-1}\left(\frac{\sin 2\lambda}{\cos 2\alpha + \cos 2\lambda}\right).$$

19. A smooth sphere of mass m_1 impinges obliquely with velocity u_1 to another smooth sphere of mass m_2 moving with velocity u_2. If the coefficient of restitution is e, find the velocities after impact.

20. If the displacement of a moving point at any time is given by an equation of the form $x = a\cos wt + b\sin wt$, show that the motion is a simple harmonic motion. If $a = 3$, $b = 4$ and $w = 2$, determine the period, amplitude, maximum velocity and maximum acceleration of the motion.

21. A point P describes a curve with constant velocity and its angular velocity about a given fixed point O varies inversely as the distance form O. Show that the curve is an equiangular spiral whose pole is O.

22. A particle moves in an ellipse under a force which is always towards its forces. Find the law of force and the velocity at any point of the path.

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9. A smooth sphere of mass m impinges obliquely on a smooth sphere of mass M which is at rest. Show that, if $m = eM$ the directions of motion after impact are at right angles (e is the coefficient of restitution).

10. Prove that the composition of two simple harmonic motions of the same period and in the same straight line is also a simple harmonic motion of the same period.

11. The velocities of a particle along and perpendicular to a radius vector from a fixed origin are λr^2 and $\mu\theta^2$, where μ and λ are constants. Show that the accelerations along and perpendicular to the radius vector are $2\lambda^2 r^3 - \frac{\mu^2\theta^4}{r}$ and $\mu\left(\lambda r\theta^2 + \frac{2\mu\theta^3}{r}\right)$.

12. Find the pedal equation of the central orbit.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. ABC is a given triangle, forces P, Q, R acting along the lines OA, OB, OC are in equilibrium, prove that

$$P:Q:R = a^2(b^2+c^2-a^2):b^2(c^2+a^2-b^2):c^2(a^2+b^2-c^2),$$ if O is the circumcentre of the triangle.

14. Forces of magnitudes $P-Q, P, P+Q$ act at a point in directions parallel to the sides of an equilateral triangle taken in order. Show that the resultant is of magnitude $Q\sqrt{3}$ acting perpendicular to the direction of the second force.

15. Find the resultant of two unlike parallel forces acting on a rigid body. Find also the position of the resultant.

16. A uniform rod of length 'a' hangs against a smooth vertical wall being supported by means of a string of length l tied to one end of the rod, the other end of the string attached to a point in the wall. Show that the rod can rest inclined to the wall at an angle θ give by $\cos^2\theta = \frac{l^2 - a^2}{3a^2}$.

17. Obtain the range of a particle on an inclined plane.

[P.T.O.]

18. Find the constant 'a' so that $u(x, y) = ax^2 - y^2 + xy$ is harmonic. Find an analytic function $f(z)$ for which u is the real part. Also find its harmonic conjugate.

19. (a) Show that by means of the inversion $w = \frac{1}{z}$ the circle given by $|z-2| = 7$ is mapped into the circle $\left|w + \frac{2}{45}\right| = \frac{7}{45}$.

(b) Prove that any bilinear transformation preserves cross ratio.

20. State and prove Cauchy's theorem.

21. (a) Show that $\int_0^{\pi} \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$ $[a > 1]$.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$.

22. (a) Prove that $\int_0^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}$.

(b) Evaluate $\int_c \tan z\, dz$ where C is $|z| = 2$.

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Paper VI — REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — $(8 \times 5 = 40$ marks$)$

Answer any EIGHT questions.

1. Let $x, y \in R^2$ then $x = (x_1, x_2)$ and $y = (y_1, y_2)$ where $x_1, x_2, y_1, y_2 \in R$. We define $d(x, y) = |x_1, -y_1| + |x_2 - y_2|$ then d is a metric on R^2.

2. Prove that in any metric space every closed ball is a closed set.

3. Prove that a subset A of a complete metric space M is complete iff A is closed.

4. Prove that the function $f : (0,1) \to R$ defined by $f(x) = \frac{1}{x}$ is not uniformly continuous.

5. If A and B are connected subsets of a metric space M and if $A \cap B \neq \phi$. Prove that $A \cup B$ is connected.

6. Prove that a closed subspace of a compact metric space is compact.

7. The points z_1 and z_2 are reflection points for the line $\bar{\alpha}z + \alpha\bar{z} + \beta = 0$ iff $\bar{\alpha}\ z_1 + \alpha\,\bar{z}_2 + \beta = 0$.

8. If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$ prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$.

9. Find the bilinear transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = -1$ into the points $w_1 = i$, $w_2 = 1$ and $w_3 = 0$ respectively. What are the invariant points in this transformation?

10. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2\, z^2}{(z-1)(z-z)} d^2$ where C is the circle $|z| = 3$.

11. Expand $\cos z$ into a Taylor's series about the point $2 = \frac{\pi}{2}$ and determine the region of convergence.

12. Calculate the residue of $\frac{z+1}{z^2 - 2^2}$ at its poles.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. If d and ρ are metrics on M and if there exists $k > 1$ such that $\frac{1}{k}\ \rho(x, y) \leq d(x, y) \leq k\rho(x, y)$ for all $x, y \in M$. Prove that d and ρ are equivalent metrics.

14. Show that l_2 is complete.

15. If $f : R \to R$ is continuous at $a \in R$ iff $w(f, a) = 0$

16. Prove that a metric space (M, d) is totally bounded iff every sequence in M has Cauchy subsequence.

17. (a) Prove that the function

$$f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & if\ \ 2 \neq 0 \\ 0 & if\ \ z = 0 \end{cases}$$

Satisfies C-R equations at the origin but $f'(0)$ does not exist.

(b) If $f(z)$ is analytic prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ $|f(z)|^2 = 4|f(z)|^2$.

18. Examine for consistency the following equations :

$$2x + 6y + 11 = 0$$
$$6x + 20y - 6z + 3 = 0$$
$$6y - 18z + 1 = 0.$$

19. Prove that every composite number can be resolved into prime factors and this can be done only in one way.

20. If n is a prime, prove that n_{c_r} is divisible by n .

21. State and prove Fermat's theorem.

22. (a) Find the remainder obtained in dividing 2^{460} by 47.

(b) If $M = 1.3.5.....(p-2)$ where p is an odd prime, show that $M^2 \equiv (-1)^{\frac{p+1}{2}} \mod (p)$.

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4192/M32

MAY 2010

Paper VII — LINEAR ALGEBRA AND NUMBER SYSTEM

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Let V be a vector space over a field F. Then prove that

(a) $\alpha(v_1 - v_2) = \alpha v_1 - \alpha v_2 \forall \alpha \in F$, $v_1, v_2 \in V$.

(b) $\alpha v = \beta v$ and $v \neq 0 \Rightarrow \alpha = \beta$.

2. Prove that any subset of a linearly independent set is linearly indepedent.

3. Let V be a vector space over a field F. Prove that a non-empty subset W of V is a subspace of V iff $u, u \in W$ and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

4. Compute the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

5. Let A be a square matrix. Prove the following :

(a) A is Hermitian iff $A = A^{-T}$

(b) A is skew Hermitian iff $A = -A^{-T}$.

6. If V is an inner product space, prove that $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$, $u, v \in V$.

7. Find the rank of the matrix $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{bmatrix}$.

8. Prove that similarity is an equivalence relation on the set S of all $n \times n$ matrices.

9. Find the highest power of 3 dividing 1000!.

10. Find the smallest number with 18 divisors.

11. Show that $3^{2n-1} + 2^{n+1}$ is divisible by 7.

12. State and prove Wilson's theorem.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Let V be a finite dimensional vector space over a field F. Let A, B be two subspaces of V. Prove that $\dim(A+B) = \dim A + \dim B - \dim(A \cap B)$.

14. State and prove the fundamental theorem of homomorphism for vector spaces.

15. (a) Prove that every square matrix satisfies its characteristic equation.

(b) Find the eigen roots and eigen vectors of the matrix A given below :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

16. Explain Gram-Schmidt orthogonalization process.

17. (a) If A and B are two $m \times n$ matrices prove that $(A^T)^T = A$ and $(A+B)^T = A^T + B^T$,

(b) If A is an $m \times n$ matrix, B is an $n \times p$ matrix, prove that $(AB)^T = B^T A^T$.

(7 pages)

4193/M33 **MAY 2010**

Paper VIII — LINEAR PROGRAMMING AND OPERATIONS RESEARCH

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Solve graphically the LPP :

Maximize $Z = 3x_1 + 5x_2$

Subject to

$x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0$

$x_2 \geq 0.$

2. Write the dual of the following LPP.

Minimize $Z = 4x_1 + 6x_2 + 18x_3$

Subject to

$x_1 + 3x_2 \geq 3$

$x_1 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0.$

3. Explain the travelling salesman problem.

4. Solve the following assignment problem

Machines

		X	Y	Z
Jobs	P	17	25	31
	Q	10	25	16
	R	12	14	11

(cost in Rs. per unit)

5. Obtain an initial basic feasible solution to the following T P using Matrix minima method

Available

		D	E	F	C	
Requirements	A	11	13	17	14	250
	B	16	18	14	10	300
	C	21	24	13	10	400
		200	225	275	250	

6. Write the limitations of O.R.

7. Let X and Y be two random variables defined over the same sample space. Prove that $E(XY) = E(X)E(Y)$ if X and Y are independent.

8. Derive the fundamental EOQ formula.

(a) What is the probability that a person arriving at the booth will have to wait?

(b) The telephone department will install a second booth when convinced that an arrival would expect waiting for alteast 3 minutes for phone. By how much should the flow of arrivals increase in order to justify a second booth.

(c) Find the average number of units in the system.

(d) Estimate the fraction of a day that the phone will be in use.

(e) What is the probability that it will take him more than 10 minutes altogether to wait for phone and complete his call?

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16. Solve the following assignment problem

Task

		E	F	G	H
Men	A	18	26	17	11
	B	13	28	14	26
	C	38	19	18	15
	D	19	26	24	10

17. Solve the following transportation problem

	D	E	F	G	H	Available
A	5	8	6	6	3	800
B	4	7	7	6	5	500
C	8	4	6	6	4	900
Requirements	400	400	500	400	800	

18. Consider the inventory system with the following data in usual notations :

$R = 1000$ units per year ; $I = 0.30$

$P =$ Rs. 0.50/unit ; $C_s =$ Rs. 10 and lead time $L = 2$ years

Determine the following :

(a) EOQ

(b) Reorder point

(c) Minimum average cost.

19. A machine is purchased for Rs. 5,000

Year :	1	2	3	4	5	6	7	8
Running Costs/ year :	1500	1600	1800	2100	2500	2900	3400	4000
Resale value :	3500	2500	1700	1200	800	500	500	500

From the above particulars find at what year is the replacement due?

20. Find the sequence of Jobs that minimizes the total elapsed time (in hours) required to complete the following jobs in two machines M_1 and M_2 in the order M_1, M_2.

Job :	J_1	J_2	J_3	J_4	J_5	J_6
M_1 :	1	3	8	5	6	3
M_2 :	5	6	3	2	2	10

21. Solve the following game graphically :

Player B

$$\text{Player A} \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix}$$

22. Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponential, with mean 3 minutes.

9. The capital cost of a machine is Rs. 80,000. The expected maintenance costs and resale price in different years are as given under

Year :	1	2	3	4
Maintenance cost :	1000	1200	1600	2400
Resale value :	75000	72000	70000	65000

After what time interval should the machine be replaced?

10. Solving the following game

$$\begin{array}{cc} & B \\ A & \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix} \end{array}$$

11. Explain the terms :

(a) Pay off matrix

(b) Pure and mixed strategie.

12. What is inventory? What are the advantages and disadvantages of having inventory?

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Solve the following problem using Simplex method

Maximize $Z = 6x_1 + 9x_2$

Subject to

$2x_1 + 2x_2 \leq 24$

$x_1 + 5x_2 \leq 44$

$6x_1 + 2x_2 \leq 60$

$x_1, x_2 \geq 0.$

14. Solve the following LPP using Big-M method

Maximize $Z = 2x_1 + 3x_3$

Subject to

$x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1, x_2, x_3 \geq 0.$

15. Using Dual simplex method solve the LPP

Minimize $Z = 2x_1 + x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0.$

4194/M34 **MAY 2010**

Paper IX — PROGRAMMING IN C AND C++

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Give all types of C operators. Explain any two of them.

2. Explain formatted input with example.

3. Give the general form of switch statement and explain with example.

4. Explain Macros and preprocessor.

5. Name the four storage classes in C. Explain any one of them with example.

6. Write a C program to calculate the average of n numbers using arrays.

7. Explain strcpy and strlen.

8. Write a program to display the contents of the pointer variables using arithmetic operation.

9. Explain 'Arrays of structures' with an example.

10. Explain the general format for opening and closing a data file.

11. How to implement Queues in C? Explain.

12. Give the advantages and disadvantages of Linked List.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Explain the different types of data types in C.

14. (a) Explain the nested if statement in C with example.

(b) Explain the simple if statement in C and give an example.

15. Explain 'categories of functions' with example.

16. Explain

(a) Nesting of functions

(b) Recursion with examples.

17. Explain 'Functions in multiple programs' with example.

18. Write a program to arrange given names in alphabetical order.

19. Explain the usuage of pointers in a function.

20. Write a program to count vowels, consonants, digits, white spaces and other characters in a line of text.

21. (a) Write a program to assign some values to the member of the structure using a points structure operator.

(b) How to create a data file? Explain.

22. Explain 'Operation on Linked List'.

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19. Prove that

$$\sum_{x=1}^{n} y_x = nc_1 y_1 + nc_2 \Delta y_1 + nc_3 \Delta^2 y_1 + ... + \Delta^{n-1} y_1 \quad \text{and}$$

use it to show that $\sum_{1}^{3} x^3 = \dfrac{n^2(n+1)^2}{4}$.

20. Find the first and second derivative of $y=\sqrt{x}$ at $x=15$ from the following table :

x :	15	17	19	21	23	25
$y=\sqrt{x}$:	3.873	4.123	4.359	4.583	4.796	5.000

21. Evaluate $\int_{4}^{5.2} \log_e x$ using

(a) Simpson's $\frac{1}{3}$ rule and

(b) Simpson's $\frac{3}{8}$ rule.

22. Solve : $y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$.

4195/M3A **MAY 2010**

Paper X — THEORY OF EQUATIONS AND NUMERICAL ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Frame an equation with rational coefficients one of whose roots is $\sqrt{5}+\sqrt{2}$.

2. Find the sum of the fourth powers of the roots of the equation $x^4 - x^3 - 7x^2 + x + 6 = 0$.

3. Transform the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ whose roots are each increased by 7.

4. Prove that the equation $x^6 - x^5 + 3x^4 + 4x - 1 = 0$ has atleast two imaginary roots.

5. Find a real root of the equation $x^3 - x - 1 = 0$ correct to three decimal places by using false position method.

6. Solve the equations $x+2y+5z=23$, $3x+y+4z=26$, $6x+y+7z=47$, using Gauss-Jordan method.

7. Calculate the value of y when $x=0.47$ from the following data :

x :	0	0.1	0.2	0.3	0.4	0.5
y :	1.000	1.110	1.242	1.399	1.583	1.797

8. Find the divided differences of y given the following table :

x :	1	2	7	8
y :	1	5	5	4

9. Find the missing term in the following table :

x :	7	9	11	13	15	17
y :	32	78	–	144	257	381

10. Prove :

(a) $E\nabla=\Delta=\nabla E$

(b) $\delta E^{1/2}=\Delta$.

11. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h=0.2$.

12. Solve : $y_{x+1}-3y_x=2$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Show that the roots of the equation $x^3+px^2+qx+r=0$ are in arithmetical progression if $2p^3-9pq+27r=0$.

14. Solve : $6x^5+11x^4-33x^3-33x^2+11x+6=0$.

15. Find the condition that the cubic equation $ax^3+3bx^2+3cx+d=0$ has two equal roots and when the condition is satisfied, find the equal roots.

16. Find the positive root of $x-\cos x=0$ by bisection method in three decimal places.

17. Use Gauss-Seidal method, to solve $8x-3y+2z=20$, $4x+11y-z=33$, $6x+3y+12z=35$.

18. Using Lagrange's interpolation formula, find $y(10)$ from the following data :

x :	5	6	9	11
y :	12	13	14	16

6. Solve the equations $x+2y+5z=23$, $3x+y+4z=26$, $6x+y+7z=47$, using Gauss-Jordan method.

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x :	0	0.1	0.2	0.3	0.4	0.5
y :	1.000	1.110	1.242	1.399	1.583	1.797

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Answer any SIX questions.

13. Show that the roots of the equation $x^3+px^2+qx+r=0$ are in arithmetical progression if $2p^3-9pq+27r=0$.

14. Solve : $6x^5+11x^4-33x^3-33x^2+11x+6=0$.

15. Find the condition that the cubic equation $ax^3+3bx^2+3cx+d=0$ has two equal roots and when the condition is satisfied, find the equal roots.

16. Find the positive root of $x-\cos x=0$ by bisection method in three decimal places.

17. Use Gauss-Seidal method, to solve $8x-3y+2z=20$, $4x+11y-z=33$, $6x+3y+12z=35$.

18. Using Lagrange's interpolation formula, find $y(10)$ from the following data :

x :	5	6	9	11
y :	12	13	14	16

18. If $A = (a_{ij})$ is the adjacency matrix of a graph G, prove that the number of walks of length n from v_i to v_j is equal to the (i, j)th entry in A^n.

19. Prove that every element of $L(\rho)$ is a (edge) disjoint union of cycles.

20. State and prove Euler's theorem on planar graphs. Also prove that $|V|-|E|+|F| = K+1$ if G has partitioned into k – components.

21. If G is a graph on p vertices, then prove that

(a) $2\sqrt{P} \leq \chi(G) + \chi(\overline{G}) \leq p+1$;

(b) $P \leq \chi(G)\chi(\overline{G}) \leq \frac{(p+1)^2}{4}$.

22. Prove that every tournament D contains a directed Hamiltonian path.

4196/M3B

MAY 2010

Paper XI — GRAPH THEORY

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — $(8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. (a) Show that the sum of the degree of all the vertices of a graph is even.

(b) Prove that for any graph G the number of points of odd degree is even.

2. Prove that any connected (p, q) – graph contains a cycle iff $q \geq p$.

3. Show that every non trivial graph contains atleast two vertices which are not cut – vertices.

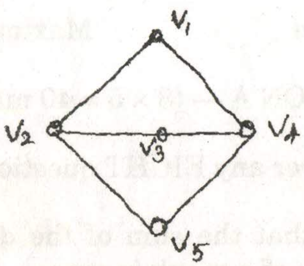
4. Prove that any non trivial connected graph is eulerian if it has no vertex of odd – degree.

5. Define weighted graph. Give an example of a graph with weight 100.

6. Prove that any (p, q) – graph G is a bipartite graph if it contains no odd cycles.

7. Write Kruskal's algorithm and Explain it with an example.

8. Define adjacency matrix and find the adjacency matrix of the following graph.



9. Let G be a connected graph. If A and B are two non – empty subsets whose union is $V(G)$ such that $<A>$ and $<B>$ are connected, then prove that $F=[A,B]$ is a cut – set of G.

10. Prove that K_5 is non – planar.

11. For any graph G, prove that $\chi(G)\leq\Delta(G)+1$.

12. If a digraph D is strongly connected then prove that D contains a directed closed walk containing all its vertices.

SECTION B — ($6\times 10=60$ marks)

Answer any SIX questions.

13. (a) Define isomorphism of graphs. Give an example. Also give two non – isomorphic graphs with the same number of vertices and same number of edges.

(b) Define a cubic graph and prove that every cubic graph has an even number of points.

14. Prove that the maximum number of lines among all p point graphs with no triangles is $\lfloor p^2/4\rfloor$.

15. In a connected graph G, prove that there is an eulerian trail iff the number of vertices of odd – degree is either zero or two.

16. If G is a graph with $p\geq 3$ vertices and $\delta\geq p/2$, then prove that G is Hamiltonian.

17. Let G be a (p, q) – graph. Prove that the following statement are quivalent.

(a) G is a tree

(b) Every two points of G are joined by a unique path.

(c) G is connected and $p=q+1$

(d) G is acyclic and $p=q+1$.