22/1

# 2005

## STATISTICS

### Paper 1

*Time : 3 Hours ]* *[ Maximum Marks : 300*

## INSTRUCTIONS

*Candidates should attempt* ***all*** *the questions in Parts A, B & C. However, they have to choose only* ***three*** *questions in Part D. The number of marks carried by each question is indicated at the end of the question.*

*Answers must be written in English.*

*This paper has four parts :*

| | |
|---|---|
| **A** | 20 marks |
| **B** | 100 marks |
| **C** | 90 marks |
| **D** | 90 marks |

*Marks allotted to each question are indicated in each part.*

*Assume suitable data if considered necessary and indicate the same clearly.*

*Notations and symbols used are as usual.*

## PART A

$4 \times 5 = 20$

*Each question carries 5 marks.*

1. (a) Define probability measure. If $A \subset B$ are two events, then prove that $P(B - A) = P(B) - P(A)$.

   (b) Define unbiased estimator. If u and v are unbiased estimators of $\theta$, then prove that $(au + bv)$ is unbiased for $\theta$, if $a + b = 1$.

   (c) Let $H_0 : X \sim f_0(x) = 1,\ 0 < x < 1$, and $H_1 : X \sim f_1(x) = 2x,\ 0 < x < 1$. Based on a single observation obtain most powerful test for $H_0$ against $H_1$, when $\alpha = 0{\cdot}1$.

   (d) If X and Y are independent standard normal variates then prove that $X + Y$ and $X - Y$ are independent normal variates.

## PART B

*10×10=100*

*Each question carries 10 marks.*

1. Let (XY) have the density function $f(x, y) = k$, for $0 < x, y$ and $x + y \leq 2$
   (i) Find k
   (ii) Obtain the marginal density of X

2. A committee of three members has been formed by selecting the members at random, from a group of 3 Statisticians, 2 Economists and 1 Doctor.
   (a) Find the probability that the commitee consists of 1 Statistician, 1 Economist and 1 Doctor.
   (b) Find the probability that committee does not consist of an economist.

3. Define characteristic function. (c.f.). Let $P[X = -1] = P[X = 1] = 1/2$. Find c.f. of X and hence find all moments of X.

4. Let $X_1, X_2, \dots X_n, \dots$ be iid $u(0, \theta)$ random variables and $X_{(n)} = \max\{x_1, \dots x_n\}$. Prove that $X_{(n)}$ converges to $\theta$ in probability.

5. State Factorization Theorem. Let $x_1 \dots x_n$ be iid random variables with p.d.f $\exp\{-(x - \theta)\}$, for $x > \theta$. Obtain non-trivial sufficient statistic for $\theta$.

6. Let $x_1, \dots x_n$ be iid random variables having $u(\theta, \theta + 1)$ distribution. Obtain maximum likelihood estimator for $\theta$. Is it unique ? Justify.

7. Let $x_1, \dots x_n$ be iid random variables having $N(\theta, 1)$ distribution. Develop LRT test for testing $\theta = 1$.

8. Describe sign test for location. State the assumptions.

9. Consider the linear model $Y = X\beta + \varepsilon$. Obtain least square estimator of $\beta$ based on n observations.

10. Define $N(\mu, \Sigma)$ distibution. If X has $N(\mu, \Sigma)$ distribution, obtain the distribution of $AX + b$.

## PART C

$6 \times 15 = 90$

*Each question carries 15 marks.*

1. State and prove Borel Centeth lemma.

2. Let X have d.f.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ (1+x)/5 & \text{if } 0 \le x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$$

   (i) Sketch the graph of F.

   (ii) Find E(X) and V(X).

3. Describe weak and strong laws of large numbers. Let $\{X_n\}$ be a sequence of iid random variables with finite second moment. Examine whether $\{X_n\}$ obeys weak law of large numbers.

4. State and prove CR inequality.

5. Describe monotone likelihood ratio property. Obtain UMP test for testing $\theta = 1$ against the alternative $\theta > 1$, where $\theta$ is the mean of exponential distribution.

6. Define multiple correlation coefficient and obtain an expression for the same.

## PART D

$3 \times 30 = 90$

*Answer any* **three** *of the following questions. Each question carries 30 marks.*

1. Define various modes of convergences of a sequence of random variables. Give an example of a sequence of random variables which converges in probability but not almost surely. State and prove a necessary and sufficient condition for almost sure convergence of a sequence of random variables.

2. Describe SPRT. Develop SPRT to test the parameter of Bernoulli distribution. Obtain approximate expressions for OC and ASN functions.

3. Distinguish between parametric and non-parametric procedures. Describe one and two sample(s) problems. Show that one sample Kolmogerov Smirnov statistic is distribution free.

4. State Gauss-Markoff theorem. Describe the model for two way analysis of variance, carry out the complete analysis and give the analysis of variance table.

5. Define students t-statistic and Hotellings $T^2$-statistic. Indicate various applications of these statistics in statistical inference.