FACULTY RECRUITMENT TEST

CATEGORY-C

Formal School Education/XI, XII

MATHEMATICS

PAPER – A

Time: 60 Minutes. ***Maximum Marks: 40***

Name: .... **Subject:**	**Marks:**	

Instructions:

- Attempt all questions.
- This question paper has two **Parts, I and II.** Each question of **Part I carries 2 marks** and of **Part II carries 5 marks.**
- Calculators and log tables are not permitted

PART–I

1. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$.

2. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

3. Consider a branch of the hyperbola, $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then find the area of the triangle ABC.

4. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

5. Find the value of $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\int_{2}^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$.

6. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1}dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1}dx$. Then, find the value of J – I.

7. If a, b, c are in A.P., prove that cos A cot A/2, cosB. cot B/2, cosCcot C/2 are in A.P.

8. Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

9. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then find the value of $\dfrac{a_{10} - 2a_8}{2a_9}$.

10. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then find the value of $\vec{r} \cdot \vec{b}$.

PART–II

1. If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and AX = U has infinitely many solution.
Prove that BX = V has no unique solution, also prove that if afd $\neq$ 0, then BX = V has no solution.

2. Let g(x) = *l*n *f*(x) where f(x) is a twice differentiable positive function on (0, ∞) such that f(x + 1) = x f(x). Then for N = 1, 2, 3 find $g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$.

3. Using permutation or otherwise, prove that $\dfrac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer.

4. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in$ í, where $f'(x)$ denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then find the value of y(2).

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