MATHEMATICS
(Option II)

1. The function $f(x) = \frac{x - |x|}{x}$ is continuous for :

   (A) all x

   (B) all x except zero

   (C) $x = 0$ only

   (D) None of the above

2. The value of c in $(0, 4)$ such that the function $f(x) = (x - 1)(x - 2)(x - 3)$ satisfies Lagrange's mean value theorem is :

   (A) $1 \pm \frac{2}{\sqrt{3}}$

   (B) $2 \pm \frac{2}{\sqrt{3}}$

   (C) $3 \pm \frac{2}{\sqrt{3}}$

   (D) $2 \pm \sqrt{\frac{2}{3}}$

3. What is the value of $\lim_{x \to \infty} \left(\frac{1}{x}\right)^{1/x}$?

   (A) -1

   (B) 0

   (C) 1

   (D) None of the above

4. What is the $(n + 1)$th derivative of $y = x^n \log x$?

   (A) $\frac{n!}{x}$

   (B) $\frac{(n+1)!}{x}$

   (C) $\frac{n!}{x^2}$

   (D) $\frac{(n+1)!}{x^2}$

5. What is the radius of curvature at any point of the cardiod $2ap^2 = r^3$?

(A) $\frac{3ap}{r^2}$

(B) $\frac{4ar}{3p}$

(C) $\frac{4pr}{3a}$

(D) $\frac{4ap}{3r}$

6. What is the number of asymptotes of the curve $y^2 = 8x$?

(A) zero

(B) one

(C) two

(D) three

7. What is the modulus of the complex number $1 - \cos 2\alpha + i \sin 2\alpha$?

(A) $2\cos\alpha$

(B) $\cos\frac{\alpha}{2}$

(C) $2\sin\alpha$

(D) $\sin\frac{\alpha}{2}$

8. What is the continued product of the four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4}$?

(A) 0

(B) 1

(C) 2

(D) None of the above

9. What is period of $\tanh x$?

(A) πi

(B) $2\pi i$

(C) $\frac{\pi i}{2}$

(D) None of the above

10. What is the sum of the series ?

$$\sin^2\alpha + \sin^2\alpha \sin 2\alpha + \sin^3\alpha \sin 3\alpha + \ldots$$

to infinity ?

(A) $\frac{\sin^2\alpha}{1 + \sin 2\alpha + \sin^2\alpha}$

(B) $\frac{\sin^2\alpha}{1 - \sin 2\alpha + \sin^2\alpha}$

(C) $\frac{\sin^2\alpha}{1 - \sin 2\alpha - \sin^2\alpha}$

(D) None of the above

11. If a tangent to the parabola $y^2 = 12x$ makes an angle of $60°$ with the axis, then the point of contact is :

(A) $\left(1, 2\sqrt{3}\right)$

(B) $\left(2, 2\sqrt{3}\right)$

(C) $\left(1, \sqrt{3}\right)$

(D) None of the above

12. What is the equation of the director circle to the ellipse $9x^2 + 4y^2 = 1$?

(A) $36(x^2 - y^2) = 13$

(B) $36(x^2 + y^2) = 13$

(C) $13(x^2 + y^2) = 36$

(D) None of the above

13. What is the sum of the eccentric angles of the feet of the normals drawn from any point to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$?

(A) $2n\pi$

(B) $4n\pi$

(C) $(2n + 1)\pi$

(D) None of the above

14. The difference of the eccentric angles of the extremities of two conjugate diameters to the ellipse is :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

15. The equation of the normal to the hyperbola $5x^2 - 4y^2 = 20$ at $(2, \sqrt{3})$ is :

(A) $2\sqrt{3}x + 5y + 9\sqrt{3} = 0$

(B) $2\sqrt{3}x - 5y + 9\sqrt{3} = 0$

(C) $2\sqrt{3}x + 5y - 9\sqrt{3} = 0$

(D) None of the above

16. What is the latus rectum of the parabola $x^2 + 2xy + y^2 - 2x - 1 = 0$?

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{5}}$

(C) $\frac{2}{\sqrt{3}}$

(D) None of the above

17. If a straight line makes an angle of $\frac{\pi}{4}$ with each of x-axis and y-axis, then the angle which it makes with z-axis is :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) None of the above

18. What is the equation of the plane through the point (2, 3, 4) and parallel to the plane $5x - 6y + 7z = 3$?

(A) $5x + 6y + 7z = 20$

(B) $5x + 6y - 7z = 20$

(C) $5x - 6y - 7z = 20$

(D) None of the above

19. When two straight lines are said to be skew ?

(A) they are parallel

(B) they are intersecting

(C) they are neither parallel nor intersecting

(D) None of the above

20. What is the equation of the sphere which passes through the origin and makes equal intercepts of unit length on the axes ?

(A) $x^2 + y^2 + z^2 = x + y + z$

(B) $x^2 - y^2 - z^2 = x + y + z$

(C) $x^2 - y^2 + z^2 = x + y + z$

(D) $x^2 + y^2 - z^2 = x + y + z$

21. What is the equation of the plane which cuts the sphere $x^2 + y^2 + z^2 = a^2$ in a circle with centre (1, 2, 3) ?

(A) $x - 2y + 3z = 14$

(B) $x + 2y + 3z = 14$

(C) $x - 2y - 3z = 14$

(D) None of the above

22. If two spheres of radii 3 and 4 cut orthogonally, then the radius of the common circle is :

(A) $\frac{5}{12}$

(B) $\frac{12}{5}$

(C) $\frac{12}{\sqrt{5}}$

(D) $\frac{\sqrt{5}}{12}$

23. The equation of the cone whose vertex is (0, 0, 0) and whose base is the curve $x^2 + y^2 = 4, z = 2$ is :

(A) $x^2 + y^2 = z^2$

(B) $x^2 - y^2 = z^2$

(C) $x^2 + z^2 = y^2$

(D) $y^2 + z^2 = 2x^2$

24. What is the condition for the line $\frac{x-2}{l} = \frac{y-1}{m} = \frac{z-3}{n}$ to touch the ellipsoid $3x^2 + 8y^2 + z^2 = c^2$?

(A) $6l + 8m + 3n = 0$

(B) $6l - 8m + 3n = 0$

(C) $6l + 8m - 3n = 0$

(D) None of the above

25. What is the number of normals that can be drawn from any point to the conicoid $ax^2 + by^2 + cz^2 = 1$?

(A) 3

(B) 4

(C) 5

(D) 6

26. $\int \frac{e^x}{x}(1 + x \log x)\, dx$ equals :

(A) $e^x \log x + c$

(B) $\log \log x + c$

(C) $e^x + \log x + c$

(D) $e^x - \log x + c$

27. $\int \frac{dx}{(x+2)\sqrt{x-1}}$ equals :

(A) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x-1}{3}\right) + c$

(B) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\sqrt{\frac{x-1}{3}}\right) + c$

(C) $\frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{x-1}{3}\right) + c$

(D) $\frac{\sqrt{3}}{2} \tan^{-1}\left(\sqrt{\frac{x-1}{3}}\right) + c$

28. If $f_n = \int_0^{\pi/4} \tan^n x\, dx$, then $f_n + f_{n-2}$ equals :

(A) $\frac{1}{n-3}$

(B) $\frac{1}{n-2}$

(C) $\frac{1}{n-1}$

(D) $\frac{1}{n}$

29. $\lim_{n\to\infty} \sum \left(\frac{n!}{n^n}\right)^{1/n}$ equals :

(A) e

(B) $\frac{1}{e}$

(C) e^2

(D) $\frac{1}{e^2}$

30. The area of a loop of the curve $3ay^2 = x(x-a)^2$ is :

(A) $\frac{8a^2}{15\sqrt{3}}$

(B) $\frac{8a^2}{15\sqrt{5}}$

(C) $\frac{8a^2}{5\sqrt{3}}$

(D) $\frac{8a^2}{3\sqrt{5}}$

31. A vector f is said to be irrotational if :

(A) div $f = 0$

(B) curl $f = 0$

(C) grad $f = 0$

(D) None of the above

32. What is the integrating factor of the differential equation :

$$x \cos x\left(\frac{dy}{dx}\right) + y(x \sin x + \cos x) = 1 \text{ ?}$$

(A) $x \sec x$

(B) $x \tan x$

(C) $x \sin x$

(D) $x \cos x$

33. What is the particular integral of the differential equation :

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x \text{ ?}$$

(A) xe^x

(B) x^2e^x

(C) $\frac{1}{x}e^x$

(D) $\frac{1}{x^2}e^x$

34. What is the singular solution of the differential equation $p^2 + y^2 = 1$?

(A) $y = 1, y = -1$

(B) $y = 1, y = 2$

(C) $y = 0, y = -1$

(D) None of the above

35. What is the solution of the differential equation $y = 2px + y^2p^3$?

(A) $y^2 = cx^2 + \frac{1}{8}c^3$

(B) $y = cx + \frac{1}{8}c^3$

(C) $y^2 = cx + \frac{1}{8}c^3$

(D) $y = cx^2 + \frac{1}{8}c^3$

36. If $A = \begin{bmatrix} 1 & k & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then the value of k so that $A^2 - 4A - 5I = 0$ is :

(A) 1

(B) 2

(C) zero

(D) –1

37. If A = B + iC, where A is Hermitian and B is real symmetric, then what is C ?

(A) real, symmetric

(B) real, skew-symmetric

(C) imaginary, symmetric

(D) None of the above

38. If $A = \begin{bmatrix} 0 & 2+3i \\ -2-3i & 0 \end{bmatrix}$, then A is :

(A) skew Hermitian

(B) symmetric

(C) skew-symmetric

(D) None of the above

39. The rank of the matrix of order 4×5 whose every element is unity is :

(A) 1

(B) 2

(C) 3

(D) 4

40. If A and B are two square matrices of order n (each), then Rank (AB) = k, where :

(A) $k \geq$ Rank A + Rank B + n

(B) $k \geq$ Rank + Rank B − n

(C) $k \geq$ Rank − Rank B − n

(D) None of the above

41. What is the sum of the characteristic roots of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$?

(A) 7

(B) 6

(C) 5

(D) 4

42. If A is a square matrix of order 7 with rank 5, then the number of linearly independent solutions of the equation $AX = 0$ is :

(A) 2

(B) 5

(C) 7

(D) 12

43. If the sum of any two roots of the equation $x^3 - px^2 + qx - r = 0$ is zero, then :

(A) $pr = q$

(B) $pq = r$

(C) $qr = p$

(D) $pqr = 1$

44. What are the roots of the reciprocal equation ?

$$x^4 - 10x^3 + 10x^2 - 10x + 1 = 0$$

(A) $3 \pm 2\sqrt{2}, 2 \pm \sqrt{5}$

(B) $3 \pm 2\sqrt{2}, 2 \pm \sqrt{3}$

(C) $3 \pm \sqrt{2}, 2 \pm \sqrt{3}$

(D) None of the above

45. If α, β and γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then the value of $\sum \frac{1}{\alpha^2\beta^2}$ is :

(A) $\frac{p^2 - 2q}{r^2}$

(B) $\frac{p^2 + 2q}{r^2}$

(C) $\frac{q^2 - 2p}{r^2}$

(D) None of the above

46. To remove the second term of the cubic $x^3 - 12x^2 - 6x - 10 = 0$, the roots are to be diminished by :

(A) -2

(B) 2

(C) -4

(D) 4

47. What is the supremum of the set :

$$\left\{-1, \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{4}, ...\right\} ?$$

(A) -1

(B) 0

(C) 1

(D) None of the above

48. Which of the following is *not* true for the real numbers x and y ?

(A) $|x + y| \leq |x| + |y|$

(B) $|x - y| \geq ||x| - |y||$

(C) $|x - y| \leq |x| - |y|$

(D) None of the above

49. Which of the following is both closed and open ?

(A) the set of real numbers R

(B) the set of rational numbers Q

(C) $\{x : 0 < x < 1, x \in R\}$

(D) None of the above

50. If $\{a_n\}$ converges and $\{b_n\}$ diverges, then $\lim\left(\frac{a_n}{b_n}\right)$ equals :

(A) ∞

(B) 1

(C) 0

(D) -1

51. What is the value of $\lim_{x \to \infty} \log|x|$?

(A) ∞

(B) 0

(C) 1

(D) $-\infty$

52. The function $f(x) = |x| + |x - 1|$ is derivable at :

(A) 0

(B) 1

(C) 2

(D) None of the above

53. If $f(x)$ is bounded and integrable on $[a, b]$, then which of the following is *not* true ?

(A) $|f(x)|$ is bounded on $[a, b]$

(B) $|f(x)|$ is integrable on $[a, b]$

(C) $|f(x)|^2$ is integrable on $[a, b]$

(D) None of the above

54. If $f(x, y) = \sqrt{|xy|}$, then $f_x(0, 0)$ equals :

(A) -1

(B) 0

(C) 1

(D) 2

55. What is the inverse of an element a in the set R of real numbers with the binary operation * defined by $a * b = a + b + ab$?

(A) $\frac{a}{a+1}$

(B) $\frac{-a}{a+1}$

(C) $\frac{a}{a-1}$

(D) None of the above

56. What is the order of the alternating group A_5 of all even permutations of degree 5 ?

(A) 5

(B) 15

(C) 30

(D) 60

57. What is the number of subgroups of order 3 in a group of order 6 ?

(A) at most one

(B) at least two

(C) four

(D) five

58. Which of the following groups is non-abelian ?

(A) group of order 3

(B) group of order 4

(C) group of order 5

(D) none of the above

59. Which of the following *cannot* be the characteristic of an integral domain ?

(A) 0

(B) 2

(C) 3

(D) 4

60. If W_1 and W_2 are subspaces of a vector space V(F), then $L(W_1 \cup W_2)$ equals :

(A) $W_1 + W_2$

(B) $W_1 - W_2$

(C) W_1

(D) W_2