(For Mathematics Student)

1. For what value of k, the inifinitesimals $\tan^{-1}3x$ and $k \sin x$, as $x \to 0$, are equivalent ?

   (A) Zero

   (B) 1

   (C) 3

   (D) 6

2. Let

$$f(x) = \begin{cases} x, \text{ when } x \text{ is rational} \\ -x, \text{ when } x \text{ is irrational} \end{cases}$$

and

$$g(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

then which of the following is *true* ?

   (A) $f(x)$ and $g(x)$ are both continuous at $x = 0$

   (B) $f(x)$ is not continuous at $x = 0$, but $g(x)$ is continuous at $x = 0$

   (C) $f(x)$ and $g(x)$ are both discontinuous at $x = 0$

   (D) None of the above

3. Assuming the validity of the expansion

$$\log (1 + e^x) = \log 2 + \alpha x + \beta x^2 + \ldots\ldots$$

what is the value of the pair (α, β) ?

(A) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

(B) $\left(\frac{1}{2}, 1\right)$

(C) $\left(\frac{1}{4}, -\frac{1}{8}\right)$

(D) $\left(\frac{1}{2}, \frac{1}{8}\right)$

4. Let

$$u = \tan^{-1}\left\{\frac{x^2 + y^2}{x + y}\right\}$$

then which of the following is *true* ?

(A) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - \sin 2u = 0$

(B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - \frac{1}{2}\sin 2u = 0$

(C) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} + \frac{1}{2}\sin 2u = 0$

(D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \sin u = 0$

5. For the curve $f(r, \theta) = 0$, the derivative of arc s with respect to θ is given by $\left(\frac{ds}{d\theta}\right)^2 =$

(A) $1 + r^2 \frac{d\theta}{dr}$

(B) $r^2 + \frac{d\theta}{dr}$

(C) $r^2 + \left(\frac{dr}{d\theta}\right)^2$

(D) $1 + r^2 \left(\frac{d\theta}{dr}\right)^2$

6. How many asymptote does the curve $y^2 = ax$ have ?

(A) One

(B) Two

(C) Infinite

(D) None

7. For any complex number z, which of the following is always *true* ?

(A) $\text{amp}(z) + \text{amp}(\bar{z}) + n\pi = 0$

(B) $\text{amp}(z) + \text{amp}(\bar{z}) - 2n\pi = 0$

(C) $\text{amp}(z) - \text{amp}(\bar{z}) + n\pi = 0, \quad n \in I$

(D) None of the above

8. If $z = \cos\theta + i \sin\theta$, then $(1 + z)^n + (1 + \bar{z})^n$ equals :

(A) $2^{n+1} \cos^n \frac{\theta}{2} \cos \frac{n\theta}{2}$

(B) $2^{n+1} \sin^n \frac{\theta}{2} \cos \frac{n\theta}{2}$

(C) $2^{n+1} \cos^n \frac{\theta}{2} \sin \frac{n\theta}{2}$

(D) $i2^{n+1} \sin^n \frac{\theta}{2} \sin \frac{n\theta}{2}$

9. If

$$\frac{(1 + i)^{p+iq}}{(1 - i)^{p-iq}} = a + ib$$

then $\tan^{-1}\left(\frac{b}{a}\right) =$

(A) $q\frac{\pi}{2} + p \log 2$

(B) $q\frac{\pi}{2} - p \log 2$

(C) $p\frac{\pi}{2} + q \log 2$

(D) None of the above

10. If $\tan(x + iy) = \alpha + i\beta$, then $\tan 2x$ equals :

(A) $\dfrac{2\alpha}{(1 + \alpha^2 + \beta^2)}$

(B) $\dfrac{2\beta i}{(1 - \alpha^2 - \beta^2)}$

(C) $\dfrac{2\alpha i}{(\alpha^2 + \beta^2 + 1)}$

(D) $\dfrac{2\alpha}{(1 - \alpha^2 - \beta^2)}$

11. Which of the following is the locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$ which are at right angles to one another ?

(A) $y + a = 0$

(B) $x + a = 0$

(C) $x + y = a$

(D) $x - y = a$

12. Which of the following is the equation of the Director Circle of the ellipse :

$$\frac{x^2}{4} + \frac{y^2}{25} = 1 \text{ ?}$$

(A) $x^2 + y^2 = \sqrt{29}$

(B) $x^2 + y^2 = 7$

(C) $x^2 + y^2 = 29$

(D) $x^2 + y^2 = 21$

13. Which of the following points is the centre of the conic ?

$$x^2 - 5xy + y^2 + 8x - 20y + 15 = 0 \text{ ?}$$

(A) (4, 0)

(B) (0, –4)

(C) (–4, 0)

(D) None of the above

14. What is the eccentricity of an ellipse whose latus rectum is equal to one half of its minor axis ?

(A) $\frac{\sqrt{2}}{\sqrt{3}}$

(B) $\frac{\sqrt{3}}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{3}$

(D) $\frac{\sqrt{3}}{2}$

15. If e_1, e_2 are the eccentricities of hyperbola and of conjugate hyperbola, then which of the following is *true* ?

(A) $e_1^2 + e_2^2 = 1$

(B) $e_1^2 + e_2^2 = e_1^2 e_2^2$

(C) $e_1^2 = e_2^2 - e_1^2 e_2^2$

(D) None of the above

16. Which of the following is a condition for diameters $y = \alpha x$ and $y = \beta x$ of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

to be conjugate ?

(A) $a^2\alpha = b^2\beta$

(B) $\alpha b^2 = \beta a^2$

(C) $a^2\alpha\beta = b^2$

(D) $b^2\alpha\beta = a^2$

17. Which of the following is the equation of right circular cone whose vertex is origin, axis of cone as z-axis and semi-vertical angle θ ?

(A) $x^2 + y^2 + z^2 = z^2 \sec^2 \theta$

(B) $x^2 + y^2 = 2z^2 \tan^2 \theta$

(C) $2(x^2 + y^2) = z^2 \sec^2 \theta$

(D) None of the above

18. Which of the following is the necessary and sufficient condition for cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2hxy + 2gzx = 0$$

to have three mutually perpendicular generators ?

(A) $a + b + c = 1$

(B) $a^{-1} + b^{-1} + c^{-1} = 0$

(C) $a + b + c = 0$

(D) None of the above

19. Which of the following is the equation of cylinder whose generators are parallel to the z-axis and intersect the curve

$$ax^2 + by^2 = 2z,\ lx + my + nz = p\ ?$$

(A) $n(lx + my) + 2(ax^2 + by^2) = 2p$

(B) $n(my^2 + nz^2) + 2z = p$

(C) $nz + 2(ax^2 + by^2) = p$

(D) $n(ax^2 + by^2) + 2(lx + my) = 2p$

20. How many normals can be drawn to the central conicoid $ax^2 + by^2 + cz^2 = 1$ from any given point ?

(A) 3

(B) 6

(C) 2

(D) infinite

21. What is degree (lowest) of an equation with real and rational coefficients whose one root is $\sqrt{2} + \sqrt{3} + i$?

(A) 2

(B) 4

(C) 8

(D) 16

22. The roots of equation

$$2x^3 - 15x^2 + 37x - 30 = 0$$

are in Arithmetic progression, what are its roots ?

(A) $\frac{5}{2}, 3, 2$

(B) 3, 2, 1

(C) $\frac{5}{2}, 3, \frac{7}{2}$

(D) None of the above

23. Which one of the following is the equation whose roots are the square of the roots of the equation :

$$x^3 - 6x^2 + 11x - 6 = 0 \text{ ?}$$

(A) $x^3 - 28x^2 + 245x - 36 = 0$

(B) $x^3 - 14x^2 + 49y - 36 = 0$

(C) $x^3 - 14x^2 + 28x^2 - 36 = 0$

(D) None of the above

24. If α, β, γ are the roots of the equation

$$x^3 - x^2 + 3x - 2 = 0,$$

then the value of $\alpha^3 + \beta^3 + \gamma^3 =$

(A) 2

(B) –5

(C) 6

(D) None of the above

25. If $G^2 + 4H^2 < 0$, then roots of the cubic

$$x^3 + 3Hx + G = 0$$

are :

(A) Real and unequal

(B) Real and equal

(C) One real and two imaginary

(D) None of the above

26. Let $f : X \to Y$ and $g : Y \to Z$ be two one-one and onto mappings, then which of the following is true ?

(A) $(g \circ f)^{-1} = g^{-1} \circ f^{-1}$

(B) $g \circ f$ is one-one and onto

(C) $f \circ g$ is one-one and onto

(D) None of the above

27. Consider the group G of all rational numbers other than 1 with operation * defined by

$$a * b = a + b - ab,$$

then the inverse of $a \in G - \{1\}$ is :

(A) 0

(B) $a - 1$

(C) $\frac{a}{a-1}$

(D) None of the above

28. Which of the following statements is *not* always correct ?

(A) Every subgroup of an abelian group is normal

(B) The subgroup N of a group G is normal iff $gN = Ng$, $g \in G$

(C) G/N is abelian group

(D) None of the above

29. Every n cycle is of order :

(A) n

(B) $n(n + 1)/2$

(C) $n!$

(D) None of the above

30. Which of the following statements is always *true* ?

(A) Every integral domain is field

(B) Every commutative ring is an integral domain

(C) The ring of integers modulo n, where n is any positive integer, is a field

(D) None of the above

31. Which of the following is *not* equal to $\int \frac{1}{\sqrt{a^2 + x^2}} dx$?

(A) $\sinh^{-1}\left(\frac{x}{a}\right)$

(B) $\log\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right)$

(C) $\coth^{-1}\left(\frac{x}{a}\right)$

(D) None of the above

32. The value of the integral $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$ is :

(A) $\frac{35}{256}$

(B) $\frac{16}{35}$

(C) $\frac{15}{48}$

(D) None of the above

33. Given that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2}$, the value of $\int_0^{\pi} x \log \sin x \, dx$ is :

(A) $\frac{\pi}{2} \log \frac{1}{2}$

(B) $\frac{\pi^2}{2} \log \frac{1}{2}$

(C) $-\frac{\pi}{2} \log 2$

(D) None of the above

34. The area of the cardioid $r = (1 - \cos \theta)$ is :

(A) $\frac{3\pi}{2}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

35. The length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ is :

(A) $\log (1 + \sqrt{3})$

(B) $\log (2 + \sqrt{3})$

(C) $\log \sqrt{3}$

(D) $\log 2$

36. The value of $\iint x \, dxdy$ over the part of the region bounded by the parabola $y = 2x - x^2$ and the line $y = x$ is :

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{12}$

(D) None of the above

37. The solution of exact differential equation

$(5x^4 + 6x^2y^2 - 8xy^3)\, dx + (4x^3y - 12x^2y^2 - 5y^4)dy = 0$ is :

(A) $x^5 + 3x^2y - 4x^2y^3 - y^5 + c = 0$

(B) $x^5 + 3x^2y - 4x^2y^2 + c = 0$

(C) $x^4y - 4x^3y^2 - 5xy^4 + c = 0$

(D) $x^4y + 3x^2y^3 - 4xy^4 + c = 0$

38. What is the value of

$$\frac{1}{D^2 + a^2}(\cos ax - \sin ax) \ ?$$

(A) 0

(B) $\frac{x^2}{2!}(\sin ax - \cos ax)$

(C) $\frac{x^2}{2a}(\sin ax + \cos ax)$

(D) $\frac{x}{2a}(\sin ax + \cos ax)$

39. The solution of differential $\frac{d^2y}{dx^2} + \frac{4a^2}{y^2} = 0$, given that $y = 2a$ and $\frac{dy}{dx} = 1$ when $x = a$ is :

(A) $x^2 = 4ay$

(B) $y^2 = 4ax$

(C) $y^2 = 2a(x + x^2)$

(D) $x^2 = 2a(y + y^2)$

40. If $P_n(x)$ denotes the Legendre polynomial of order n, then for n odd, $P_n(-1)/P_n(1)$ equals :

(A) 0

(B) 1

(C) $n(n + 1)/2$

(D) None of the above

41. Let

$$T = \{x \in R : 0 \leq x \leq 1\}, \text{ then}$$

which of the following is *true* ?

(A) Supremum of T belongs to T but infimum does not

(B) Infimum of T belongs to T but supremum does not

(C) Both supremum and infimum of T belongs to T

(D) Neither supremum nor infimum of T belongs to T

42. Which of the following is always *true* ?

(A) Every bounded sequence has a limit point

(B) Every bounded sequence is convergent

(C) Every monotonic increasing sequence is convergent

(D) None of the above

43. For what value of x, the series

$$\sum_{n=1}^{\infty} \frac{1.3.5....(4n-3)}{2.4.6....(4n-2)} \frac{x^{2n}}{4n}$$

converges :

(A) $1 < x < 2$

(B) $-2 < x < 1$

(C) $x \geq 1$

(D) $|x| \leq 1$

44. Let

$$f(x) = x \sin \frac{1}{x}, x \neq 0$$

$$= 0 \quad , x = 0$$

and $g(x) = x$,

then at $x = 0$, which of the following is *true* ?

(A) $f(x)\ g(x)$ is not derivable but $f(x)$ is derivable

(B) $f(x)\ g(x)$ is derivable but $f(x)$ is not derivable

(C) both $f(x)\ g(x)$ and $f(x)$ is derivable

(D) neither $f(x)\ g(x)$ nor $f(x)$ is derivable

45. If a function $f(x)$ is (*i*) continuous on $[a, b]$ (*ii*) derivable on (a, b) and (*iii*) $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is :

(A) Constant on $[a, b]$

(B) Strictly increasing on $[a, b]$

(C) Strictly decreasing on $[a, b]$

(D) None of the above

46. Let

$f(x) = 0$, when x is rational

$= 1$, when x is irrational,

then lower Riemann integral $\int_{-0}^{1} f(x)dx$ is :

(A) 0

(B) 1

(C) –1

(D) None of the above

47. For bounded Riemann integrable function, let

$$\int_a^b f(x)dx = \mathrm{L} \text{ and } \int_a^b |f(x)|dx = \mathrm{M},$$

then which of the following is *true* ?

(A) $\mathrm{L} > \mathrm{M}$

(B) $\mathrm{L} < -\mathrm{M}$

(C) $-\mathrm{L} < \mathrm{M} < \mathrm{L}$

(D) $-\mathrm{M} < \mathrm{L} < \mathrm{M}$

48. Let

$$f(x, y) = \frac{x^2y^2}{x^2y^2 + (x - y)^2}, (x, y) \neq (0, 0)$$

$$= 0 \qquad , (x, y) = (0, 0)$$

and

$$g(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}, (x, y) \neq (0, 0)$$

$$= 0 \qquad , (x, y) = (0, 0),$$

then at the point (0, 0) :

(A) $f(x, y)$ is continuous but $g(x, y)$ is *not* continuous

(B) $f(x, y)$ is not continuous but $g(x, y)$ is continuous

(C) Both $f(x, y)$ and $g(x, y)$ are continuous

(D) Neither $f(x, y)$ nor $g(x, y)$ is continuous

49. Let (a, b) be a stationary point of a function $f(x, y)$. Let, at the point (a, b)

$$f_{xx} \cdot f_{yy} - f_{xy}^2 = \alpha$$

then $f(x, y)$ has maxima at (a, b) if :

(A) $f_{xx}(a, b) < 0$ and $\alpha < 0$

(B) $f_{xx}(a, b) > 0$ and $\alpha > 0$

(C) $f_{xx}(a, b) < 0$ and $\alpha > 0$

(D) $f_{xx}(a, b) > 0$ and $\alpha < 0$

50. Given the Beta function $\beta(p, q)$, $p > 0$, $q > 0$,

$$\beta(p + 1, q) + \beta(p, q + 1) =$$

(A) $\beta(p, q)$

(B) $\beta(p + 1, q + 1)$

(C) $(p + q)\ \beta(p, q)$

(D) None of the above

51. If A and B are Hermitian matrices of same order such that $A^2 + B^2 = 0$, then which of following is *true* ?

(A) $A \neq 0, B \neq 0$

(B) $A \neq 0, B = 0$

(C) $A = 0, B \neq 0$

(D) $A = 0, B = 0$

52. If A, C are non-singular matrices such that $\begin{bmatrix} A^{-1} & O \\ T & C^{-1} \end{bmatrix}$ is the inverse of $\begin{bmatrix} A & O \\ B & C \end{bmatrix}$, then the matrix T equals :

(A) $C^{-1}B^{-1}A^{-1}$

(B) $-C^{-1}BA^{-1}$

(C) $CB^{-1}A^{-1}$

(D) None of the above

53. The characteristic roots of the matrix

$$\begin{bmatrix} 2 & 4 & 7 \\ 0 & -1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

are :

(A) 2, 4, 7

(B) 7, −1, 3

(C) 2, −1, 3

(D) 0, 0, 3

54. If A is the non-singular matrix of order n and B is a matrix of rank r such that AB is defined, what is the rank of AB ?

(A) r

(B) n

(C) nr

(D) $n - r$

55. The number of linearly independent solutions of the linear system $XA = 0$, where A is a 3×4 matrix of rank 2, is :

(A) None

(B) One

(C) Two

(D) Three

56. The system of non-homogeneous equations $AX = B$ has a unique solution if :

(A) Rank (A) = Rank ([A : B])

(B) Rank (A) $\neq$ Rank ([A : B]) and $|A| \neq 0$

(C) Rank (A) = Rank ([A : B]) and $|A| = 0$

(D) Rank (A) = Rank ([A : B]) and $|A| \neq 0$

57. A non-empty subset W of a vector space V_f is a subspace of V if :

(A) $\alpha x \in W, \alpha \in F, x \in W$

(B) $x + y \in W, x, y \in W$

(C) $\alpha x + \beta y \in W, \alpha, \beta \in F, x, y \in W$

(D) None of the above

58. Which of the following is *not* a subspace of $M_n(F)$, the vector space of $n \times n$ matrices over a field F ?

(A) All upper triangular matrices of order n

(B) All non-singular matrices of order n

(C) All symmetric matrices of order n

(D) All matrices of order n, the sum of whose diagonal entries is zero

59. Let V_F be a vector space of dimension n, then which of the following is *true* ?

(A) Any $(n + 1)$ members of V_F are linearly dependent

(B) Any $(n + 1)$ members of V_F are linearly independent

(C) If $z_1, z_2, \ldots\ldots, z_n$ span V, then they are linearly dependent

(D) None of the above

60. If U and W are subspaces of a finite dimensional vector space V_F such that dim U = p, dim V = q and dim (U + W) = r, then dim $(U \cap W)$ =

(A) $p + r - q$

(B) $q + r - p$

(C) $p + q - r$

(D) None of the above