

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

17th May 2010

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1) (a)** A construction company is considering building a commercial complex. The company expects revenues to be generated from the project in years 3, 4 and 5. If the project is a success, then the revenues will be Rs.60 crores per annum. Otherwise the revenues will be Rs.30 crores per annum. The probability of success is $\frac{4}{5}$.

The initial cost of the project is Rs.72 crores. If the weather remains fine, it will be completed within one year. Bad weather will cause construction to drag into year 2 with additional cost of Rs. 36 crores. The probability of bad weather is $\frac{1}{3}$. The company also faces risk of possible strike action by the workforce. If a strike occurs, it will increase the cost in each year of construction by 20%. The probability of strike is $\frac{1}{2}$ if the weather is good but only $\frac{1}{4}$ if the weather is bad.

Assume all cash-flows occur at the start of each year. Calculate the expected NPV of the project at a risk discount rate of 10% p.a. (9)

- (b)** What is the probability of the best scenario, i.e., minimum cost and maximum revenue? (1)
[10]

- Q. 2) i)** A 1000 par value 20-year bond with annual coupons and redeemable at maturity at 1050 is purchased for P to yield an annual effective rate of 8.25%. The first coupon is 75. Each subsequent is 3% greater than the preceding coupon.

Determine P. (3)

- ii)** A Company has issued a security, redeemable at par at the end of n years, which promises to pay annual coupons at a rate of 20% at the end of each year during its term. An investor not subject to tax has calculated that, if the possibility of default is ignored, purchasing Rs.100 of the stock at a price of Rs.195.50 will give an effective annual yield of 8.50%.

a) Find n.

The investor subsequently decides that there could be default in payment and that the probability of payment being made at the end of the t^{th} year is 0.94017^t , (where $1 \leq t \leq n$) independently of the other payments. (7)

- b) Calculate the present value of the income stream produced by a holding of Rs.100 of the stock, assuming an interest rate of 10% per annum. Ignore the possibility of partial payments. [10]

- Q. 3)** There are two types of annuities being offered by a company with the following features :

Annuity 1: A 10-year decreasing annuity, with annual payments of 10, 9, 8, . . . ,1, the first payment being made at end of first year.

Annuity 2: A yearly perpetuity, which pays 1 at end of year 1, 2 at end of year 2, 3 at end of year 3, . . . , and 11 at end of year 11 . After year 11, the payments remain constant at 11.

At an annual effective interest rate of i , the present value of Annuity 2 is twice the present value of Annuity 1. Calculate the value of Annuity 1. [10]

- Q. 4)** i) A loan of Rs. Y was taken for a period of n years at 7.75% p.a. convertible monthly. The loan was to be repaid through equated monthly installments (EMIs) of Rs.14612.884. The capital content of the 16th installment was Rs.3433.056.
(a) Find n and Y. (4)
- (b) Immediately after payment of 36 monthly installments, the borrower makes a part-repayment of the loan by paying an amount of Rs.300,000/-. Hence, he gets a rebate on interest rate and now is charged only 7.5% p.a. effective. Calculate the revised EMI if the loan o/s is to be repaid over the remaining period. (5)
- ii) I have borrowed Rs. X for 10 years at an annual effective rate of 6.5% . If I pay the principal and accumulated interest in one lump sum at the end of 10 years, I would pay 486.091 more in interest than if I had repay the loan with 10 level payments at the end of each year. Calculate X. (5)
- [14]**
- Q. 5)** i) Define force of interest. (1)
- ii) If $i^{(2)}=0.0775$
Calculate $i, \delta, d^{(12)}, i^{(1/2)}$. (4)
- iii) If $\sum_1^{30} \log_{10}(1+(1/\ddot{a}_{t|}) - d) = 1.169228$ find i . (5)
- iv) Derive, from first principles, the formula for $(\ddot{Ia})_{n|}$ for $i>0$ (3)
- v) Ria deposits a lump-sum in an account today in order to fund her retirement. She would like to receive payments of 50000 per year, in real terms, at the end of each year for a total of 12 years, with the first payment occurring seven years from now. The inflation rate will be 0.0% for the next 6 years and 1.2% p.a. thereafter. The annual effective rate of return is 6.3%.
Calculate the lump-sum that she invests today. (3)
- iv) A fund has an amount of Rs. 3,00,000 presently. Find the probability that the amount will become at least Rs. 5,00,000 after 10 years assuming that the annual growth rates vary independently from year to year and have log-normal distribution with parameters $\mu = 0.060$ and $\sigma^2 = 0.15^2$. (3)
- [19]**
- Q. 6)** An investor has following 3 options:
- Option 1:** To invest in a bank and obtain interest during the next 10 years at 8% p.a. compounding annually
- Option 2:** To invest in a fixed interest security for a period of 10 years, redeemable at the rate of 110% with 6% annual coupons and annual term structure of interest rates 7%, 7%, 8%, 8%, 9%, 9%, 10%, 10%..... and so on.

Option 3: To invest in a fixed interest security at a price of Rs. 90 per 100 nominal for a period of 5 years, redeemable at the rate of 105% and annual coupons of 5% p.a. The redemption proceeds at time 5 are further reinvested for 5 years and 5 year forward rate at time 5 is 10% p.a.

- i) Which of the above 3 options should he select in order to maximize his IRR? Ignore taxation. (10)
- ii) What are the other criteria that can be used to decide between alternative investment projects. (1)
- [11]**

Q. 7) Given below is the data relating to the assets of an investment fund:
(Figures are in crores)

| <u>Date</u> | <u>Fund Value</u> | <u>Net Cashflows</u> |
|-------------|-------------------|----------------------|
| 01.01.2008 | 8.6 | |
| 31.03.2008 | 8.4 | |
| 01.04.2008 | | + 0.6 |
| 31.12.2008 | | - 1.0 |
| 01.01.2009 | 10.5 | |
| 01.07.2009 | 11.8 | |
| 31.12.2009 | 12.0095 | |

For the period from 1.1.2008 to 31.12.2009 calculate

- a) the money weighted rate of return (2)
- b) the time weighted rate of return (3)
- c) the linked annual rate of return (using year-long linking periods)
Express your answers as annual rates. (3)
- d) Without any further calculations, explain giving reason how
- (i) the money weighted rate of return would change if the positive cashflow of 0.6 occurs on 1.4.2009 instead of 1.4.2008. (1)
- (ii) the time weighted rate of return would change if the net cashflow on 31.12.2008 is (+1.0) instead of (-1.0). (1)

Assume that there is no change in the Fund Values given above.

[10]

- Q. 8)** i) Describe the characteristics of Corporate Bonds. (2)
- ii) A company issued bonds with a coupon of 10% p.a. payable half-yearly in arrears which can be redeemed at par at the option of the investor at any time between 15 to 20 years from the date of issue.

Calculate the maximum price that Investor X, who is subject to income tax of 20% and capital gain tax of 10% and wishes to obtain net yield of 8% p.a., should pay for Rs. 100 nominal of this bond. (5)

- iii)** After 2 years and 2 months, Investor X wants to sell the bonds. Investor Y is prepared to purchase the bonds provided he earns net yield of at least 9% p.a. Calculate the maximum price that Investor Y should pay to Investor X if Y is subject to capital gain tax of 30% and income tax of 25%. (7)
- iv)** What is the actual net yield obtained by Investor X if he sells the bonds to Y at price calculated in (iii) above? (2)

[16]
