Actuarial Society of India

EXAMINATIONS

22nd May 2006

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***

4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q1) The force of interest $\delta(t) = 0.004t + 0.0002t^2$ for all t

(i) Find present value of a sum of Rs.1000 payable at the end of 10 year from now. (3)

(ii) Find the constant annual effective rate of interest over ten year period (2)

[5]

Q2)

(i) Explain the relationship d = iv by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of interest. (2)

(ii) Calculate nominal rate of discount per annum convertible half yearly which is equivalent to

(a) An effective rate of discount of 2.25% per quarter (2)

(b) A nominal rate of discount of 5% convertible every 2 years. (2)

(iii) A 91-day government bill provides the investor with an annual effective rate of return of 5%. Calculate the annual simple discount rate at which the bill is discounted. (2)

[8]

Q3)

(i) Define "No arbitrage". (2)

(ii) A one-year forward contract is issued on 1 January 2005 on a share with price of Rs.550 on that date. Dividends of Rs.60 are expected on 30 June 2005 and 31 December 2005. The risk-free spot rates for 6-month and 12-month are 5% and 5.5% per annum effective respectively on 1 January 2005.

Calculate the forward price at issue, assuming no arbitrage. (4)

[6]

Q4) A housewife buys a refrigerator at Rs.15,000/- from a dealer under instalment scheme to be repaid in 36 equal monthly instalments payable in arrear. The amount of each monthly instalment is Rs.550/-.

(i) Calculate the flat rate of interest p.a. (2)

(ii) Calculate Annual Percentage Rate (APR) (3)

[5]

Q5)

(i) Describe risk characteristics of fixed interest government bond. (2)

(ii) 20-year fixed interest bond with coupon of 6% per annum payable half yearly was issued on 1 July 2005. The bond is redeemable at 120% at maturity date. An investor who pays both income tax and capital gains tax @ 30% purchased the bond on date of issue. The investor's net yield to redemption was 8% per annum effective on the purchase.

(a) Calculate the price paid by the investor. (4)

(b) Calculate the duration of the net payments from the fixed interest bond for an investor, who does not pay any tax , at rate of interest 6% per annum convertible half yearly. (4)

[10]

Q6)

(a) List the factors that cause interest rates vary over time. (3)

(b) How do you measure 10-year spot rate, y_{10}? (1)

(c) What is forward interest rate $f_{t,r}$ where t>0? (1)

(d) The 3-year and 7-year spot rates are 6.5% p.a. and 5.5% pa respectively. 3-year forward rate from time 4 is 5.7% p.a. Calculate

(i) f_3 (ii) y_4 (4)

[9]

Q7) 5 years ago, a borrower took a home loan of Rs.600,000 from a bank at rate of interest of 10% per annum effective. Loan is repayable by monthly installments of Rs. X each paid in arrears over 15-year period.

(i) Calculate X. (3)

(ii) Calculate the interest and capital components of 19th installment. (4)

(iii) The rates of interest on home loan have fallen during past 5 years. In order to retain the customer, the bank makes an offer to the borrower to take a new loan equal to the outstanding loan at the end of year 5 at rate of interest of 8% per annum effective. The bank charges a processing fee equal to 4% of the original loan. The first loan is repaid and new loan is repayable over remaining 10 years period by monthly instalments in arrear.

(a) Calculate the revised monthly instalment of the new loan. (4)

(b) Calculate the present value of reduction in monthly instalment at the rate of interest of 8% per annum effective and show that the offer is profitable to borrower. (3)

[14]

Q8)

(i) Briefly describe Call Option and Put Option. (2)

(ii) Explain why selling a call option is not the same as buying put option? (2)

(iii) An investor purchases an ordinary share that pays annual dividend at price of Rs.600. The next dividend is due in exactly six months time and is expected to be Rs.30. It is expected that subsequent dividends will grow at the rate of 8% per annum effective and are expected to continue in perpetuity. The inflation is expected to be 5% per annum.

Calculate the expected real rate of return per annum effective for the investor. (5)

[9]

Q9) A company ABC is considering investing in a project. The cost of project will be Rs.10 crore at the beginning of the first year. There are further costs at the beginning of second to fifteenth year. The cost at the beginning of second year will be Rs.1.5 crore and will increase every year thereafter at a rate of 5%.

The income will be Rs.3.5 crore at the end of each of first 5 years, increasing by Rs.50 lakh at the end of each of sixth to fifteenth year.

The company ABC sets a discount rate of 15% pa to evaluate this project.

(i) Explain what is meant by discounted payback period of a project (2)

(ii) Calculate net present value of the project (7)

(iii) Calculate the discounted pay back period for the project (4)

[13]

Q10) A Life Insurance Company sold a single premium bond with term of 10 years. The bond gives a maturity payment of 150% of single premium at the end of 10 years. Company's investment strategy for this bond is such that the expected annual effective rate of interest from investment is 8% and standard deviation of annual return is 11%. Annual returns are independent and $(1+i_t)$ follows lognormal distribution where i_t is the return in t^{th} year. The company has received a premium of Rs.20,000.

(i) Calculate the expected value and standard deviation of an investment of Rs. 20,000 at the end of 10th year, deriving all the formula used. (9)

(ii) Calculate the probability that the accumulated value of the investment will be less than 60% of its expected value in 10 years time. (8)

(iii) How much the company should invest at time t=0 such that accumulated value at time 10 exceeds maturity payout with 95% probability (4)

[21]