Test Code : RE I / RE II (Short Answer type) 2004
(Junior Research Fellowship in Economics)

The candidates for Junior Research Fellowship in Economics are required to take two short answer type tests - RE I (Mathematics) in the forenoon session and RE II (Economics) in the afternoon session.

Syllabus for RE I

1. Permutations and combinations.
2. Elementary set theory; Functions and relations; Matrices, coordinate geometry.
3. Convergence of sequences and series.
4. Functions of one and several variables : limits, continuity, differentiation, applications, integration of elementary functions, definite integrals.
5. Constrained and unconstrained optimization, convexity of sets and concavity and convexity of functions.
6. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, distributions of functions of random variable.

Syllabus for RE II

Note : This test will have questions on Economics, Statistics and Mathematics. However, there will be sufficient number of alternative questions to answer from any one subject, if the candidate so desires.

1. Theory of consumer behaviour; theory of production; market structure; general equilibrium and welfare economics; international trade and finance; public economics.
2. Macroeconomic theories of income distribution, economic growth and in-equality.
3. Game Theory: Normal and extensive forms, Nash and subgame-perfect equilibrium.
4. Statistical inference, regression analysis, least squares and maximum like-lihood estimation, specification bias, endogeneity, instrumental variables, elementary time-series analysis.

Sample Questions for RE I

1. Examine the continuity of the function

$$f(x) = \lim_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, \quad x>0$$

2. Express the polynomial $f(x) = x^3 + 3x - 3$ as a function of y where $y = (x-1)$.

3. Prove that the function

$$f(x) = \frac{|x|}{1+|x|}$$ is strictly increasing for $\{x | x \geq 0\}$ and strictly decreasing for $\{x | x \leq 0\}$. Graph the function.

4. A fair coin is tossed three times. Let X denote the number of heads in the first two tosses and let Y denote the number of heads in the last two tosses. Find the correlation between X and Y.

5. Let $f(x) = 1$, $0<x<1$, and zero elsewhere, be the probability density function of X.
   (a) Find the distribution function and the probability density function of $Y = \sqrt{X}$.
   (b) Find the median of the distribution of X and of the distribution of Y.

6. Find the number of subsets of $\{1,2, \ldots,9\}$ containing at least one odd integer.

7. Let $\{a_n\}$ be the sequence defined by

$$a_1 = \frac{3}{2} \text{ and } a_{n+1} = 2 - \frac{1}{a_n}, \; n \geq 1.$$

Show that $\{a_n\}$ is monotonic decreasing, bounded below and converges to 1.

8. Find the inverse of the following matrix $\begin{bmatrix} 1 & 2 & 0.5 \\ 3 & 7 & 8 \\ 8 & 16 & 4 \end{bmatrix}$

9. Prove that $\log x \le x-1$. Draw the region $x-|y| \le 1$ and $y = \log x$ on the same graph.

10. Find the value of x at which the function $y = x^{1/x}, x \ne 0$, attains its maximum.

11. Three groups of children comprise 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys, respectively. One child is selected at random from each group. Find the probability that the three selected children consist of 1 girl and 2 boys.

12. Prove or disprove: f and g are two functions defined from R to R which are not differentiable at 0. Then $f+g$ is not differentiable at 0.

13. Show that $\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x+a}-\sqrt{x}\right) = \frac{1}{2}a.$

14. $\max_a I(a) = \int_{a-1}^{a+1} e^{-|x|} dx.$

15. Show that the equation $\frac{2}{3}x \ Sinx = 1$ has four roots between $-\pi$ and π

16. (a) Give an example to show that any monotone increasing transformation of a concave function is not necessarily concave.

(b) Does a concave function always have a decreasing average? Prove your claim or give a counter example.

17. How many 6 letter words can be formed using the letters A, B and C so that each letter appears at least once in the word?

18. Suppose that X and Y are independent Poisson random variables with

parameters λ_1 and λ_2 respectively such that Var.(X)+Var.(Y)=3. Evaluate P[X+Y<2].

19. Suppose $f(x)$ is a function such that $f' > 0$. If $f(\lambda x_1 + (1-\lambda)x_2) < \lambda f(x_1) + (1-\lambda) f(x_2)$ for $0 < \lambda < 1$, then show that $f'(x)$ is increasing.

20. Find the equation of a circle which passes through the origin , whose radius is 'a' and for which $y = mx$ is a tangent.

21. Let $\lim_{x \to \xi} f(x) = 0$, $\lim_{x \to \xi} g(x) = 0$, $f'(\xi)$ and $g'(\xi)$ exist and $g'(\xi) \neq 0$. Then show that

$$\lim_{x \to \xi} \frac{f(x)}{g(x)} = \frac{f'(\xi)}{g'(\xi)}.$$

22. If n men, including A and B, stand in a row, what is the probability that there will be exactly r men between A and B?

23. X and Y have joint probability distribution as follows

Y / X	1	2	3
		p_{ij}	
1	1/12	2/12	1/12
2	1/12	0	1/12
3	2/12	0	4/12

Find (a) the marginal distribution of X, (b) the marginal distribution of Y and (c) the expectations of X and Y.

JRF Exam 2004

SAMPLE PAPER FOR RE-II

DO ANY FOUR QUESTIONS

1. Consider a competitive industry with two types of firms. There are 100 identical firms, each of which has a cost function

$$C_1(q) = 0.25 + q^2$$

There are 5000 identical firms with the cost function

$$C_2(q) = 2q$$

The aggregate demand function is

$$Q = A - 2p$$

(a) Find the market clearing price if (i)A=200 (ii)A=76.5.

(b) What are the quantities supplied under (i) and (ii)?

[12+13]

2. There are two countries in the world economy, A and B. There aer two goods, 1 and 2. There is one factor of production, labor. The amount of labor required to produce one unit of goods 1 and 2 are 10 and 20 respectively in country A and 20 and 50 respectively in country B. There are constant returns to scale in producing each good in each country. The total labor endowments in countries A and B are 240 and 400 respectively. Preferences in both countries for the two goods are identical and described by the utility function

$$U(C_1, C_2) = C_1.C_2$$

where C_1 and C_2 are the quantities of goods 1 and 2 consumed respectively.

(a) If these two countries did not trade with each other, how much would each country produce and consume?

(b) Now suppose they open up trade. There are no barriers to trade or transport costs. Which country will export which good and how much of each good will each country produce and consumer? What will be the world terms of trade?

[10+15]

3. Consider the following two-person game in normal form. There are two players, 1 and 2 each of whom has two strategies, Top and Bottom. If both players play Top, they both get a payoff of x; if both play Bottom, they both get a; if player 1 plays Top and 2 plays Bottom, then player 1 gets b and player 2 gets y; if player 1 plays Bottom and 2 plays Top, then player 1 gets y and player 2 gets b. Find the set of pure and mixed strategy Nash equilibria in this game in each of the following cases:

(a) $y > x > a > b$

(b) $y = x > a > b$

(c) $x > y > a > b$

[8+9+8]

4. (a) The Pareto distribution is frequently used as a model to study the income distribution. It has the following CDF:

$$F(x;\theta_1,\theta_2) = 1 - \left(\frac{\theta_1}{x}\right)^{\theta_2} \qquad \text{for all } x \geq \theta_1$$
$$= 0 \qquad \text{elsewhere}$$

Assume that θ_1 and θ_2 are both positive. Given a random sample of n observations, find the maximum likelihood estimators of θ_1 and θ_2.

(b) Consider any unbiased estimator $\hat{\theta}$ of θ, and show that $\hat{\theta}^2$ is not an unbiased estimator of θ^2. Is the bias positive or negative? Explain.

[18+7]

5. Let W, E and A denote a worker's wage, education and age respectively. Given data on W, E and A for a random sample of 500 workers of small-scale industrial units, you are asked to examine whether or not the marginal return to education declines with the age of a worker.

(a) i. Formulate the linear regression model that you will estimate in each of the following situations: (i) W, E and A are all measured as quantitative variables in units of Rs./month, number of years of schooling and number of completed years since birth respectively. (ii) While W is measured as specified in (i) above, E and A are measured in qualitative terms as illiterate/literate for E and above and below age 35 for A.

ii. In each case, specify the linear restrictions on parameters that you will test to verify your hypothesis.

(b) Consider the model

$$
\begin{aligned}
y_i &= \beta x_i + \epsilon_i, & i = 1, 2, ...N \\
\epsilon_i &= u_i + \delta u_{i-1} & |\delta| < 1 \\
E(u_i) &= 0 \text{.................for all } i \text{ and} \\
E(u_i, u_j) &= \sigma_u^2, \quad i = j \\
&= 0 \text{.......} i \neq j.
\end{aligned}
$$

Assuming δ is known, obtain the generalized least squares estimator of β.

[15+10]

6. (a) A random sample of n observations $x_1, x_2 ...., x_n$ of a random variable X is drawn. X has a probability density function proportional to $x^{\alpha}(1 - x)$ defined in the range $0 \leq x \leq 1$, where the parameter α is unknown. Show that α^*, the appropriate maximum likelihood estimate of α, is given by the expression

$$\alpha = -\frac{(3g + 2) + (g^2 + 4)^{\frac{1}{2}}}{2g}$$

where g is the natural logarithm of the geometric mean of the sample.

(b) From an extensive record collected over several years, it is found that only 2% of a certain type of cancer patients are cured by surgery. A chemotherapist claims that his non-surgical method of treating these m patients is more successful than surgery. To obtain experimental evidence to support his claim, he uses his method on 200 patients, 6 of whom are cured. The chemotherapist asserts that the observation of a 3% cure rate in a large sample is sufficient evidence to support his claim.

i. Formulate the hypothesis in terms of the parameter m, the expected number of cures out of 200 patients.

ii. With the level of significance $\alpha = 0.05$, determine the rejection region. Also determine if the claim is convincing.

iii. If the chemotherapy actually produces a cure rate of 4.5%, what is the probability that the test will support the claim that chemotherapy is more successful than surgery?

[10+15]

7. An $n \times n$ chessboard is coloured in the following way: the (i, j) square (that is, the square on the ith row and jth column) is coloured white if $(i + j)$ is even and black if $(i + j)$ is odd. A coin placed on the (i, j) square can be moved to (i', j') square if $(i - i')^2 + (j - j')^2$ is an even number.

(a) i. Prove that if a coin can be moved from one square to another, then the squares must be of the same colour.

ii. Suppose n is even. Show that it is not possible for a coin to travel from the square $(1, 1)$ to the square $(1, n)$ by a sequence of moves.

(b) If a polynomial in x consists of a number of terms connected by + signs followed by a number of terms connected by − signs, show that it has one positive root and no more.

[15+10]

8. Suppose that there are used cars of two types: peaches (cars of good quality) and lemons (cars of bad quality). A peach, if it is know to be a peach, is worth Rs. 3,000 to a buyer and Rs 2,500 to a seller. A lemon, on the other hand, is worth Rs. 2,000 to a buyer and Rs. 1,000 to a seller. Assume that the supply of cars is fixed and the number of possible buyers is infinite. There are twice as many lemons as peaches. What can be said about equilibrium prices and quantitites of cars traded in each of the following cases?

(a) Both buyers and sellers can examine a car to determine whether it is a peach or lemon before it is traded.

(b) Neither buyer nor seller knows whether a car is a peach or lemon before trade.

(c) Only the seller knows whether a car offered for sale is a peach or a lemon.

(d) Would you revise your conclusion in (c) above if there were two peaches to every lemon?

(e) Explain why it is difficult for a person above the age of 65 to purchase insurance.

[5+5+5+5+5]

9. In an experiment of tossing an unbiased coin repeatedly, a person gets 1 point if a head occurs and two points if a tail occurs. Let $p(k)$ denote the probability of obtaining a sum of k points. Show that

$$p(k) = \frac{p(k-1) + p(k-2)}{2}$$

[25]

10. In a set of objects, each object has one of two colours and one of two shapes. There is at least one object of each colour and at least one object of each shape. Prove that there exists two objects in the set that differ both in colour and in shape.

[25]