

SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E - EEE

Title of the paper: Engineering Mathematics - III

Semester: III

Sub.Code: 514301

Date: 04-05-2007

Max. Marks: 80

Time: 3 Hours

Session: AN

PART – A

(10 x 2 = 20)

Answer ALL the Questions

1. Find the Transform of $\left(\frac{\sin t}{t}\right)$

2. Find $L^{-1}\left(\frac{s-1}{s^2+3s+2}\right)$

3. Find Root Mean square value of $f(x) = x - x^2$ in $-1 < x < 1$.

4. Write complex form of fourier series for $f(x)$.

5. Solve $\frac{\partial^2 u}{\partial x^2} = xy$

6. Find the particular integral of $(D^2 + 2DD' + D'^2)z = \sinh(x+y)$

7. Classify the partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

8. Write the solutions of one dimensional heat equation obtained by the method of separation of variables.

9. If $F\{f(x)\} = F(s)$ then prove that $F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} F(s)$

10. Find fourier sine transform of $\frac{1}{x}$

PART – B (5 x 12 = 60)
Answer All the Questions

11. (a) Find the Laplace transform of half sine wave rectifier function

$$f(t) = \begin{cases} a \sin \omega t & 0 \leq t \leq \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} \leq t \leq 2 \frac{\pi}{\omega} \end{cases}$$

(b) Find $L^{-1}\left(\tan^{-1}\left(\frac{2}{s^2}\right)\right)$

(or)

12. (a) Find $L^{-1}\left(\frac{s^2}{(s^2 + 9)(s^2 + 4)}\right)$ using convolution theorem.

(b) Solve $y'' + 4y = \sin$ at given $y(0) = 0$ and $y'(0) = 0$.

13. (a) Obtain Fourier series of the periodic function defined by

$$f(x) = \begin{cases} -\pi & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(b) Find half range cosine series for $f(x) = (x - 1)^2$ in $0 < x < 1$.

Hence show that $\pi^2 = 6 \left\{ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots \right\}$

(or)

14. (a) Expand $f(x) = \pi x - x^2$ in a half range sine series in the interval $(0, \pi)$.

(b) Compute the first three harmonics for the fourier series of $f(x)$ given by the following table:

| | | | | | | | |
|------|---|-----------------|------------------|-------|------------------|------------------|--------|
| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | π | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | 2π |
| f(x) | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1 |

15. (a) Form pde by eliminating arbitrary functions.

From $z = f(x + ct) + \phi(x - ct)$.

(b) Solve $x(y - z)p + y(z - x)q = z(x - y)$.

(or)

16. (a) Solve $p^2 + q^2 = z^2(x^2 + y^2)$

(b) Solve $(D^2 - 3DD' + 2D'^2)z = z \sin x \cos y$.

17. A taut string of length $2l$ is fastened at both ends. The mid point of the string is taken to a height a and then released from the rest in that position. Find the displacement of the string.

(or)

18. A bar, 10cm long with insulated sides, has its ends A and B kept at 20° and 40°C respectively until steady-state conditions

prevail. The temperature at A is then suddenly raised to 50°C and at the same time B is lowered to 10°C. find the temperature distribution $u(x,t)$ at any time.

19. (a) Show that the transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$ by finding the fourier transform of $e^{-a^2x^2}$ $a > 0$.

(b) Find the fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence

find the value of $\int_0^{\infty} \frac{\sin^4 t}{t^4} dt$.

(or)

20. (a) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the fourier sine transform of $xe^{-a^2x^2}$

(b) Evaluate $\int_0^{\infty} \frac{dx}{(a^2+x^2)(b^2+x^2)}$