

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20th May 2011

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
- 2) *Mark allocations are shown in brackets.*
- 3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
- 4) *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** (i) Describe the forward rate $F(t, T, S)$. (2)

You are the head of the portfolio risk management department of your company. Your investment team has subscribed to fresh issue of two uncorrelated bonds as follows:

- INR 1 million face value of an A3 rated bond, Annual coupon 10% per annum (with annual compounding), Term 4 years
- INR 1 million face value of a B2 rated bond, Annual coupon 15% per annum (with annual compounding), Term 4 years

These are the only two bonds in the portfolio.

The investment department provided the following *rating* specific rates $F(0, 1, S)$. Rates are quoted at per annum with continuous compounding.

Rating	S=2	S=3	S=4
A1	9%	10%	11%
A2	9.5%	11%	12%
A3	10.25%	12%	13.5%
B1	11%	13%	15.5%
B2	12%	14%	18%

- (ii) Assuming there are no rating changes during the year and the investment department believes that at the end of the first year $F(1, 1, S) = F(0, 1, S)$, calculate the expected value of the portfolio at the end of the first year just before the first coupon payment. (4)

You use a 6 state (5 ratings and default) JLT model for credit risk assessment. Your team has estimated the following one year transition probability matrix.

		Rating at the end of the year					
		A1	A2	A3	B1	B2	Default
Rating at the beginning of the year	A1	90%	6%	4%	0%	0%	?
	A2	5%	85%	7.5%	2.5%	0%	?
	A3	1%	2%	80%	9%	8%	?
	B1	0%	1%	7.5%	75%	12%	?
	B2	0%	1%	4%	10%	70%	?

- (iii) Estimate the probabilities of default? (2)

Assume a recovery rate of 30% on defaulted bonds.

- (iv) Describe the worst possible scenario for the portfolio in one year's time and calculate its likelihood. (3)

- (v) Calculate the 99% VaR over one year for the bond portfolio relative to the position in 1(ii). (4)

[15]

- Q. 2)** (i) State the expression for the delta of a call option on a non-dividend paying stock. Define all terms used. (2)

You are a long term investor and hold 1,000 units of a non-dividend paying stock in your portfolio.

You have observed that the markets are excessively volatile during the cricket world cup.

- (ii) When are stock prices considered to be “excessively volatile”? (2)

The stock is priced at 100 and the price of a 6 month at-the-money European call option on the stock is 9.1990.

- (iii) Calculate the implied volatility to 0.5% of accuracy if the risk free force of interest is 6% per annum (with continuous compounding). (6)

Long term outlook of the stock you hold remains stable during this period.

- (iv) Construct a delta neutral portfolio using 6 month call options on the stock to benefit from excessive volatility? Describe the challenge you will face in maintaining a delta neutral portfolio. (5)

The volatility reaches its long term mean level of 10% after a month and you unwind your derivative positions. The interest rates and stock price remains the same.

- (v) Assuming that you unwind the position after one month, calculate the profit made assuming you do not perform further trades after constructing the delta neutral portfolio in (iv). State the assumptions made. (5)

[20]

- Q. 3)** Based on general reasoning derive the upper and lower bounds on the price of a European call option on a non-dividend paying share. Define all terms used. (4)

- Q. 4)** Describe the state price deflator approach to modeling term structure of interest rates. (4)

- Q. 5)** Consider a three-period binomial model for a non-dividend paying stock whose current price is 100. You buy a special option on the stock that pays 10 units of cash if the spot price of the stock is either more than 120 or less than 90 at maturity.

Assume that:

- the risk free force of interest is 5% per period (with continuous compounding).
- over each period the stock price can either move up by a factor of 1.1 or go down by a factor of 0.9.

Prove that the market is arbitrage free and derive the price of the option.

[7]

- Q. 6)** (i) Describe three key assumptions underlying the empirical verification of the validity of the CAPM. (3)
- (ii) In the context of the CAPM discuss the concept of systematic risk for a risky security. (3)
- (iii) You are given the following historical information for a share in ABC company and for a portfolio of 100 shares:

	<i>Expected Return (% p.a.)</i>	<i>Standard deviation (% p.a.)</i>	<i>Beta of return</i>
Portfolio	10.5	16	1.1
ABC	8.5	20	0.7

Under which conditions, the figures in the above table are consistent with the CAPM. (3)

- (iv) A student has commented that ABC's lower return and higher standard deviation, relative to the 100 share portfolio, contradicts the predictions of the CAPM.

Discuss the student's comment. (3)

[12]

- Q. 7)** Suppose that a stock price S follows geometric Brownian motion with expected return μ and volatility σ

$$dS = \mu S dt + \sigma S dz$$

where Z_t is a standard Brownian motion process

- (i) Describe and discuss the plausibility of the assumptions behind this equation when it is used as a model of share prices. (4)
- (ii) Show that S^n also follows geometric Brownian motion (5)
- (iii) Consider Arvind Mills stock which follows geometric Brownian motion as described above. Suppose Arvind Mills stock has an expected return of 16% per annum (with continuous compounding) and a volatility of 30% per annum (with continuous compounding). When the stock price of Arvind Mills is Rs. 50 per share, calculate the following
- (1) The expected stock price in one trading day (1)
- (2) The standard deviation of the stock price in one trading day (2)
- (3) 95% confidence interval for the stock in one trading day (4)

[16]

- Q. 8)** (i) Describe the forms of the Efficient Markets Hypothesis (EMH). (3)
- (ii) The existence of fund managers who sell their services based on their alleged ability to select over-performing sectors and stocks and so add value to portfolios demonstrates that capital markets are not efficient.
- Discuss the above statement (4)
- (iii) Define a multifactor model of security returns. All the notations you use must be clearly defined. What is the goal of the builders of such a model. (2)
- (iv) Explain the type of factors on the basis of which multifactor models can be classified. (3)

[12]

- Q. 9)** You are contemplating an investment with a return of Rs X, where:

$$X = 300,000 - 500,000U$$

where U is a uniform $[0, 1]$ random variable.

Calculate each of the following four measures of risk:

- a) variance of return (1)
- b) downside semi-variance of return (1)
- c) shortfall probability, where the shortfall level is Rs 100,000 (1)
- d) Discuss the usefulness of downside semi-variance as a measure of investment risk for an investor (2)

[5]

Q. 10) An individual who prefers more to less has a quadratic utility function and initial wealth of 100. She faces a random loss that is normally distributed with mean 5 and standard deviation 10. She is indifferent between facing this loss and paying 5.5 to fully protect herself from the loss.

a) Find the form of her utility function. (2)

b) The individual is considering entering a lottery in which the first prize is 1000. Explain whether the utility function from (a) can be used to calculate how much should be paid for the ticket. (3)

[5]
