



Answer: (C)

Exp: Total question

$$44 \times 2 = 88$$

$$44 \times 3 = 132$$

$$144 = 88$$

$$\begin{array}{r} 132 \\ \hline 308 \end{array}$$

$$\text{Total marks obtained} = (21 \times 2) + (15 \times 3) + (23 \times 2) = 133$$

$$\text{Total Number of students} = 44$$

$$\text{Average} = \frac{133}{44} = 3.02$$

### Q. No. 6 – 10 Carry One Mark Each

6. A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is
- (A) Students should come at 9.00 a.m. and parents should come at 10.00 a.m.
- (B) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.
- (C) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.
- (D) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

Answer: (B)

7. By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage?

- (i) Our understanding of the universe changes based on the positions of stars
- (ii) Paradigm shifts usually occur at the beginning of centuries
- (iii) Stars are important objects in the universe
- (iv) Experimental evidence was important in confirming this paradigm shift

(A) (i), (ii) and (iv)    (B) (iii) only    (C) (i) and (iv)    (D) (iv) only

Answer: (D)





3. Let  $G$  be a group with 15 elements. Let  $L$  be a subgroup of  $G$ . It is known that  $L \neq G$  and that the size of  $L$  is at least 4. The size of  $L$  is \_\_\_\_\_.

Answer: (5)

Exp: Order of subgroup divides order of group (Lagrange's theorem).

3, 5 and 15 can be the order of subgroup. As subgroup has atleast 4 elements and it is not equal to the given group, order of subgroup can't be 3 and 15. Hence it is 5.

4. Which one of the following statements is TRUE about every  $n \times n$  matrix with only real eigenvalues?

(A) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

(B) If the trace of the matrix is positive, all its eigenvalues are positive.

(C) If the determinant of the matrix is positive, all its eigenvalues are positive.

(D) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

Answer: (A)

Exp: If the trace of the matrix is positive and the determinant of the matrix is negative then atleast one of its eigen values is negative.

Since determinant = product of eigen values.

5. If  $V_1$  and  $V_2$  are 4-dimensional subspaces of a 6-dimensional vector space  $V$ , then the smallest possible dimension of  $V_1 \cap V_2$  is \_\_\_\_\_.

Answer: (2)

Exp: Let the basis of 6-dimensional vector space be  $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ . In order for  $V_1 \cap V_2$  to have smallest possible dimension  $V_1$  and  $V_2$  could be, say,  $\{e_1, e_2, e_3, e_4\}$  and  $\{e_3, e_4, e_5, e_6\}$  respectively. The basis of  $V_1 \cap V_2$  would then be  $\{e_3, e_4\}$ .  $\Rightarrow$  Smallest possible dimension = 2.

6. If  $\int_0^{2\pi} |x \sin x| dx = k\pi$ , then the value of  $k$  is equal to \_\_\_\_\_.

Answer: (4)

$$\text{Exp: } \int_0^{2\pi} |x \sin x| dx = K\pi \Rightarrow \int_0^{\pi} x \sin x dx + \int_{\pi}^{2\pi} -(x \sin x) dx = K\pi \left[ \begin{array}{l} \because |\sin x| = -\sin x \\ \pi < x < 2\pi \end{array} \right]$$

$$\Rightarrow x(-\cos x) - 1(-\sin x) \Big|_0^{\pi} - (-x \cos x + \sin x) \Big|_{\pi}^{2\pi} = K\pi$$

$$\Rightarrow (-\pi \cos \pi + \sin \pi) - 0 - [(-2\pi \cos 2\pi + \sin 2\pi) - (-\pi \cos \pi + \sin \pi)] = K\pi$$

$$\Rightarrow \pi + 0 - [-2\pi + 0 - (\pi + 0)] = K\pi \Rightarrow 4\pi = K\pi \Rightarrow k = 4$$

7. Consider the following minterm expression for F.

$$F(P, Q, R, S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$$

The minterms 2, 7, 8 and 13 are ‘do not care terms’. The minimal sum of-products form for F is

- (A)  $Q\bar{S} + \bar{Q}S$   
 (B)  $\bar{Q}\bar{S} + QS$   
 (C)  $\bar{Q}\bar{R}\bar{S} + \bar{Q}R\bar{S} + Q\bar{R}S + QRS$   
 (B)  $\bar{P}\bar{Q}\bar{S} + \bar{P}QS + PQS + P\bar{Q}\bar{S}$

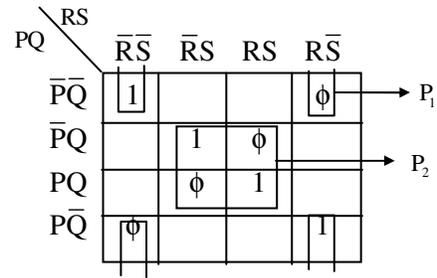
Answer: (B)

Exp: The K-map for the function F is as follows:-

$$P_1 = \bar{Q}\bar{S} \text{ and } P_2 = QS$$

$$\therefore F(P, Q, R, S) = P_1 + P_2$$

$$= \bar{Q}\bar{S} + QS$$



8. Consider the following combinational function block involving four Boolean variables x, y, a, b where x, a, b are inputs and y is the output.

$f(x, y, a, b)$   
 {  
     if (x is 1) y = a;  
     else y = b;  
 }

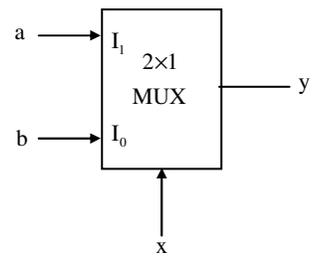
Which one of the following digital logic blocks is the most suitable for implementing this function?

- (A) Full adder      (B) Priority encoder      (C) Multiplexor      (D) Flip-flop

Answer: (C)

Exp:  $y = \bar{x}b + xa$

‘x’ is working as selection line, where the two input lines are ‘a’ and ‘b’, so the function  $F(x, y, a, b)$  can be implemented using  $(2 \times 1)$  multiplexer as follows:



9. Consider the following processors (ns stands for nanoseconds). Assume that the pipeline registers have zero latency.

- P1: Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.  
 P2: Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.  
 P3: Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.  
 P4: Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

- (A) P1      (B) P2      (C) P3      (D) P4

Answer: (C)

Exp: Clock period (CP) = max stage delay + overhead

$$\text{So } CP_{p_1} = \text{Max}(1, 2, 2, 1) = 2\text{ns}$$

$$CP_{p_2} = \text{Max}(1, 1.5, 1.5, 1.5) = 1.5\text{ns}$$

$$CP_{p_3} = \text{Max}(0.5, 1, 1, 0.6, 1) = 1\text{ns}$$

$$CP_{p_4} = \text{Max}(0.5, 0.5, 1, 1, 1, 1) = 1.1\text{ns}$$

As frequency  $\propto \frac{1}{\text{C.P}}$ , so least clock period will give the highest peak clock frequency.

$$\text{So, } f_{p_3} = \frac{1}{1\text{ns}} = 1\text{GHz}$$

10. Let A be a square matrix size  $n \times n$ . Consider the following pseudocode. What is the expected output?

```
C = 100;
for i = 1 to n do
  for j = 1 to n do
```

```
{
  Temp = A[i][j] + C;
  A[i][j] = A[j][i];
  A[j][i] = Temp - C;
}
```

```
for i = 1 to n do
  for j = 1 to n do
    output (A[i][j]);
```

- (A) The matrix A itself
- (B) Transpose of the matrix A
- (C) Adding 100 to the upper diagonal elements and subtracting 100 from lower diagonal elements of A
- (D) None of these

Answer: (A)

Exp: In the computation of given pseudo code for each row and column of Matrix A, each upper triangular element will be interchanged by its mirror image in the lower triangular and after that the same lower triangular element will be again re-interchanged by its mirror image in the upper triangular, resulting the final computed Matrix A same as input Matrix A.

11. The minimum number of arithmetic operations required to evaluate the polynomial  $P(X) = X^5 + 4X^3 + 6x + 5$  for a given value of X, using only one temporary variable is \_\_\_\_\_.

Answer: (7)

Exp:  $P(x) = x^5 + 4x^3 + 6x + 5$  can be re written as follows

$$P(x) = x^3(x^2 + 4) + 6x + 5$$

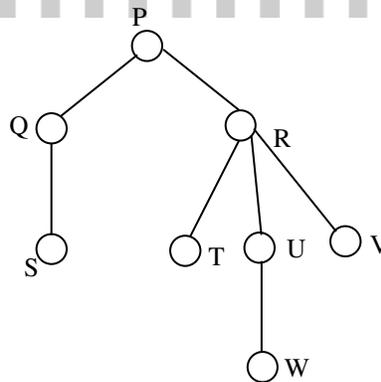
Now using only one temporary variable  $t$  and any number of data transfer as well as memory related operation the polynomial can be evaluated as follows

1.  $t = x * x$  [Evaluate  $x^2$  and store in memory]
2.  $t = t + 4$  [Evaluate  $(x^2 + 4)$  and store in memory]
3.  $t = x^2$  [Retinue  $x^2$  from memory]
4.  $t = t * X$  [Evaluate  $x^3$  and store in memory]
5.  $t = t * (x^2 + 4)$  [Evaluate  $x^3(x^2 + 4)$  and store in memory]
6.  $t = 6 * x$  [Evaluate  $6x$  and store in memory]
7.  $t = t + 5$  [Evaluate  $(6x + 5)$  and store in memory]
8.  $t = t + x^3(x^2 + 4)$  [Retrieve  $x^3(x^2 + 4)$  from memory and evaluate  $\{x^3(x^2 + 4) + 6x + 5\}$ ]

In the above 8 steps of evaluation, the total number of arithmetic operations required and 7 [4 Multiplications, 3 Additions]

So answer is 7 arithmetic operations.

12. Consider the following rooted tree with the vertex labelled P as the root



The order in which the nodes are visited during an in-order traversal of the tree is

- |              |              |
|--------------|--------------|
| (A) SQPTRWUV | (B) SQPTUWRV |
| (C) SQPTWUVR | (D) SQPTRUWV |

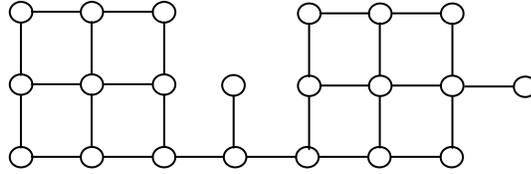
Answer: (A)

Exp: The In order Traversal of Ternary Tree is done as follows:

Left → Root → Middle → Right

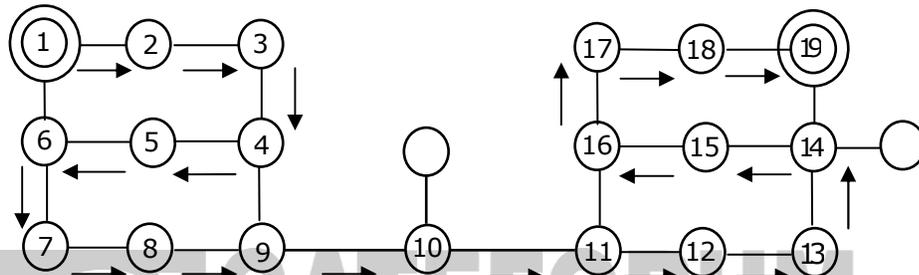
So the nodes are visited in SQPTRWUV order.

13. Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is \_\_\_\_\_.



Answer: 19

Exp:



Suppose, we start DFS at vertex-numbered as 1 and continue calling recursive function for DFS on subsequent nodes numbered in ascending order.

The recursive calling sequence is shown as marked line in the above diagram which shows maximum possible recursion depth including the initial call is 19.

14. You have an array of  $n$  elements. Suppose you implement quick sort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is

(A)  $O(n^2)$       (B)  $O(n \log n)$       (C)  $\theta(n \log n)$       (D)  $O(n^2)$

Answer: (A)

Exp: The Worst case time complexity of quick sort is  $O(n^2)$ . This will happen when the elements of the input array are already in order (ascending or descending), irrespective of position of pivot element in array.

15. The length of the shortest string NOT in the language (over  $\Sigma = \{a, b\}$ ) of the following regular expression is \_\_\_\_\_.  $a^*b^*(ba)^*a^*$

Answer: (3)

Exp: R.E =  $a^*b^*(ba)^*a^*$

Length 0 is present as it accepts  $\epsilon$  all length 1 strings are present (a,b) also aa, ab, ba, bb are present, But 'bab' is not present. So it is 3

16. Let  $\Sigma$  be a finite non-empty alphabet and let  $2\Sigma^*$  be the power set of  $\Sigma^*$ . Which one of the following is **TRUE**?
- (A) Both  $2\Sigma^*$  and  $\Sigma^*$  are countable
  - (B)  $2\Sigma^*$  is countable  $\Sigma^*$  is uncountable
  - (C)  $2\Sigma^*$  is uncountable and  $\Sigma^*$  is countable
  - (D) Both  $2\Sigma^*$  and  $\Sigma^*$  are uncountable

Answer: (C)

Exp:  $2^{\epsilon^*}$  is the power set of  $\epsilon^*$

$\epsilon^*$  is countably infinite.

The power set of countably infinite set is uncountable.

So  $2^{\epsilon^*}$  is uncountable, and  $\epsilon^*$  is countable.

17. One of the purposes of using intermediate code in compilers is to
- (A) make parsing and semantic analysis simpler.
  - (B) improve error recovery and error reporting
  - (C) increase the chances of reusing the machine-independent code optimizer in other compilers.
  - (D) improve the register allocation.

Answer: (C)

Exp: Intermediate code is machine independent code which makes it easy to retarget the compiler to generate code for newer and different processors.

18. Which of the following statements are **CORRECT**?
- 1) Static allocation of all data areas by a compiler makes it impossible to implement recursion.
  - 2) Automatic garbage collection is essential to implement recursion.
  - 3) Dynamic allocation of activation records is essential to implement recursion.
  - 4) Both heap and stack are essential to implement recursion.
- (A) 1 and 2 only      (B) 2 and 3 only      (C) 3 and 4 only      (D) 1 and 3 only

Answer: (D)

Exp: To implement recursion, activation record should be implemented by providing dynamic memory allocation. This dynamic allocation is done from runtime stack. Heap is essential to allocate memory for data structures at run-time, not for recursion.

So statement 1 and 3 are correction.

19. In the context of modular software design, which one of the following combinations is desirable?
- (A) High cohesion and high coupling
  - (B) High cohesion and low coupling
  - (C) Low cohesion and high coupling
  - (D) Low cohesion and low coupling

Answer: (B)

Exp: Cohesion is a measure of internal strength within a module, whereas coupling is a measure of inter dependency among the modules. So in the context of modular software design there should be high cohesion and low coupling.

20. A system uses 3 page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

Answer: (6)

Exp: Page reference string is

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

Implementing LRU using 3 page frames

Total page faults = 6

4	1
7	2
6	7

21. What is the optimized version of the relation algebra expression  $\pi_{A_1}(\pi_{A_2}(\sigma_{F_1}(r)))$ , where  $A_1, A_2$  are sets of attributes in with  $A_1 \subset A_2$  and  $F_1, F_2$  are Boolean expressions based on the attributes in  $r$ ?

(A)  $\pi_{A_1}(\sigma(F_1 \wedge F_2)(r))$

(B)  $\pi_{A_1}(\sigma(F_1 \vee F_2)(r))$

(C)  $\pi_{A_2}(\sigma(F_1 \wedge F_2)(r))$

(D)  $\pi_{A_2}(\sigma(F_1 \vee F_2)(r))$

Answer: (A)

Exp:  $\pi$  is used to select a subset of attributes and  $\sigma_p$  is used to select subset of tuples matching the predicate  $P$ .

$$\pi_{A_1}(\pi_{A_2}(\sigma_{F_1}(\sigma_{F_2}(r)))) = \pi_{A_1}(\pi_{A_2}(\sigma(F_1 \wedge F_2)(r)))$$

and as  $A_1 \subset A_2$ , so final relation will be displaying values for attributes present in set  $A$ .

$$\pi_{A_1}(\pi_{A_2}(\sigma(F_1 \wedge F_2)(r))) = \pi_{A_1}(\sigma(F_1 \wedge F_2)(r))$$

22. A prime attribute of a relation scheme  $R$  is an attribute that appears

(A) in all candidate keys of  $R$ .

(B) in some candidate key of  $R$ .

(C) in a foreign keys of  $R$ .

(D) only in the primary key of  $R$ .

Answer: (B)

Exp: A prime attribute or key attribute of a relation scheme  $R$  is an attribute that appears in any of the candidate key of  $R$ , remaining attributes are known as non-prime or non-key tribute

23. In the following pairs of OSI protocol layer/sub-layer and its functionality, the **INCORRECT** pair is

(A) Network layer and Routing

(B) Data Link Layer and Bit synchronization

(C) Transport layer and End-to-end process communication

(D) Medium Access Control sub-layer and Channel sharing

Answer: (B)

- Exp: (a) One of the main functionality of Network Layer is Routing. So Option (a) is CORRECT.  
 (b) Bit Synchronization is always handled by Physical Layer of OSI model but not Data Link Layer. So Option (b) is INCORRECT.  
 (c) End – to – End Process Communication is handled by Transport Layer. So Option (c) is CORRECT.  
 (d) MAC sub layer have 3 types of protocols (Random, Controlled and Channelized Access).

24. A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is  
 (A) 0111110100 (B) 0111110101  
 (C) 0111111101 (D) 0111111111

Answer: (B)

Exp: Given 8 – bit delimiter pattern of 01111110.

Output Bit string after stuffing is 01111100101

↓

StuffedBit

Now, Input String is 0111110101

25. Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP V4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?  
 (i) TTL (ii) Checksum (iii) Fragment Offset  
 (A) (i) only (B) (i) and (ii) only  
 (C) (ii) and (iii) only (D) (i), (ii) and (iii)

Answer: (D)

Exp: While an IP Datagram is transferring from one host to another host, TTL, Checksum and Fragmentation Offset will be changed.

**Q. No. 26 – 55 Carry Two Marks Each**

26. An IP router implementing Classless Inter-domain routing (CIDR) receives a packet with address 131.23.151.76. The router’s routing table has the following entries:

Prefix	Output Interface Identifier
131.16.00/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is \_\_\_\_\_.

Answer: (1)

Exp: Given address 131.23.151.76 coming to the first field of given routing table

⇒ 131.16.0.0/12

131.0001 0111.151.76

131.0001 0000.0.0 (∴ given mask bits = 12)

⇒ 131.16.0.0 Matched

Coming to the 2<sup>nd</sup> field of given Routing table

⇒ 131.28.0.0/14

131.0001 0111.151.76

131.0001 0100.0.0 (∴ given mask bits = 14)

⇒ 131.20.0.0 Not matched.

Coming to the 3<sup>rd</sup> field of given Routing table

Error! Not a valid link. 131.19.0.0/16

131.0001 0111.151.76

131.0001 0111.0.0 (∴ given mask bits = 16)

⇒ 131.23.0.0 Not matched

Coming to the 4<sup>th</sup> field of given Routing table

⇒ 131.22.0.0/15

131.0001 0111.151.76

131.0001 0110.0.0 (∴ given mask bits = 15)

⇒ 131.22.0.0 Matched

We are getting 1<sup>st</sup> and 4<sup>th</sup> entries are matched so among them we have to pick up the longest mask bit, so output interface identifier is 1.

27. Every host in an IPv4 network has a 1-second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50-bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?

Answer: (256)

Exp: Given that each host has a globally unique IPv4 Address and we have to design 50 – bit unique Id. So, 50 – bit in the sense (32 + 18). So, It is clearly showing that IP Address (32 – bit) followed by 18 bits.

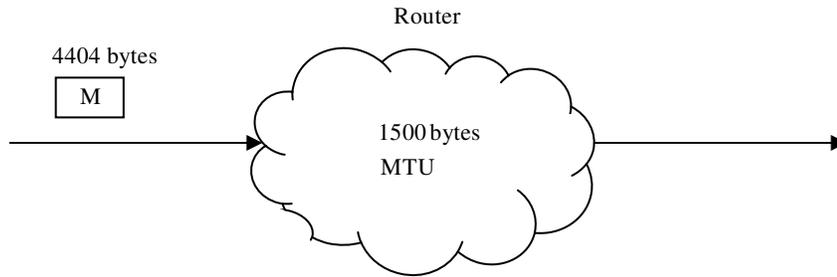
1000 unique Ids ⇒ 1Sec

$2^{18}$  unique Ids ⇒  $2^{18} / 1000 = 2^8 = 256$

28. An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are
- (A) MF bit: 0, Datagram Length: 1444; Offset: 370  
 (B) MF bit: 1, Datagram Length: 1424; Offset: 185  
 (C) MF bit: 1, Datagram Length: 1500; Offset: 370  
 (D) MF bit: 0, Datagram Length: 1424; Offset: 2960

Answer: (A)

Exp:



DataLength	1480	1480	1444
	+	+	
HeaderLength	20	20	
MF	1	1	0

29. Consider the transactions T1, T2, and T3 and the schedules S1 and S2 given below.

T1 : r1 (X) ; r1 (z) ; w1 (X) ; w1 (z)

T2 : r2 (X) ; r2 (z) ; w2 (z)

T3 : r3 (X) ; r3 (X) ; w3 (Y)

S1: r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)

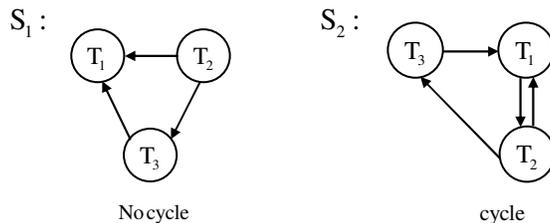
S2: r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)

Which one of the following statements about the schedules is **TRUE**?

- (A) Only S1 is conflict-serializable.
- (B) Only S2 is conflict-serializable.
- (C) Both S1 and S2 are conflict-serializable.
- (D) Neither S1 nor S2 is conflict-serializable.

Ans: (A)

Exp: Precedence graph for S<sub>1</sub> & S<sub>2</sub> are as follows



∴ Only S<sub>1</sub> is conflict serializable.

30. Consider the relational schema given below, where eId of the relation dependent is a foreign key referring to empId of the relation employee. Assume that every employee has at least one associated dependent in the dependent relation.

employee (empId, empName, empAge)

dependent (depId, eId, depName, depAge)

Consider the following relational algebra query

$$\Pi_{empId} (employee) - \Pi_{empId} (employee \bowtie_{(empId=eID) \wedge (empAge \leq depAge)} dependent)$$

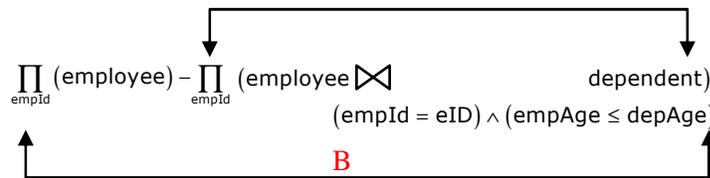
The above query evaluates to the set of empIds of employees whose age is greater than that of

- (A) some dependent. (B) all dependents.  
(C) some of his/her dependents. (D) all of his/her dependents

Answer: (D)

A

Exp:



Part A of the above given relational algebra query will give the set of empIds of those employees whose age is less than or equal to the age of some of his/her dependents.

Now when set of empIds of all employees minus set of empIds obtained from part A is done, then we get the set of empIds of employees whose age is greater than that of all of his/her dependents.

31. A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is \_\_\_\_\_.

Answer: (7)

Exp:	Maximum	Allocate	Need	Available
P <sub>1</sub>	-3	2	1	1
P <sub>2</sub>	-3	2	1	
P <sub>3</sub>	-3	2	1	

With the above given data, after allocating 2 units of tape to each process, with 1 available unit any of the 3 process can be satisfied in such a way, that No dead lock will be there.

So answer is 7 tape units.

- Q.32 An operating system uses *shortest remaining time first* scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

Process	Arrival Time	Burst Time
P1	0	12
P2	2	4
P3	3	6
P4	8	5

The average waiting time (in milliseconds) of the processes is \_\_\_\_\_.

Answer: (5.5)

Exp: The Gantt chart for SRTF scheduling algorithm is as follows:



$$\text{Average waiting time} = \frac{15+0+3+4}{4} = \frac{22}{4} = 5.5$$

33. Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is \_\_\_\_\_.

Answer: (122)

$$\text{Exp: } T_{\text{ave}} = H_1 \times (T_{\text{TLB}} + T_M) + (1 - H_1) \times (T_{\text{TLB}} + 2 \times T_M)$$

$$T_{\text{TLB}} = \text{time to search in TLB} = 10\text{ms}$$

$$T_M = \text{time to access physical memory} = 30\text{ms}$$

$$H_1 = \text{TLB hit ratio} = 0.6$$

$$T_{\text{ave}} = 0.6 \times (10 + 80) + (1 - 0.6) \times (10 + 2 \times 80)$$

$$T_{\text{ave}} = 0.6 \times 90\text{ms} + 0.4 \times (170)\text{ms}$$

$$T_{\text{ave}} = 54\text{ms} + 68\text{ms} = 122\text{ms}$$

34. Consider the basic block given below.

$$a = b + c$$

$$c = a + d$$

$$d = b + c$$

$$e = d - b$$

$$a = e + b$$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

- (A) 6 and 6                      (B) 8 and 10                      (C) 9 and 12                      (D) 4 and 4

Answer: (A)

Exp:

The given basic block can be rewritten as

$$a = b + c \qquad a = b + c$$

$$c = a + d \qquad c = b + c + d$$

$$d = b + c \Rightarrow d = b + b + c + d = 2b + c + d$$

$$e = d - b \qquad e = \cancel{b} + b + c + d - \cancel{b} = b + c + d$$

$$a = e + b \qquad a = b + b + c + d = 2b + c + d$$

From above simplification it is visible that e is same as c and final value of a is same as d. So the final basic block can be written as follows:

$$a = b + c$$

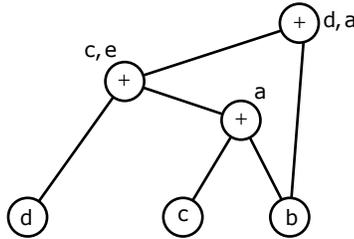
$$c = a + d$$

$$d = 2b + c + d$$

$$e = c$$

$$a = d$$

The DAG generated for the above basic block in as



Maximum number of nodes and edges in above DAG is (6,6)

35. Which one of the following problems is undecidable?
- (A) Deciding if a given context-free grammar is ambiguous.
  - (B) Deciding if a given string is generated by a given context-free grammar.
  - (C) Deciding if the language generated by a given context-free grammar is empty.
  - (D) Deciding if the language generated by a given context-free grammar is finite.

Answer: (A)

Exp: There were algorithms to find the membership of CFG (using CYK algorithm) and finiteness of CFG (using CNF graph) and emptiness. But there is no algorithm for ambiguity of CFG, so it is undecidable.

36. Consider the following languages over the alphabet  $\Sigma = \{0,1,c\}$

$$L_1 = \{0^n 1^n \mid n \geq 0\}$$

$$L_2 = \{wcw^r \mid w \in \{0,1\}^*\}$$

$$L_3 = \{ww^r \mid w \in \{0,1\}^*\}$$

Here  $w^r$  is the reverse of the string  $w$ . Which of these languages are deterministic Context-free languages?

- (A) None of the languages
- (B) Only  $L_1$
- (C) Only  $L_1$  and  $L_2$
- (D) All the three languages

Answer: (C)

Exp: For the languages  $L_1$  and  $L_2$  we can have deterministic push down automata, so they are DCFL's, but for  $L_3$  only non-deterministic PDA possible. So the language  $L_3$  is not a deterministic CFL.

37. Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers  $i, j$  with  $i < j$ . Given a shortcut  $i, j$  if you are at position  $i$  on the number line, you may directly move to  $j$ . Suppose  $T(k)$  denotes the smallest number of steps needed to move from  $k$  to 100. suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let  $y$  and  $z$  be such that  $T(9) = 1 + \min(T(y), T(z))$ . Then the value of the product  $yz$  is \_\_\_\_\_.

Answer: (150)

Exp: By definition,  $T(9) = \text{Dist. From 9 to 100}$

As given,  $T(9) = 1 + \min(T(y), T(z)) = 1 + \min(\text{Dist. from } y \text{ to } 100, \text{Dist. From } z \text{ to } 100)$

$\Rightarrow 1 = \text{Dist. from } 9 \text{ to } y / \text{Dist. From } 9 \text{ to } z$

$\Rightarrow$  There are only two such values-one is the simple one step on number line i.e. 10, and the other is the shortcut associated with 9 i.e. 15.

$\Rightarrow$  Therefore,  $y$  and  $z$  are 10 and 15 (in any order)

$\Rightarrow$  Product  $yz = 150$ . Answer

38. Consider the decision problem 2CNFSAT defined as follows:

{  $\phi \mid \phi$  is a satisfiable propositional formula in CNF with at most two literal per clause }

For example,  $\phi = (x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_3) \wedge (x_2 \vee x_4)$  is a Boolean formula and it is in 2CNFSAT.

The decision problem 2CNFSAT is

(A) NP-Complete.

(B) solvable in polynomial time by reduction to directed graph reachability.

(C) solvable in constant time since any input instance is satisfiable.

(D) NP-hard, but not NP-complete.

Answer: (B)

Exp: 2 SAT is in P. This we can prove by reducing 2 SAT to directed graph reachability problem which is known to be in P.

Procedure for reducing 2 SAT to reachability problem:

1. Let  $\phi$  be CNF with clauses of length 2 and let  $P$  be the set of propositional variables (literals) in  $\phi$
2. Build a graph  $G=(V,E)$  with  $V = P \cup \{\neg p \mid p \in P\}$  and  $(x,y) \in E$  iff there is a clause in  $\phi$  that is equivalent to  $x \rightarrow y$  (all the clauses are converted to equivalent implications and the graph built is called as implication graph)
3. Observe that  $\phi$  is unsatisfiable iff there is a  $p \in P$  such that there is both a path from  $p$  to  $\neg p$  and from  $\neg p$  to  $p$  in  $G$ .

This condition can be tested by running the reachability algorithm several times.

39. Suppose we have a balanced binary search tree  $T$  holding  $n$  numbers. We are given two numbers  $L$  and  $H$  and wish to sum up all the numbers in  $T$  that lie between  $L$  and  $H$ . Suppose there are  $m$  such numbers in  $T$ . If the tightest upper bound on the time to compute the sum is  $O(n^a \log^b n + m^c \log^d n)$ , the value of  $a + 10b + 100c + 1000d$  is \_\_\_\_\_.

Answer: (110)

Exp: It takes  $(\log n)$  time to determine numbers  $n_1$  and  $n_2$  in balanced binary search tree T such that

1.  $n_1$  is the smallest number greater than or equal to L and there is no predecessor  $n'_1$  of  $n_1$  such that  $n'_1$  is equal to  $n_1$ .
2.  $n_2$  is the largest number less than or equal to H and there is no successor of  $n'_2$  of  $n_2$  such that is equal to  $n_2$ .

Since there are m elements between  $n_1$  and  $n_2$ , it takes 'm' time to add all elements between  $n_1$  and  $n_2$ .

So time complexity is  $O(\log n + m)$

So the given expression becomes  $O(n^0 \log n + m^0 \log n)$

$$\text{And } a + 10b + 100c + 1000d = 0 + 10 \cdot 1 + 100 \cdot 1 + 1000 \cdot 1 = 10 + 100 = 110$$

Because  $a = 0, b = 1, c = 1$  and  $d = 0$

40. Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

- (A)  $(97 \times 97 \times 97)/100^3$  (B)  $(99 \times 98 \times 97)/100^3$   
 (C)  $(97 \times 96 \times 95)/100^3$  (D)  $(97 \times 96 \times 95)/(3! \times 100^3)$

Answer: (A)

Exp:A

$$P(\text{First insertion in such a way that first 3 slots are unfilled}) = \frac{{}^{97}C_1}{{}^{100}C_1} = \frac{97}{100}$$

**B**

$$P(\text{second insertion in such a way that first 3 slots are unfilled}) = \frac{{}^{97}C_1}{{}^{100}C_1} = \frac{97}{100} \quad [ \because \text{chaining is used to resolve collision, so second insertion can be done at same index as first index } ]$$

**C**

$$P(\text{Third insertion in such a way that first 3 slots are unfilled}) = \frac{{}^{97}C_1}{{}^{100}C_1} = \frac{97}{100} \quad [ \because \text{Third insertion can be done at same index as first or second index } ]$$

So Total prob.  $P(A) \times P(B) \times P(C)$

$$= \frac{97}{100} \times \frac{97}{100} \times \frac{97}{100} = \frac{(97 \times 97 \times 97)}{100^3}$$

41. Consider the pseudocode given below. The function Dosomething () takes as argument a pointer to the root of an arbitrary tree represented by the leftMostChild-rightSibling representation. Each node of the tree is of type treeNode.

```
typedef struct treeNode* treeptr;
Struct treeNode
{
    Treeptr leftMostchild, rightSibiling;
};
Int Dosomething (treeptr tree)
{
    int value =0;
    if (tree != NULL) {
        If (tree -> leftMostchild == NULL)
    else
        value = Dosomething (tree->leftMostchild);
    value = value + Dosometing (tree->rightsibiling);
}
return (value);
}
```

When the pointer to the root of a tree is passed as the argument to **DoSomething**, the value returned by the function corresponds to the

- (A) number of internal nodes in the tree.
- (B) height of the tree.
- (C) number of nodes without a right sibling in the tree.
- (D) number of leaf nodes in the tree.

**Answer: (D)**

**Exp:** The key to solving such questions is to understand or detect where/by what condition the value (or the counter) is getting incremented each time.

Here, that condition is if (tree→leftMostchild == Null)

- ⇒ Which means if there is no left most child of the tree (or the sub-tree or the current node-as called in recursion)
- ⇒ Which means there is no child to that particular node (since if there is no left most child, there is no child at all).
- ⇒ Which means the node under consideration is a leaf node.
- ⇒ The function recursively counts, and adds to value, whenever a leaf node is encountered.
- ⇒ The function returns the number of leaf nodes in the tree.

42. Consider the C function given below. Assume that the array listA contains n (> 0) elements, sored in ascending order.

```
int ProcessArray (int * listA, int x, int n)
```

```

{
  Int l, j, k;
  i = 0;
  j = n - 1;
  do {
    k = (i + j) / 2;
    if (x <= listA [k])
      j = k - 1;
    If (listA [k] <= x)
      i = k+1;
  }while (l <= j);
  If (listA [k] == x)
    return (k) ;
  else
    return -1;
}

```

Which one of the following statements about the function **ProcessArray** is **CORRECT**?

- (A) It will run into an infinite loop when x is not in listA.
- (B) It is an implementation of binary search
- (C) It will always find the maximum element in listA.
- (D) It will return - 1 even when x is present in listA.

**Answer: (B)**

**Exp:** By the logic of the algorithm it is clear that it is an attempted implementation of Binary Search. So option C is clearly eliminated. Let us now check for options A and D.

A good way to do this is to create small dummy examples (arrays) and implement the algorithm as it is. One may make any array of choice. Running iterations of the algorithm would indicate that the loop exits when the x is not present. So option A is wrong. Also, when x is present, the correct index is indeed returned. D is also wrong. Correct answer is B. It is a correct implementation of Binary Search.

43. An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 ns, and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are  $P$  and  $Q$  nanoseconds, respectively. The value of  $P/Q$  is \_\_\_\_\_

Answer: ( 1.54 )

Exp:

	No. of stages	Stall cycle	Stall frequency	Clock period	Avg. access time
Old design	5	2	20%	2.2ns	P
New design	8	5	20%	1 ns	Q

$$P = \left[ 80\% (1\text{clock}) + 20\% \left( \frac{1}{\text{completion}} + \frac{2}{\text{stall clock}} \right) \right] \times T_{c-p}$$

$$P = (.8 + .6) \times 2.2\text{ns} = 3.08\text{ns}$$

$$Q = \left[ 80\% (1\text{clock}) + 20\% \left( \frac{1}{\text{completion}} + \frac{5}{\text{stall clock}} \right) \right] \times T_{c-p}$$

$$Q = (.8 + .12) \times 1\text{ns} = 2\text{ns}$$

$$\text{So the value of } \frac{P}{Q} = \frac{3.08\text{ns}}{2\text{ns}} = 1.54$$

44. The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit-ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is \_\_\_\_\_.

Answer: (1.68)

$$\text{Exp: Total instruction} = \frac{100 \text{ instruction}}{\text{fetch operation}} + \frac{60 \text{ memory}}{\text{operand read operation}} + \frac{40 \text{ memory}}{\text{operand write operation}}$$

$$= 200 \text{ instructions (operations)}$$

Time taken for fetching 100 instructions (equivalent to read)

$$= 90 * 1\text{ns} + 10 * 5\text{ns} = 140\text{ns}$$

$$\begin{aligned} \text{Memory operand Read operations} &= 90\% (60) * 1\text{ns} + 10\% (60) * 5\text{ns} \\ &= 54\text{ns} + 30\text{ns} = 84\text{ns} \end{aligned}$$

$$\begin{aligned} \text{Memory operands write operation time} &= 90\% (40) * 2\text{ns} + 10\% (40) * 10\text{ns} \\ &= 72\text{ns} + 40\text{ns} = 112\text{ns} \end{aligned}$$

$$\text{Total time taken for executing 200 instructions} = 140 + 84 + 112 = 336\text{ns}$$

$$\therefore \text{Average memory access time} = \frac{336\text{ns}}{200} = 1.68\text{ns}$$



I. By Trapezoidal rule

$$\int_0^1 x^2 dx = \frac{h}{2} [(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$= \frac{0.25}{2} [(0+1) + 2(0.0625 + 0.25 + 0.5625)] = 0.34375$$

II. By Simpson's  $\frac{1}{3}$  rule

$$\int_0^1 x^2 dx = \frac{h}{3} [(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)]$$

$$= \frac{0.25}{3} [(0+1) + 2(0.25) + 4(0.0625 + 0.5625)] = \frac{1}{3}$$

Exact value  $\int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}$

47. The value of the integral given below is

$$\int_0^\pi x^2 \cos x dx$$

(A)  $-2\pi$  (B)  $\pi$  (C)  $-\pi$  (D)  $2\pi$

Answer: (A)

Exp:  $\int_0^\pi x^2 \cos x dx = x^2(\sin x) - 2x(-\cos x) + 2(-\sin x) \Big|_0^\pi$

$$= (\pi^2 \sin \pi + 2\pi \cos \pi - 2 \sin \pi) - (0+0+0) = -2\pi$$

48. Let S be a sample space and two mutually exclusive events A and B be such that  $A \cup B = S$ . If  $P(\cdot)$  denotes the probability of the event, the maximum value of  $P(A)P(B)$  is \_\_\_\_\_

Answer: (0.25)

Exp: Given

$$A \cup B = S$$

$$\Rightarrow P(A \cup B) = P(S) = 1$$

$$\Rightarrow P(A) + P(B) = 1 \quad (\because A \text{ \& } B \text{ are mutually exclusive})$$

$$\Rightarrow P(B) = 1 - P(A)$$

$$\text{Maximum value of } P(A)P(B) = ?$$

$$\text{Maximum value of } P(A) [1 - P(A)] = ?$$

$$\text{Let } P(A) = X$$

$$\text{Let } f(x) = x(1-x) = x - x^2$$

for  $f(x)$  maximum  $\Rightarrow f'(x)=0 \Rightarrow 1-2x=0 \Rightarrow x=\frac{1}{2}$

$$f''(x) = -2; \quad f''\left(\frac{1}{2}\right) < 0$$

$\therefore f(x)$  has maximum

At  $x = \frac{1}{2}$  and maximum value

$$= f\left(\frac{1}{2}\right) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$$

49. Consider the set of all functions  $f : \{0,1,\dots,2014\} \rightarrow \{0,1,\dots,2014\}$  such that  $f(f(i)) = i$ , for  $0 \leq i \leq 2014$ . Consider the following statements.

P. For each such function it must be the case that for every  $i$ ,  $f(i) = i$ ,

Q. For each such function it must be the case that for some  $i$ ,  $f(i) = i$ ,

R. Each such function must be onto.

Which one of the following is CORRECT?

(A) P, Q and R are true (B) Only Q and R are true

(C) Only P and Q are true (D) Only R is true

Answer: (B)

Exp: Let us consider a function (counter example) as

$$f(0) = 1, f(1) = 0, f(2) = 3, f(3) = 2, \dots, f(2012) = 2013,$$

$$f(2013) = 2012 \text{ and } f(2014) = 2014$$

Clearly  $f(f(i)) = i$  for  $0 \leq i \leq 2014$

Here  $f(i) \neq i$  for every  $i$  and  $f(i) = i$  for some  $i$

Also  $f$  is onto

Hence, only Q and R are true

50. There are two elements  $x, y$  in a group  $(G, *)$  such that every element in the group can be written as a product of some number of  $x$ 's and  $y$ 's in some order. It is known that

$$x * x = y * y = x * y * x * y = y * x * y * x = e$$

where  $e$  is the identity element. The maximum number of elements in such a group is

\_\_\_\_\_.

Answer: (4)

Exp:  $x * x = e \Rightarrow x$  is its own inverse

$y \times y = e \Rightarrow y$  is its own inverse  
 $(x \times y) \times (x \times y) = e \Rightarrow (x \times y)$  is its own inverse  
 $(y \times x) \times (y \times x) = e \Rightarrow (y \times x)$  is its own inverse  
 also  $x \times x \times e = e \times e$  can be rewritten as follows  
 $x \times y \times y \times x = e \times y \times y \times e = e [\because y \times y = e]$   
 $(x \times y) \times (y \times x) = e$  shows that  $(x \times y)$  and  $(y \times x)$   
 Are each other's inverse and we already know that  
 $(x \times y)$  and  $(y \times x)$  are inverse of its own.  
 As per  $(G, *)$  to be group any element should have  
 only one inverse element (unique)  
 This process  $x \times y = y \times x$  (is one element)  
 So the elements of such group are 4 which are  $\{x, y, e, x \times y\}$

51. If  $G$  is a forest with  $n$  vertices and  $k$  connected components, how many edges does  $G$  have?

- (A)  $\lfloor n/k \rfloor$  (B)  $\lceil n/k \rceil$  (C)  $n - k$  (D)  $n - k + 1$

Answer: (C)

Exp: Let  $n_1, n_2, \dots, n_k$  be the number of vertices respectively in  $K$  connected components of a forest  $G$ , then  $n_1 - 1, n_2 - 1, \dots, n_k - 1$  be the number of edges respectively in  $K$  connected components and  $n_1 + n_2 + \dots + n_k = n$  (number of vertices in  $G$ )

Hence, number of edges in  $G =$  number of edges in  $K$  connected components  
 $= (n_1 - 1) + (n_2 - 1) + \dots + (n_k - 1) = n - k$

52. Let  $\delta$  denote the minimum degree of a vertex in a graph. For all planar graphs on  $n$  vertices with  $\delta \geq 3$ , which one of the following is **TRUE**?

- (A) In any planar embedding, the number of faces is at least  $\frac{n}{2} + 2$   
 (B) In any planar embedding, the number of faces is less than  $\frac{n}{2} + 2$   
 (C) There is a planar embedding in which the number of faces is less than  $\frac{n}{2} + 2$   
 (D) There is a planar embedding in which the number of faces is at most  $\frac{n}{\delta + 1}$

Answer: (A)

Exp: We know that  $v + r = e + 2 \Rightarrow e = n + r - 2 \dots(1)$

Where  $V = n$  (number of vertices);  $r =$  number of faces and

$e =$  number of edges

Given,  $\delta \geq 3$  then  $3n \leq 2e$

$$\Rightarrow e \geq \frac{3n}{2}$$

$$\Rightarrow n + r - 2 \geq \frac{3n}{2} \text{ (using (1))}$$

$$\Rightarrow r \geq \frac{3n}{2} - n + 2 \Rightarrow r \geq \frac{n}{2} + 2$$

$\therefore$  Number of faces is atleast  $\frac{n}{2} + 2$

53. The CORRECT formula for the sentence, “not all rainy days are cold” is

(A)  $\forall d ( \text{Rainy}(d) \wedge \sim \text{Cold}(d) )$  (B)  $\forall d ( \sim \text{Rainy}(d) \rightarrow \text{Cold}(d) )$

(C)  $\exists d ( \sim \text{Rainy}(d) \rightarrow \text{Cold}(d) )$  (D)  $\exists d ( \text{Rainy}(d) \wedge \sim \text{Cold}(d) )$

Answer: (D)

Exp: Given statement is  $\sim \forall d [ r(d) \rightarrow c(d) ]$

$$\equiv \sim \forall d [ \sim r(d) \vee c(d) ]$$

$$\equiv \exists d [ r(d) \wedge \sim c(d) ]$$

(Since  $p \rightarrow q \equiv \sim p \vee q$  and let  $r(d)$  be rainy day,  $c(d)$  be cold day)

54. Consider the following relational schema:

Employee (empId, empName, empDept)

Customer (custId, custName, salesRepId, rating)

SalesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

SELECT empName

FROM employee E

WHERE NOT EXISTS (SELECT custId

FROM customer C

WHERE C. salesRepId = E. empId

AND C. rating < > 'GOOD')

- (A) Names of all the employees with at least one of their customers having a 'GOOD' rating.
- (B) Names of all the employees with at most one of their customers having a 'GOOD' rating.
- (C) Names of all the employees with none of their customers having a 'GOOD' rating.
- (D) Names of all the employees with all their customers having a 'GOOD' rating.

Answer: (D)

Exp: The outer query will return the value (names of employees) for a tuple in relation E, only if inner query for that tuple will return no tuple (usage of NOT EXISTS).

The inner query will run for every tuple of outer query. It selects cust-id for an employee e, if rating of customer is NOT good. Such an employee should not be selected in the output of outer query.

So the query will return the names of all those employees whose all customers have GOOD rating.

55. Let  $\oplus$  denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q.

$$F(P,Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

The equivalent expression for F is

- (A)  $P+Q$
- (B)  $\overline{P+Q}$
- (C)  $P \oplus Q$
- (D)  $\overline{P \oplus Q}$

Answer: (D)

Exp:

$$\begin{aligned} F(P,Q) &= ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0)) \\ &= (\overline{P} \oplus (P\overline{Q} + \overline{P}Q)) \oplus ((P\overline{Q} + \overline{P}Q) \oplus Q) \\ &= [\overline{P}(PQ + \overline{P}\overline{Q}) + P(P\overline{Q} + \overline{P}Q)] \oplus [(PQ + \overline{P}\overline{Q})Q + (P\overline{Q} + \overline{P}Q)\overline{Q}] \\ &= (\overline{P}\overline{Q} + P\overline{Q}) \oplus (PQ + P\overline{Q}) = \overline{Q} \oplus P = PQ + \overline{P}\overline{Q} = \overline{P \oplus Q} \end{aligned}$$