Test Code MS (Short answer type) 2005
Syllabus for Mathematics

Permutations and Combinations. Binomials and multinomial theorem. Theory of equations. Inequalities.

Determinants, matrices, solution of linear equations and vector spaces.

Trigonometry, Coordinate geometry of two and three dimensions.

Geometry of complex numbers and De Moivre's theorem. Elements of set theory.

Convergence of sequences and series. Power series. Functions, limits and continuity of functions of one or more variables.

Differentiation, Leibnitz formula, maxima and minima, Taylor's theorem. Differentiation of functions of several variables. Applications of differential Calculus.

Indefinite integral, Fundamental theorem of Calculus, Riemann integration and properties. Improper integrals. Differentiation under the integral sign. Double and multiple integrals and Applications.

Syllabus for Statistics

Probability and Sampling Distributions

Notions of sample space and probability, combinatorial probability, conditional probability and independence, random variable and expectations, moments, standard discrete and continuous distributions, sampling distributions of statistics based on normal samples, central limit theorem, approximation of Binomial to Normal or Poisson law. Bivariate normal and multivariate normal distributions.

Descriptive Statistics

Descriptive statistical measures, graduation of frequency curves, product-moment, partial and multiple correlation, Regression (bivariate and multivariate).

Inference

Elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Tests of regression. Elements of non-parametric inference.

Design of Experiments and Sample Surveys

Basic designs (CRD/RBD/LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques (SRSWR/SRSWOR) including stratification; ratio and regression methods of estimation.

Sample Questions

1. Let A be a $n \times n$ upper triangular matrix such that $AA^T = A^T A$. Show that A is a diagonal matrix.

2. Let X and Y be independent random variables with X having a binomial distribution with parameters 5 and 1/2 and Y having a binomial distribution with parameters 7 and 1/2. Find the probability that $|X - Y|$ is even.

3. Let A be a $n \times n$ orthogonal matrix, where n is even and suppose $|A| = -1$, where $|A|$ denotes the determinant of A. Show that $|I - A| = 0$, where I denotes the $n \times n$ identity matrix.

4. Let f be a non-decreasing, integrable function defined on $[0, 1]$. Show that

$$\left(\int_0^1 f(x)dx \right)^2 \leq 2 \int_0^1 x(f(x))^2 dx.$$

5. Suppose X and U are independent random variables with

$$P(X = k) = \frac{1}{N+1}, \qquad k = 0, 1, 2, \ldots, N,$$

and U having a uniform distribution on $[0, 1]$. Let $Y = X + U$.

a) For $y \in \mathbb{R}$, find $P(Y \leq y)$.

b) Find the correlation coefficient between X and Y.

6. Consider a randomized block design with v treatments, each replicated r times. Let t_i be the treatment effect of the i-th treatment. Find $Cov(\sum l_i \hat{t}_i, \sum m_i \hat{t}_i)$ where $\sum l_i \hat{t}_i$ and $\sum m_i \hat{t}_i$ are the best linear unbiased estimators of $\sum l_i t_i$ and $\sum m_i t_i$ respectively and $\sum l_i = \sum m_i = \sum l_i m_i = 0$.

7. Suppose that $X_1, \ldots, X_n$ is a random sample of size $n \geq 1$ from a Poisson distribution with parameter λ. Find the minimum variance unbiased estimator of $e^{-\lambda}$.

8. Suppose $X_1, \ldots, X_n$ constitute a random sample from a population with density

$$f(x, \theta) = \frac{x}{\theta^2} \exp\left(-\frac{x^2}{2\theta^2}\right), \quad x > 0, \theta > 0.$$

Find the Cramer-Rao lower bound to the variance of an unbiased estimator of θ^2.

9. Let X_1, X_2, X_3 be independent random variables such that X_i is uniformly distributed in $(0, i\theta)$ for $i = 1, 2, 3$. Find the maximum likelihood estimator of θ and examine whether it is unbiased for θ.

10. Suppose that in 10 tosses of a coin we get 7 heads and 3 tails. Find a test at level $\alpha = 0.05$ to test that the coin is fair against the alternative that the coin is more likely to show up heads. Find the power function of this test.

11. Let X and Y be two random variables with joint probability density function

$$f(x, y) = \begin{cases} 1 & \text{if } -y < x < y,\ 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the regression equation of Y on X and that of X on Y.