DIPLOMA IN NAUTICAL SCIENCE

Term-End Examination

June, 2007

BNA-011 : APPLIED MATHEMATICS

Time : 2 hours   Maximum Marks : 70

Note : All questions are **compulsory**. Use of non-programmable scientific calculator is allowed.

1. (a) If $1, \omega, \omega^2$ are cube roots of unity, show that
   $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = 32$ 5

   (b) Evaluate

   $$\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$ 5

2. (a) Solve :

   $(e^x + e^{-x})\dfrac{dy}{dx} = (e^x - e^{-x})$ 5

   (b) If $y = \sqrt{\dfrac{1-x}{1+x}}$ prove that $(1 - x^2)\dfrac{dy}{dx} + y = 0.$ 5

3. (a) Solve :

   $$\int x^2 \tan^{-1} x \, dx$$ 5

(b) Evaluate :

$$\int_0^{\sqrt{2}} \sqrt{2 - x^2} \ dx$$

5

4. (a) Find a vector of magnitude 19 which is perpendicular to both the vector $4\,\hat{i} - \hat{j} + 8\,\hat{k}$ and $-\hat{j} + \hat{k}$.

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(b) Find the equation of the circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4).

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5. (a) In a spherical triangle PQR angles P, Q and R are 58° 30′, 100° 24′ and 74° 00′ respectively. Calculate side p.

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(b) In a quadrantal spherical triangle ABC, side b = 90°, angles A and B are 65° 30′ and 75° 15′ respectively. Calculate side c and angle C.

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6. (a) Find the vector equation of the plane through the point $5\,\hat{i} - 2\,\hat{j} - 3\,\hat{k}$ and perpendicular to each of the planes $\vec{r} \cdot (2\,\hat{i} - \hat{j} + 2\,\hat{k}) = 0$ and $\vec{r} \cdot (\hat{i} + 3\,\hat{j} - 5\,\hat{k}) + 3 = 0$.

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(b) Find the coefficients of $a^5 b^7$ in $(a - 2b)^{12}$.

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7. (a) From a group of 5 men and 4 women, 3 persons are selected at random to form a committee. Find the probability that the committee contains (i) two men and one woman (ii) no women.

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(b) Find the line of regression of x on y for the data given below :

x	4	2	3	4	2
y	2	3	2	4	4

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