S.E / Electronic & Telecom / Sem IV

Applied Maths IV

57-p3-upq-Con No. File

Con. 5585-10.

GT-6522

(3 Hours)

[Total Marks : 100

NB: 1. QUESTION NO 1 IS COMPULSORY.

2. ATTEMPT ANY FOUR QUESTIONS OUT OF REMAINING SIX QUESTIONS.

3. FIGURES TO THE RIGHT INDICATE FULL MARKS.

Q.1 a) If $\bar{F} = xy\,\hat{i} - z\,\hat{j} + x^2\hat{k}$ then evaluate $\int \bar{F} \times d\bar{r}$ over C. (20)

Where C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to 1.

b) Prove that $\int J_3(x)\,dx = -2\,\frac{J_1(x)}{x} - J_2(x)$.

c) Show that one of the Eigen values of the matrix A is zero

Where $A = \begin{bmatrix} 123 & 111 & 222 \\ 201 & -86 & -121 \\ 324 & 25 & 101 \end{bmatrix}$

d) Evaluate $\int \bar{z}\,dz$ over C where C is the upper half of the circle $r = 1$.

Q.2 a) Evaluate $\int \frac{4z-1}{(z^2+z-2)}dz$ over C where C is $\frac{x^2}{4} + \frac{y^2}{9} = 1$. (6)

b) Find the invariant points of the bilinear transformation $w = -\left(\frac{2z+4i}{iz+1}\right)$ & prove that these 2 points together with any point z & its image w form a set of points whose cross ratio is constant. (7)

c) Prove that $\int_0^1 x^{\frac{5}{2}} J_{\frac{3}{2}}(ax)\,dx = \frac{1}{a}\,J_{\frac{5}{2}}(a)$. (7)

Q.3 a) Find the Eigen values & the Eigen vectors of the matrix A & A^{-1} (6)

Where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

b) Evaluate $\int \frac{ze^z}{(z-1)^3}\,dz$ over C where C is $|z| = 2$. (7)

c) Apply Stoke's theorem to evaluate $\int y\,dx + z\,dy + x\,dz$ over C where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ & $x + z = a$. (7)

[TURN OVER

Q.4 a) Reduce the following quadratic form to canonical form & find the rank & signature **(6)**

$2x^2 + y^2 - 3z^2 + 12xy - 4xz - 8yz$.

b) Prove that $f(z) = |z|^2$ is not analytic anywhere but satisfies C-R equations at z = 0. **(7)**

c) Using Green's theorem evaluate $\int (2x^2 - y)dx + (y^2 + 2x)dy$ over C where C is the Boundary of the region bounded by $y = x^2$; $y = 1$; $x = 0$.

Q. 5 a) Expand $f(z) = \frac{1}{(z-2)(z-1)}$ in the regions **(6)**

(i) $1 < |z-1| < 2$ (ii) $1 < |z-3| < 2$

b) Using Cayley- Hamilton theorem find $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ **(7)**

where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

c) Show that $\bar{F} = (ye^{xy}\cos z)\hat{\imath} + (xe^{xy}\cos z)\hat{\jmath} - (e^{xy}\sin z)\hat{k}$ **(7)**

is irrotational & also find scalar potential for $\bar{F}$.

Q.6 a) Find the image of the area bounded between $x^2 + y^2 = 16$ & $x^2 + y^2 = 81$ in the z- plane Into the w - plane under the transformation w = log (z). **(6)**

b) If $A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$ then find $\cos^{-1} A$. **(7)**

c) Find analytic function f(z) = u + iv such that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ & $f(\frac{\pi}{2}) = 0$. **(7)**

Q. 7 a) If $B = \begin{bmatrix} 123 & 231 & 312 \\ 231 & 312 & 123 \\ 312 & 123 & 231 \end{bmatrix}$ then prove that **(6)**

(i) One of the Eigen values of B is 666.

(ii) One of the Eigen values of B is negative.

b) Using Gauss divergence theorem prove that **(7)**

$\iint (y^2z^2\hat{\imath} + z^2x^2\hat{\jmath} + z^2y^2\hat{k}) \cdot \hat{N}\, ds = \frac{\pi}{12}$.

c) Expand $f(x) = 4x - x^3$ in (0,2) in a series

$4x - x^3 = 8\sum \frac{J_2(2\lambda_n)}{\lambda_n^2\ J_2^2(2\lambda_n)} J_1(\lambda_n x)$ Where $\lambda_i' s$ are positive roots of $J_1(2\lambda) = 0$. **(7)**