

SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech- CSE/IT/ECE/EEE/MECH/M&P/ EIE/E&C/
CHEM/ CIVIL/ETCE/AERO

Title of the paper: Engineering Mathematics - I

Time: 3 Hours

Semester: I

Max. Marks: 80

Sub.Code: 6C0002

Date: 07-12-2006

Session: FN

PART – A

(10 x 2 = 20)

Answer ALL the Questions

1. Given that $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, find the eigen values of A^T .
2. Show that the square matrix $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ satisfies its own characteristics equation.
3. Find the sum of the series
 $(\log_e 2) + \frac{(\log_e 2)^2}{2!} + \frac{(\log_e 2)^3}{3!} + \dots \infty.$
4. When x is large, prove that $\sqrt{x^2+1} - \sqrt{x^2-1} \cong \frac{1}{x}$.
5. Find the radius of curvature at $(0, 1)$ on the curve $y = e^x$.
6. Find the envelop of $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ where θ is the parameter.
7. If $u = \frac{x}{y} + \frac{Y}{Z} + \frac{Z}{X}$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$.
8. Find the Jacobian $\frac{\partial(r, \vartheta)}{\partial(x, y)}$ if $x = r \cos \theta$, $y = r \sin \theta$.

9. Find the particular integral of $(D^4 - 2D^3 + D^2)y = e^x$.

10. Solve $(D - 2)^2 y = e^{2x}$ where $D = \frac{d}{dx}$

PART – B (5 x 12 = 60)
Answer ALL the Questions

11. Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -8 \\ 10 & -4 & -6 \end{bmatrix}$$

(or)

12. Diagonalise the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ by similarity transformation.

13. (a) Find the sum to infinity of $\frac{7}{72} + \frac{7.28}{72.96} + \frac{7.28.49}{72.96.120} + \dots$

(b) Find the coefficient of x^r in the expansion of $\frac{1+2x+3x^2}{e^{2x}}$

(or)

14. (a) When n is large, show that $\left(\frac{n+1}{n-1}\right)^{n/2} = e \left(1 + \frac{1}{3n^2}\right)$

(b) Find the sum to infinity of the series $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots \infty$

15. (a) Find the evolute of the parabola $x^2 = 4ay$.

(b) Find the envelop of $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relation $a^2 + b^2 = c^2$.

(or)

16. Find the center of curvature and circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

17. (a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$.

(b) Expand $e^x \cos y$ in powers of x and y upto terms of third degree using Taylor's theorem.

(or)

18. (a) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if

$$y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3},$$

(b) A rectangular box, open at the top, is to have a volume of 32 C.C. Find the dimensions of the box that requires the least material for its construction.

19. (a) Solve $(D^2 + 16)y = e^{-3x} + \cos 4x$.

(b) Solve $\frac{dx}{dt} + y - 1 = \sin t$; $\frac{dy}{dt} + x = \cos t$

(or)

20. (a) Solve the equation $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = x \log x$.

(b) Solve using the method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x.$$