

Name :

Roll No. :

Invigilator's Signature :



CS/B.Tech(ECE,EE,EIE,EEE,PWE,BME,ICE)/SEM-3/M-302/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

i) The probability that a leap-year selected at random will contain 53 sundays is

a) $\frac{3}{7}$

b) $\frac{2}{7}$

c) $\frac{5}{7}$

d) $\frac{4}{9}$

ii) If a coin is tossed 6 times in succession, the probability of getting at least one head is

a) $\frac{63}{64}$

b) $\frac{3}{64}$

c) $\frac{7}{63}$

d) None of these.

iii) The probability that the 4 children of a family have different birthdays is

- a) 0.9836 b) 0.4735
c) 0.9 d) 0.757.

iv) A tree has n vertices. The number of its edges is

- a) $n + 1$ b) $n - 1$
c) $2n$ d) none of these.

v) The value of m such that $3y - 5x^2 + my^2$ is a harmonic function is

- a) 5 b) -5
c) 0 d) 3.

vi) Let X and Y be two random variables such that $Y = a + bx$ where a and b are constants. Then, $\text{Var}(y)$ is

- a) $b^2 \text{Var}(X)$ b) $\text{Var}(X)$
c) $a^2 \text{Var}(X)$ d) $(b/a) \text{Var}(X)$.

vii) The value of $\int_C \frac{dz}{z+3}$ where C is a circle $|z| = 1$ is

- a) 0 b) 1
c) 2 d) -1.

viii) If $f(z) = \frac{1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

- a) 3 b) 2
c) 1 d) 4.

ix) The order of the pole $z = 0$ of the function $\frac{\sin z}{z^3}$ is

a) 1 b) 2

c) 3 d) 4.

x) The period of the function $f(x) = \sin 2\pi x$ is

a) $\frac{1}{2}$ b) 1

c) 0 d) $\frac{1}{3}$.

xi) If $f(x) = x \sin x$, $-\pi \leq x \leq \pi$, be presented in Fourier

series as $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$,

then the value of a_0 will be

a) 2 b) 0

c) 4 d) 1.

xii) If two variables x and y are uncorrelated, then r_{xy} is

a) 1 b) 2

c) 3 d) 0.

xiii) If $x = 4y + 5$ be a regression line of x on y then b_{xy} is

a) $\frac{1}{4}$ b) 4

c) 0 d) 1.

GROUP - B

(Short Answer Type Questions)

Answer any three of the following. $3 \times 5 = 15$

2. Show that $f(x)$ given by

$$f(x) = x; 0 < x < 1$$

$$= k - x; 1 < x < 2$$

$$= 0; \text{ elsewhere,}$$

is a probability density function for a suitable value of k .

Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.

3. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

4. Evaluate $\int_C \frac{3z^2 - 2}{z - 1} dz$, where c is the circle $|z| = \frac{1}{2}$.

5. An urn contains 3 white and 5 black balls. One ball is drawn and its colour is unnoted, kept aside and then another ball is drawn. What is the probability that it is (i) black (ii) white ?

6. Find the mean and standard deviation of a binomial distribution.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If A and B are mutually independent events, prove that A^c and B^c are also mutually independent events.
- b) There are three identical urns containing white and black balls. The first urn contains 3 white and 4 black balls, the 2nd urn contains 4 white and 5 black balls and the 3rd urn contains 2 white and 3 black balls. An urn is chosen at random and a ball is drawn from it. If the drawn ball is white, what is the probability that the 2nd urn chosen ?
- c) A random variable X has the following p.d.f.

$$f(x) = cx^2 \quad 0 \leq x \leq 1$$

$$= 0, \text{ otherwise.}$$

Find (i) c (ii) $P\left(0 \leq X \leq \frac{1}{2}\right)$. 5 + 5 + 5

8. a) Find the Fourier series expansion of the periodic function of period 2π ,

$f(x) = x^2, -\pi \leq x \leq \pi$. Hence deduce

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots + \dots = \frac{\pi^2}{12}$$

- b) The following marks have been obtained by students in Mathematics and Statistics (out of 100) :

Maths	45	55	56	58	60	65	68	70	75	80	85
Stats	56	50	48	60	62	64	65	70	74	82	90

Compute the co-efficient of correlation for the above data. Find also the equations of the lines of regrssion.

7 + 8

9. a) Solve

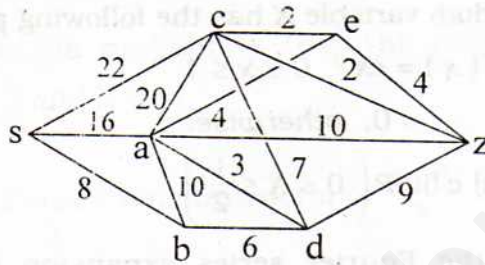
$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,$$

if $u(0, t) = 0$, $u(x, 0) = e^{-x}$, $x > 0$, $u(x, t)$ is unbounded.

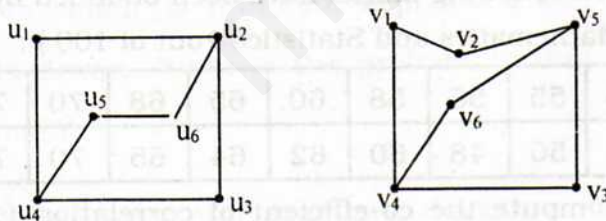
b) If $f(z)$ is a regular function of z , then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad 8 + 7$$

10. a) Apply Dijkstra's algorithm to determine a shortest path between s to z in the following graph :



b) Define isomorphism of two graphs. Examine whether the following graph G and G' are isomorphic. Give reasons.



8 + 7

11. a) Use residue theorem to evaluate $\int_C \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$

around the circle $|z| = 2$.

b) Expand the function $f(z) = \frac{1}{(z^2 + 1)(z + 2)}$ in the region $|z| < 1$.

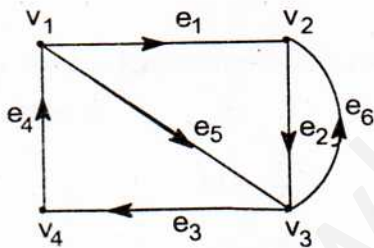
c) Show that the function $f(z) = \begin{cases} \frac{3xy^2}{x^2 + y^2} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$

is continuous at $z = 0$.

5 + 7 + 3

12. a) Show that a simple graph with n vertices and k -components can have at most $\frac{(n - k)(n - k + 1)}{2}$ edges.

b) Find the incidence matrix of the following graph.



c) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} 1 & \text{for } 0 < x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$

and hence evaluate the integral

$$\int_0^{\infty} \frac{1 - \cos p\pi}{p} \sin px dp.$$

5 + 5 + 5

