Directorate of Correspondence Course

First Year B.Sc. Degree Examinations

August /September 2010

(New Scheme)

MATHEMATICS

Paper - I

Time: 3 hrs.] [Max.Marks : 90

Note: ***Answer any SIX full questions of the following choosing at least ONE from each Part.***

PART - A

A. ***Answer the following.*** 2 Marks

1. a) i) Find $\phi(1026)$

ii) Find the greatest common divisor of 592 and 252. 2 Marks

b) Prove that the relation of congruence defined on the set Z of integers is an equivalence relation. 5 Marks

c) Find the remainder when 159^{7654} is divided by 23. 6 Marks

2. a) i) If $A = \{1, 2\}$ B [illegible] $.6\}$ find $A \times B$ and $B \times A$. 2 Marks

ii) Let $f : R \rightarrow R$ and $g : R \rightarrow R$ defined by $f(x) = 4x - 1$ and $g(x) = cos\ x$. Show that $fog \neq gof$. 2 Marks

b) Prove that the composition of two bijective functions is also bijective. 5 Marks

c) If $f : R \longrightarrow R$ and $g : R \longrightarrow R$ are defined by $f(x) = 2x - 1$ and $g(x) = 5 - 3x$. Verify $(gof)^{-1} = f^{-1}og^{-1}$ 6 Marks

PART - B

3. a) i) Define continuity of a function at $x = a$ and give an example. 2 Marks

ii) If $y = cos(ax + b)$ find y_n. 2 Marks

b) If $y = (x + \sqrt{x^2 + 1})^m$ Prove that $(x^2 + 1)y_2 + xy_1 - m^2 y = 0$ 5 Marks

c) If $cos^{-1}(y/b) = log(x/n)^n$. Prove that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + 2n^2\ y_n = 0$ 6 Marks

Contd.... 2

4. a) i) Find the angle between the radius vector and the tangent for the curve $r = ae^{\theta cot\alpha}$ where α is a constant. **2 Marks**

ii) Calculate $\frac{ds}{dx}$ for the curve $y = c\ cosh(x/c)$ **2 Marks**

b) Show that the following curves intersect orthogonally

$r = a\ cosec^2(\theta/2)$ and $r = b\ sec^2(\theta/2)$ **5 Marks**

c) Prove that the radius of curvature for the curve $r = f(\theta)$ in polar form is $\rho = \frac{(r^2+\dot{r}^2)^{3/2}}{r^2+2\dot{r}^2-r\ddot{r})}$

6 Marks

PART - C

5. a) i) If the planes $x - y - z + 1 = 0$ and $2x - ay - 2z + 3 = 0$ are parallel find 'a'. **2 Marks**

ii) Find the equation of the plane passing through $(-2, 1, 3)$ and parallel to the plane $5x - 3y + 5z + 3 = 0$. **2 Marks**

b) Find the equation to a plane passing through the points $(1, 1, 0)$, $(1, 2, 1)$ and $(-2, 2, -1)$. **5 Marks**

c) Determine the mutual positions of the lines

$L_1 : x = 1 - t, \quad y = 2 + t, \quad z = 2t$

$L_2 : x = 3 - 2s, \quad y = 4 + 2s, \quad z = 6 + 4s$ **6 Marks**

6. a) i) Find the equation of the sphere whose centre is $(2, -1, 3)$ and radius is 5 units. **2 Marks**

ii) Find the asymptotes parallel to coördinate axes for the curve $y^2(x^2 - a^2) = x$. **2 Marks**

b) Find the position and nature of the double points of the curve $(x - 1)\,(x - 2)^2 - y^2 = 0$. **5 Marks**

c) Find the surface area of the curve $r = a(1 + cos\theta)$ **6 Marks**

PART - D

7. a) i) If A and B be symmetric matrices (or skew symmetric matrices) of the same order. Then Prove that $A + B$ is also symmetric matrices. (or skew symmetric matrices) **2 Marks**

ii) Define a rank of matrix. **2 Marks**

b) Find the inverse of the matrix

$A = \begin{bmatrix} 1 & 6 & 4 \\ 0 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ using elementary grow operations. **5 Marks**

c) Solve completely the following system of equations for consistency.

$$x + 2y + 3z = 0$$
$$2x + 3y + 4z = 0$$
$$7x + 13y + 19z = 0$$

6 Marks

8. a) i) Evaluate $\int_{0}^{1} xe^{x}\, dx$ **2 Marks**

ii) Evaluate $\int e^{x} \frac{(x-1)}{(x+1)^2}\, dx$ **2 Marks**

b) Evaluate $\int \frac{x}{(x-1)\,(x^2+4)}\, dx$ **5 Marks**

c) If $I_n = \int_{0}^{\frac{\pi}{4}} tan^{n}x\, dx$. Then prove that $I_n + I_{n-2} = \frac{1}{n-1}$ where n is a +ve integer and hence evaluate $\int_{0}^{\frac{\pi}{4}} tan^{4}x\, dx$. **6 Marks**

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