

Further Mathematics SL P1

2007 May

School Level 12th IB Diploma

Programme

Board Exam

International Baccalaureate (IB
Board)

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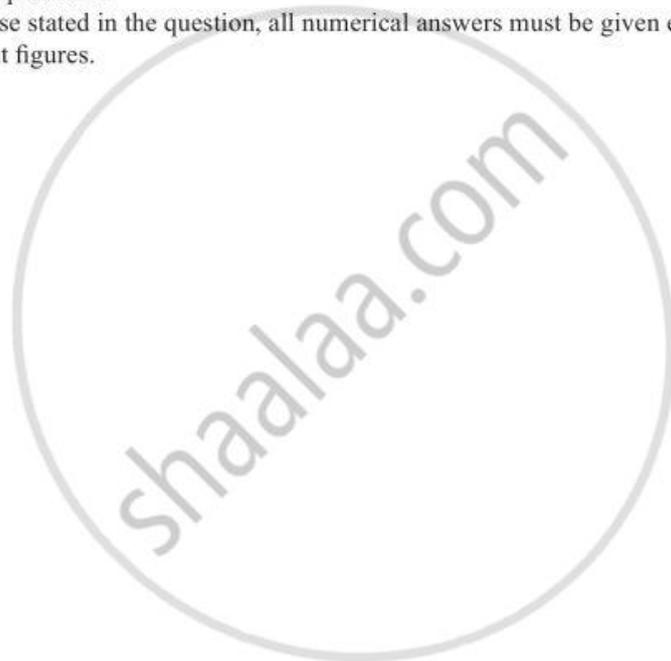
**FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1**

Wednesday 16 May 2007 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. [Maximum mark: 8]

The point $P(x, y)$ moves in such a way that its distance from the point $(1, 0)$ is three times its distance from the point $(-1, 0)$.

- (a) Find the equation of the locus of P . [4 marks]
- (b) Show that this equation represents a circle and state its radius and the coordinates of its centre. [4 marks]

2. [Maximum mark: 8]

Calculate the following limits

- (a) $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$; [3 marks]
- (b) $\lim_{x \rightarrow 0} \frac{(1+x^2)^{\frac{3}{2}} - 1}{\ln(1+x) - x}$. [5 marks]

3. [Maximum mark: 12]

- (a) Show that the set S of numbers of the form $2^m \times 3^n$, where $m, n \in \mathbb{Z}$, forms a group $\{S, \times\}$ under multiplication. [6 marks]
- (b) Show that $\{S, \times\}$ is isomorphic to the group of complex numbers $m + ni$ under addition, where $m, n \in \mathbb{Z}$. [6 marks]

4. [Maximum mark: 12]

- (a) Use the Euclidean Algorithm to show that 275 and 378 are relatively prime. [5 marks]
- (b) Find the general solution to the diophantine equation $275x + 378y = 1$. [7 marks]

5. [Maximum mark: 9]

Solve the differential equation $x \frac{dy}{dx} + 2y = \sqrt{1+x^2}$

given that $y = 1$ when $x = \sqrt{3}$.

[9 marks]

6. [Maximum mark: 11]

The weights, X kg, of male birds of a certain species are normally distributed with mean 4.5 kg and standard deviation 0.2 kg. The weights, Y kg, of female birds of this species are normally distributed with mean 2.5 kg and standard deviation 0.15 kg.

(a) (i) Find the mean and variance of $2Y - X$.

(ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird.

[6 marks]

(b) Two randomly chosen male birds and three randomly chosen female birds are placed together on a weighing machine for which the recommended maximum weight is 16 kg. Find the probability that this maximum weight is exceeded.

[5 marks]
