Code: A-01/C-01/T-01 **Subject: MATHEMATICS-I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 11 Questions in all.

- **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied.**
- **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x8)

a. The value of limit $\lim\limits_{(x,y)\to(1,1)} \frac{x^3 - y^3}{x^2 - y^2}$

(A) equals 0. **(B)** equals $\frac{1}{2}$.

(C) equals $\frac{3}{2}$. **(D)** does not exist.

b. The total differential of the function $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$ at the point (1, 1) is

(A) dx + dy. **(B)** $\frac{1}{2}(dx + dy)$.

(C) $dx - dy$. **(D)** $\frac{1}{2}(dx - dy)$.

c. Let $u(x,y) = \frac{x^3 + y^3}{x + y} + x\tan^{-1}\left(\frac{y}{x}\right), (x,y) \neq (0,0)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(A) 0. **(B)** 2 u.

(C) $\frac{x^3 + y^3}{x + y} + 2x\tan^{-1}\frac{y}{x}$. **(D)** $\frac{2(x^3 + y^3)}{x + y} + x\tan^{-1}\frac{y}{x}$.

d. The value of α so that $e^{\alpha x^2}$ is an integrating factor of differential equation $x(1-y)dx - dy = 0$ is

(A) $-\frac{1}{2}$. **(B)** - 2.

(C) $\frac{1}{2}$. **(D)** 2.

e. If method of undetermined coefficients is used for finding a particular integral of differential equation $y'' + 5y' + 6y = 3e^{-2x} + e^{3x}$ then the solution to be tried is

(A) $axe^{-2x} + be^{3x}$. **(B)** $a\,e^{-2x} + bx\,e^{3x}$.

(C) $a\,e^{-2x} + b\,e^{3x}$. **(D)** $a\,x\,e^{-2x} + bx\,e^{3x}$.

f. Let A be a non-singular matrix. Then the inverse of the matrix AA^T

(A) is symmetric. **(B)** is skew – symmetric.

(C) does not exist. **(D)** equals $A^{-1}\left(A^{-1}\right)^T$.

g. The linear transformation $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, represents

(A) reflection about x_1-axis.

(B) reflection about x_2-axis.

(C) clockwise rotation through angle $\frac{\pi}{2}$.

(D) Orthogonal projection on to x_2-axis.

h. For $P_n(x)$, the Legendre's polynomial of order n, then $P_n(-1)$ equals

(A) 0. **(B)** $(-1)^n$. **(C)** 1. **(D)** $-n$.

PART I

Answer any THREE questions. Each question carries 14 marks.

Q.2 a. If $x^x y^y z^z = c$, where c is a constant, then find $\frac{\partial^2 z}{\partial x \partial y}$ in terms of x, y, z. **(6)**

b. Expand $e^x \cos y$ in powers of x and $\left(y - \frac{\pi}{2}\right)$ as far as 3rd degree terms using Taylor's series expansion. **(8)**

Q.3 a. Let u and v be two functions of x, y. Show that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1,$ where $\frac{\partial(u,v)}{\partial(x,y)}$ denotes the Jacobian of u, v with respect to x, y. **(6)**

b. Find points of local minima and local maxima and saddle points for the function $f(x,y) = x^4 - y^4 - 2x^2 + 2y^2$. **(8)**

Q.4 a. Solve the following system of equations by using the Cramer's Rule

$$x_1 + x_2 = 1$$
$$x_2 + x_3 = 0$$
$$x_3 + x_4 = 0$$
$$x_4 + x_5 = 0$$
$$x_5 + x_1 = 0$$

(7)

b. Find the value of α so that the vectors (1, 2, 9, 8), $(2, 0, 0, \alpha)$, $(\alpha, 0, 0, 8)$, $(0, 0, 1, 0)$ are linearly independent. **(7)**

Q.5 a. Prove that similar matrices have the same eigenvalues. Also give the relationship between the eigenvectors of two similar matrices. **(6)**

b. Find the eigenvalues and the eigenvectors for the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & \frac{1}{2} & 0 \\ 1 & 0 & 4 \end{bmatrix}$. **(8)**

Q.6 a. If a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, find the matrix A^{32}, using Cayley Hamilton theorem. **(8)**

b. Let a 4×4 matrix A have eigenvalues $1, -1, 2, -2$ and matrix $B = 2A + A^{-1} - I$. Find

(i) determinant of matrix B.

(ii) trace of matrix B. **(6)**

PART II

Answer any THREE questions. Each question carries 14 marks.

Q.7 a. Change the order of integration in integral $\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$ and then evaluate the integral. **(7)**

b. Find the volume of the solid bounded by the surfaces z = 0, $3z = x^2 + y^2$ and $x^2 + y^2 = 9$. **(7)**

Q.8 a. Solve the differential equation $xy' = y^2 + y - 2$ by transforming the equation by substitution $y = 1 + \frac{1}{z}$. **(7)**

b. Find the differential equation whose general solution is $y = a e^x + b \cos x$, where a, b are arbitrary constants. **(7)**

Q.9 a. Using method of variation of parameters, show that A can always be determined so that $y = A x^2 e^{\gamma x}$ is a solution of the differential equation $y'' - 2\gamma y' + \gamma^2 y = 2 e^{\gamma x}$. **(7)**

b. Find the general solution of the equation $y'' - 5y' + 4y = 65 \sin 2x$. **(7)**

Q.10 a. Solve the differential equation $\left(D^8 + 6D^6 - 32D^2\right)y = 0$. **(6)**

b. Using Frobenius method, show that the differential equation $xy'' + y' + y = 0$, has a solution $y_1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} x^n$, near the origin. Suggest the form of 2nd solution

y_2 linearly independent of y_1.
(8)

Q.11 a. Show that under change of dependent variable y defined by the substitution $y = u/\sqrt{t}$, the Bessel's equation of order υ becomes $\frac{d^2u}{dt^2} + \left(1 + \frac{1-4\upsilon^2}{4t^2}\right)u = 0$. Hence show that for large values of t, the solutions of Bessel's equation are described approximately by the expression of the form $C_1\frac{\sin t}{\sqrt{t}} + C_2\frac{\cos t}{\sqrt{t}}$.
(7)

b. Using Rodrigues formula for Legendre polynomials $P_n(x)$, show that

$$\int_{-1}^{1} f(x)P_n(x)dx = \frac{(-1)^n}{2^n n!}\int_{-1}^{1} f^{(n)}(x)(x^2-1)^n dx$$

where f is any function integrable on interval $[-1, 1]$. Hence show that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0, m \neq n.$
(7)