Subject: MATHEMATICS-I

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2 x 10)

a. The value of $\lim\limits_{(x,y)\to(0,0)} (x+y)\sin\dfrac{1}{(x+y)}$ is

(A) limit does not exist **(B)** 0

(C) 1 **(D)** –1

b. If $u = x^2 + y^2$ then the value of $\dfrac{\partial^2 u}{\partial x \partial y}$ is equal to

(A) 0 **(B)** 2

(C) $2x + 2y$ **(D)** yx^{y-1}

c. If $u = \log\left(\dfrac{x^2}{y}\right)$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

(A) 2u **(B)** u

(C) 0 **(D)** 1

d. The value of integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ is

(A) 2π **(B)** 2.

(C) –2. **(D)** 0.

e. The solution of the differential equation $(D^4 - 6D^3 + 12D^2 - 8D)y = 0$ is given by

(A) $y = a + (b + cx + dx^2)e^{2x}$ **(B)** $y = (b + cx + dx^2)e^{2x}$

(C) $y = a + bx + cx^2 + dx^3$ **(D)** $y = a + bx + cx^2 + de^{2x}$

f. $e^{-x}\left(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right) + c_3 e^{2x}$ is the solution of

(A) $\frac{d^3y}{dx^3} + 4y = 0$ **(B)** $\frac{d^3y}{dx^3} - 8y = 0$

(C) $\frac{d^3y}{dx^3} + 8y = 0$ **(D)** $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

g. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen values of A^2 are

(A) 1,2,3 **(B)** –1,2,3

(C) 1,4,9 **(D)** –1,4,9

h. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & \gamma & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ then

(A) A is row equivalent to B only if $\alpha = 2, \beta = 3, \gamma = 4$

(B) A is row equivalent to B only if $\alpha \neq 0, \beta \neq 0, \gamma = 0$

(C) A is not row equivalent to B

(D) A is row equivalent to B for all values of α, β, γ

i. The value of $\int_{-1}^{1} P_n^2(x)dx$ is

(A) 0 **(B)** $-\frac{2}{2n+1}$

(C) $\frac{2}{2n+1}$ **(D)** –1

j. The value of the integral $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if

(A) $\alpha = \beta$ **(B)** $\alpha = -\beta$

(C) $\alpha \neq \beta$ **(D)** none of the above

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), x > 0, y > 0$ then evaluate $x^2 \frac{\partial^2 u}{\partial x^2} + 2yx \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ **(8)**

b. Find the absolute maximum and minimum values of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$ **(8)**

Q.3 a. Evaluate the integral $\iiint_T y\,dx\,dy\,dz,$ where T is region bounded by the surfaces $x = y^2, x = y + 2, 4z = x^2 + y^2, z = y + 3$ **(8)**

b. The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm, 6cm respectively. The possible error in each measurement is 0.1cm. Find approximately the maximum possible error in the values computed for the volume and the lateral surface. **(8)**

Q.4 a. The initial value problem governing the current *i* flowing in a series RL circuit when a voltage v(t)=t is applied is given by $iR + L\frac{di}{dt} = t, t \geq 0,$ $i(0) = 0$ where R and L are constants. Find the current i(t) at time t. **(8)**

b. Using the method of undetermined coefficients, find the general solution of the differential equation $y'' + 9y = \cos 3x$ **(8)**

Q.5 a. Find the general solution of the equation $y'' - 4y' + 13y = 18e^{2x} \sin 3x$. **(8)**

b. Show that the matrix A is diagonalizable, where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. **(8)**

Q.6 a. Solve the equations 4x+2y+z+3w=0, 6x+3y+4z+7w=0, 2x+y+w=0. **(8)**

b. Using Gauss Jordan Method, find the inverse of the matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

(8)

Q.7 a. Show that the transformation

$y_1 = 2x_1 + x_2 + x_3,\; y_2 = x_1 + x_2 + 2x_3,\;\; y_3 = x_1 - 2x_3$ is non-singular.

Find the inverse transformation. **(8)**

b. Prove that the following matrix is orthogonal: **(8)**

$$\begin{bmatrix} -8/9 & 4/9 & 1/9 \\ 1/9 & 4/9 & -8/9 \\ 4/9 & 7/9 & 4/9 \end{bmatrix}$$

(8)

Q.8 a. Find the first five non-vanishing terms in the power series solution of the initial value problem $(1-x^2)y'' + 2xy' + y = 0,\; y(0) = 1,\, y'(0) = 1.$ **(11)**

b. Show that $\int P_n(x)dx = \dfrac{1}{2n+1}\left[P_{n+1}(x) - P_{n-1}(x)\right]$ **(5)**

Q.9 a. Show that $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left[\dfrac{1}{x^2}\left(3 - x^2\right)\sin x - \dfrac{3}{x}\cos x\right]$ **(8)**

b. Show that $\displaystyle\int_{-1}^{1} x^2 P_{n-1}(x)P_{n+1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ **(8)**