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Note: Throughout this question paper, $\mathbb{N}$ stands for the set of all natural numbers, $\mathbb{Z}$ stands for the set of all integers, $\mathbb{Q}$ stands for the set of all rational numbers , $\mathbb{R}$ stands for the set of all real numbers and $\mathbb{C}$ stands for the set of all complex numbers.

Part A - 1 mark for each question

1. Let $\lambda = e^{\frac{30\pi i}{36}}$. Then the smallest positive integer l such that $\lambda^l = 1$ is

   (a) 6
   (b) 9
   (c) 12
   (d) 5

2. Consider the vector $(1, 1, 1)$ in $\mathbb{R}^3$. Two linearly independent vectors orthogonal to it are

   (a) (1,-1,1) and (1,1,-2)
   (b) (-2,1,1) and (1,1-2)
   (c) (1,-1,0) and (2,-2,0)
   (d) (0,1,-1) and (0,-2,2)

3. The graph of the polynomial $(X^2 - 2)(X^2 + X + 1)$ will cross the X-axis

   (a) 0 times
   (b) once
   (c) twice
   (d) 3 times

4. "There exists an integer which is not divisible by the square of a prime number". The negation of this statement is

   (a) There exists an integer which is divisible by the square of a prime
   (b) Every integer is not divisible by the square of a prime number
   (c) Every integer is divisible by the square of a prime number
   (d) There exists many integers divisible by the square of a prime number

5. Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous functions whose graphs do not intersect. Then for which function below the graph lies entirely on one side of the X-axis

   (a) f
   (b) $g + f$

(c) $g - f$

(d) gf

6. An example of a function from $\mathbb{R} \to \mathbb{R}^2$ with bounded range is

   (a) $f(t) = (t, t^2)$

   (b) $f(t) = (t, \sin t)$

   (c) $f(t) = (t, \sinh t)$

   (d) $f(t) = (\sin t, \cos t)$

7. The real root of $X^3 + X + 1 = 0$ lies between

   (a) -2 and -1

   (b) -1 and 0

   (c) 1 and 2

   (d) 2 and 3

8. Which of the following maps is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}$?

   (a) $T(a, b, c) = a(b + c)$

   (b) $T(a, b, c) = 2(a + b + c)$

   (c) $T(a, b, c) = ab + c$

   (d) $T(a, b, c) = abc$

9. The events A_1 and A_2 occur with probabilities 0.6 and 0.8 respectively. At least one of them occurs with a probablity of 0.9. The probability that both A_1 and A_2 will occur is

   (a) 0

   (b) 0.5

   (c) 1

   (d) cannot be determined from the data given

10. Two students each are randomly placed in n rooms in a hostel. If n of the 2n students are Mathematics students and n are in Statistics, the probablity that each room has Mathematics student and a statistics student is

    (a) $\frac{1}{2n!}$

    (b) $\frac{1}{2n!}$

    (c) $\frac{2^n}{{}^{2n}C_n}$

    (d) $\frac{2^n}{(n!)^2}$

11. How many positive integers a less than 24 satisfy $a^8 \equiv 1 (mod 24)$?

(a) 2
(b) 4
(c) 6
(d) 8

12. Suppose $f : [0, \infty) \longrightarrow \mathbb{R}$ is continuous

(a) Then f is uniformly continuous on $[k, \infty)$ for some $k > 0$
(b) Then $| f |$ is uniformly continuous on $[0, \infty)$
(c) If f is uniformly continuous on $[k, \infty)$, for some $k > 0$, then f is uniformly continuous on $[0, \infty)$
(d) If f is decreasing then f is uniform continuous

13. Let V be a vector space of all polynomials of degree less than or equal 4 over $\mathbb{Q}$ and $W = \{\sum_{i=0}^{4} a_i X^i \in \mathbb{Q}[X] \mid a_0 \text{ is an even integer}\}$. Then

(a) W is not a subspace of V
(b) W is a subspace and $\dim W < \dim V$
(c) W is a subspace and $\dim W = 4$
(d) W is a subspace and $\dim W = 5$

14. $x^2 = 2y^2 \log y$ is a solution of

(a) $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$
(b) $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$
(c) $\frac{dy}{dx} = \frac{2xy}{2x^2+y^2}$
(d) $\frac{dy}{dx} = \frac{2xy}{x^2+2y^2}$

15. Let C_1, C_2 be two circles in $\mathbb{R}^2$ with centres at points a, b respectively and suppose that $C_1 \cap C_2$ is a singleton set $\{c\}$. Let $|a-b|$ denote the distance between a and b. Then

(a) $|a - b| \geq |a - c|$
(b) $|a - b| = |a - c| + |c - b|$
(c) $|a - b|^2 = |a - c|^2 + |c - b|^2$
(d) None of these

16. The distance between the straight lines $3x-4y+10 = 0$ and $3x-4y-5 = 0$ is

(a) 0
(b) 3
(c) 5

(d) 15

17. $f(x) = e^x - e^{-x}$, $g(x) = e^x + e^{-x}$ Then

    (a) Both f and g are even functions
    (b) Both f and g are odd functions
    (c) f is odd , g is even
    (d) f is even, g is odd

18. $x_{n+1} = \frac{-3}{4} x_n$, $x_0 = 1$. The sequence $\{x_n\}$

    (a) diverges
    (b) x_n is monotonically increasing and converges to 0
    (c) x_n is monotonically decreasing and converges to 0
    (d) None of the above

19. $f(x)$ is an odd function, $g(x)$ an even function then

    (a) $f \circ g$ is odd
    (b) $f \circ g$ is even
    (c) $f \circ f$ is odd
    (d) $g \circ g$ is odd

20. Let X be a set and $f, g : X \to X$ be functions. We can say that $f \circ g$ is bijective if

    (a) at least one of f, g is bijective
    (b) both f and g are bijective
    (c) f is $1-1$ and g is onto
    (d) f is onto and g is $1-1$

21. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function such that $\int_{-1}^{1} f(x)dx = 0$. Then

    (a) $f \equiv 0$
    (b) f is an odd function
    (c) $\displaystyle\int_{-1/2}^{1/2} f(x)dx = 0$
    (d) None of these

22. Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions. We can conclude that $h(x) \leq f(x) \, \forall x \in \mathbb{R}$ if we define $h : \mathbb{R} \to \mathbb{R}$ as

    (a) $\min\{g(x), f(x) + g(x)\}$
    (b) $\min\{f(x), f(x) + g(x)\}$

(c) $\max\{g(x), f(x) + g(x)\}$

(d) $\max\{f(x), f(x) + g(x)\}$

23. Let X be a non-empty set, $f : X \to X$ be a function and let $A, B \subset X$. Then the identity $f(A \cap B) = f(A) \cap f(B)$ is true

(a) always

(b) if f is 1-1

(c) if f is onto

(d) if $A \cup B = X$

24. If $n \geq 1000$ is a natural number, the remainder when $n^2 + n + 1$ is divided by 4 is

(a) always 1

(b) always 3

(c) 1 or 3

(d) 0 or 2

25. Let X be a finite set with 5 elements. Then the number of 1-1 functions from $X \times X$ to $X \times X$ is

(a) 5!

(b) $(5!)^2$

(c) 25!

(d) $\frac{25!}{5!}$

Part B - 2 marks for each question

1. The number of 2×2 matrices with integer entries that satisfy the polynomial $X^2 + X + 1$ is

(a) atmost 2

(b) exactly 2

(c) infinite

(d) none

2. Let $(\mathbb{Q}, +)$ be the group of all rationals under addition and $(\mathbb{Q}^*_+, .)$ be the group of positive nonzero rationals under multiplication. Suppose $f : \mathbb{Q} \to \mathbb{Q}^*_+$ is a homomorphism. Then $f(17) =$

(a) 17^2

(b) 17

(c) $\frac{1}{17}$

(d) 1

3. Let f be a function from [-1,1] to $\mathbb{R}$

   (a) If f is differentiable at 0 with $f'(0) = 0$ then $f(0) = 0$
   (b) If $f(0) = 0$ then f is differentiable at 0
   (c) If $f(0) = 0$ then the X-axis is tangent to the graph of f at 0
      i. All three statements are false
      ii. (a) and (c) are false but (b) is true
      iii. (a) and (b) are false but (c) is true
      iv. (b) and (c) are false but (a) is true

4. Let $f_n(x) = (x + \frac{1}{n})^2$ and $f(x) = lim_{n\to\infty} f_n(x)$. Then

   (a) $lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ does not exist
   (b) $lim_{n\to\infty} \int_0^1 f_n(x)\,dx$ exists but $\int_0^1 f(x)\,dx$ does not exist
   (c) $lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 f(x)\,dx$
   (d) $lim_{n\to\infty} \int_0^1 f_n(x)\,dx \neq \int_0^1 f(x)\,dx$

5. Let $f(x) = |\cos x|$ and $g(x) = \cos|x|$. then

   (a) both f and g are differentiable at 0
   (b) f is differentiable at 0 but g is not
   (c) g is differentiable at 0 but f is not
   (d) neither f nor g are differentiable at 0

6. Let V_1 and V_2 be subspaces of $\mathbb{R}^3$ given by $V_1 = \{(a,b,c) \in \mathbb{R}^3\,|a+b = 2c\}$ and $V_2 = \{(a,b,c) \in \mathbb{R}^3\,|a+b-c = 0\}$. Then dim $(V_1 \cap V_2)$ is

   (a) 0
   (b) 1
   (c) 2
   (d) 3

7. In a bag there are 12 marbles, 11 of which are white and one is red. A child takes out 6 of them, the probability that one of these 6 is red is

   (a) Strictly greater then $\frac{1}{2}$
   (b) equal to $\frac{1}{2}$
   (c) Strictly less then $\frac{1}{3}$
   (d) equal to $\frac{1}{3}$

8. Two families of 3 members each have to be seated in a row, in how many ways can it be done so that all members of a family do not sit together?

(a) 648
(b) 504
(c) 120
(d) 324

9. Let T_1, T_2 be two linear transformations from a finite dimensional vector space V to another space W. Suppose that T_1, T_2 are onto. Then

(a) dim Ker T_1=dim Ker T_2
(b) Ker T_1= Ker T_2
(c) Ker T_1 strictly contained Ker T_2
(d) $T_1 = T_2$

10. The orthogonal trajectories of the family of curves $x + 2y^2 = c$ where c is a constant is

(a) $y = 4x$
(b) $y = -4x$
(c) $y = e^{4x}$
(d) $y = e^{-4x}$

11. A non zero vector common to the space spanned by (1,2,3), (3,2,1) and the space spanned by (1,0,1) and (3,4,3) is

(a) (1,2,3)
(b) (0,-2,-2)
(c) (3,2,0)
(d) (1,1,1)

12. A subset A of $\mathbb{C}$ is said to be balanced if whenever $a \in A$ and $t \in R$, it is true that $ae^{it} \in A$. Which one of these four subsets is balanced?

(a) The elliptic region $\{x + iy \,|\, \frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$
(b) The upper half plane $\{x + iy \,|y > 0\}$
(c) The Y-axis $\{x + iy \,|y = 0\}$
(d) The annular region $\{x + iy \,|1 \leq x^2 + y^2 \leq 2\}$

13. Let A_6 be the set of all positive integers for which 6 is not a factor. Then

(a) A_6 is closed under addition
(b) A_6 is closed under multiplication
(c) $A_6 \cup 6\mathbb{N} = \mathbb{N}$
(d) $A_6 \cup 6A_6 = \mathbb{N}$

14. Which is not a group homomorphism?

    (a) $f : (\mathbb{R}, +) \to (\mathbb{R} - \{0\}, .)$ given by $f(x) = xe^x$
    (b) $f : (\mathbb{Q} - \{0\}, .) \to (\mathbb{Q} - \{0\}, .)$ given by $f(x) = 2x$
    (c) $f : (\mathbb{N}, +) \to (\mathbb{R}, +)$ given by $f(x) = x + |x|$
    (d) $f : (\mathbb{C}, +) \to (\mathbb{C}, +)$ given by $f(x) = 2\overline{x}$

15. For a real number x, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Then

    (a) $\lfloor xy \rfloor \geq \lfloor x \rfloor \lfloor y \rfloor$ for all $x, y \in \mathbb{R}$
    (b) $\lfloor xy \rfloor \leq \lfloor x \rfloor \lfloor y \rfloor$ for all $x, y \in \mathbb{R}$
    (c) $\lfloor xy \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$ for all $x, y \in \mathbb{R}$
    (d) $\lfloor xy \rfloor \leq \lfloor x \rfloor + \lfloor y \rfloor$ for all $x, y \in \mathbb{R}$

16. Let (x_n) be a sequence of positive real numbers. A sufficient condition for (x_n) to have no convergent subsequence is

    (a) $|x_{n+2} - x_{n+1}| > |x_{n+1} - x_n| \, \forall n \in \mathbb{N}$
    (b) $\forall \, i, j \in \mathbb{N}$, the set $\{n \in \mathbb{N} : |x_i - x_n| < \frac{1}{j}\}$ is finite
    (c) $\sum_{k=1}^{\infty} x_{n_k} = \infty$ for every increasing sequence (n_k) of natural numbers.
    (d) none of the above

17. Let P be a real polynomial such that for $x \in \mathbb{R}$, $P(x) = 0$ iff $x = 2$ or 4. Then

    (a) degree of P is 2
    (b) $P(3) < 0$
    (c) $P'(x) = 0$ for some $x < 4$
    (d) $P(x)$ is of the form $c(x-2)^n(x-4)^m$ where c is a constant

18. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function with $f(0) = 0$ and let (x_n) be a sequence in $\mathbb{R}$ with $\lim_{n \to \infty} f(x_n) = 0$. Then

    (a) $\lim_{n \to \infty} x_n = 0$
    (b) $\lim_{k \to \infty} x_{n_k} = 0$, for some subsequence (x_{n_k})
    (c) (x_n) is bounded
    (d) none of the above

19. Number of generators of the group $(\mathbb{Z}_{36}, +)$ is

    (a) 1
    (b) 6

(c) 12

(d) 35

20. Let $x_n = \frac{1}{n^2+1}$ and $y_n = \frac{1}{n \log n}$. then

(a) Σx_n is convergent, Σy_n is divergent

(b) Σx_n is convergent, Σy_n is convergent

(c) Σx_n is divergent, Σy_n is convergent

(d) Σx_n is divergent, Σy_n is divergent

21. Let $A \Delta B$ denote the symmetric difference of A and B. Then $A \Delta B \Delta C$ is the same as

(a) $\{x \mid x$ belongs to all or none of the sets $A, B, C\}$.

(b) $\{x \mid x$ belongs to all or exactly one of the sets $A, B, C\}$.

(c) $\{x \mid x$ belongs to exactly one of the sets $A, B, C\}$.

(d) $\{x \mid x$ belongs to the complement of the union of A, B and $C\}$.

22. Consider the following three conditions on a set $A \subset N$:
Condition (1) : $A = \{ma+nb \mid m, n \in \mathbb{N}\}$ for some $a, b \in \mathbb{N}$ with $(a, b) = 1$.
Condition (2) : $\mathbb{N} - A$ is finite.
Condition (3) : there exists $n_o \in \mathbb{N}$ such that $A = \{n \in N / n \geq n_o\}$. Then

(a) (2) $\Rightarrow$ (3).

(b) (3) $\Rightarrow$ (1) $\Rightarrow$ (2).

(c) (1) $\Rightarrow$ (2) $\Rightarrow$ (3).

(d) (1) $\Rightarrow$ (2).

23. The function $f(x) = x^x$ on $(0, \infty)$ has

(a) a local maximum at e^{-1} but no local minimum.

(b) a local maximum at e^{-1} and a local minimum at 1.

(c) two local maxima at 1 and e^{-1} but no local minimum.

(d) neither a local maximum nor a local minimum.

24. Let $f(x) = 1 - x^{2/3}$ for $x \in [-1, 1]$. Then

(a) $f'(c) = 0$ for some $c \in (-1, 0)$.

(b) $f'(c) = 0$ for some $c \in (0, 1)$.

(c) $f'(x)$ is never zero in $(-1, 0)$.

(d) $f'(x)$ is zero in $(0, 1)$ at two points.

25. A vector of length 1 in $\mathbb{R}^3$ which is orthogonal to the vectors $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ is

(a) $-\frac{\hat{i}}{\sqrt{6}} + \frac{\sqrt{2}\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{6}}$

(b) $\frac{\hat{i}}{\sqrt{6}} - \frac{\sqrt{2}\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{6}}$

(c) $-\frac{\hat{i}}{\sqrt{6}} + \frac{\sqrt{2}\hat{j}}{\sqrt{3}} - \frac{\hat{k}}{\sqrt{6}}$

(d) $\frac{\hat{i}}{\sqrt{6}} + \frac{\sqrt{2}\hat{j}}{\sqrt{3}} - \frac{\hat{k}}{\sqrt{6}}$