Code: A-01/C-01/T-01 **Subject: MATHEMATICS-I**

December 2005

Time: 3 Hours

Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2x10)**

a. The value of limit $\lim_{(x,y)\to(0,0)} \frac{x.\sin\left(x^2+y^2\right)}{x^2+y^2}$ is

(A) 0 **(B)** 1
(C) -1 **(D)** does not exist

b. If $f(x,y)=e^{xy^2}$, the total differential of the function at the point (1, 2) is

(A) $e(dx+dy)$ **(B)** $e^4(dx+dy)$
(C) $e^4(4dx+dy)$ **(D)** $4e^4(dx+dy)$

c. Let $u(x,y)=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right), x>0, y>0$ then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ equals

(A) 0 **(B)** 2u
(C) u **(D)** 3u

d. The value of the integral $\iiint_E xyz\ dx\ dy\ dz$, over the domain E bounded by planes x = 0, y = 0, z = 0, x + y + z = 1 is

(A) $\frac{1}{20}$ **(B)** $\frac{1}{40}$
(C) $\frac{1}{720}$ **(D)** $\frac{1}{800}$

e. The value of α so that $e^{\alpha y^2}$ is an integrating factor of the differential equation $\left(e^{-\frac{y^2}{2}} - xy\right)dy - dx = 0$ is

(A) -1 (B) 1

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

f. The complementary function for the solution of the differential equation $2x^2y'' + 3xy' - 3y = x^3$ is obtained as

(A) $Ax + Bx^{-3/2}$ (B) $Ax + Bx^{3/2}$

(C) $Ax^2 + Bx$ (D) $Ax^{-3/2} + Bx^{3/2}$

g. Let $V_1 = (1, -1, 0)$, $V_2 = (0, 1, -1)$, $V_3 = (0, 0, 1)$ be elements of R^3. The set of vectors $\{V_1, V_2, V_3\}$ is

(A) linearly independent (B) linearly dependent

(C) null (D) none of these

h. The value of μ for which the rank of the matrix $A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to 3 is

(A) 0 (B) 1

(C) 4 (D) -1

i. Using the recurrence relation, for Legendre's polynomial $(n+1)$ $P_{n+1}(x) = (2n+1)\, x\, P_n(x) - n\, P_{n-1}(x)$, the value of $P_2(1.5)$ equals to

(A) 1.5 (B) 2.8

(C) 2.875 (D) 2.5

j. The value of Bessel function $J_2(x)$ in terms of $J_1(x)$ and $J_0(x)$ is

(A) $2J_1(x) - x\,J_0(x)$ (B) $\frac{4}{x}J_1(x) - J_0(x)$

(C) $2J_1(x) - \frac{2}{x}J_0(x)$ (D) $\frac{2}{x}J_1(x) - J_0(x)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that for the function $f(x,y) = \sqrt{|xy|}$, partial derivatives f_x and f_y both exist at the origin and have value 0. Also show that these two partial derivatives are continuous except at the origin. **(8)**

b. In a plane triangle ABC, if the sides a, b be kept constant, show that the variations of its angles are given by the relation

$$\frac{dA}{\sqrt{a^2 - b^2 \sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2 \sin^2 B}} = -\frac{dC}{C}.$$ **(8)**

Q.3 a. Find the shortest distance from (0, 0) to hyperbola $x^2 + 7y^2 + 8xy = 225$ in XY-plane. **(8)**

b. Express $\int_0^{\frac{a}{\sqrt{2}}} \int_0^{x} x\,dxdy + \int_{\frac{a}{\sqrt{2}}}^{a} \int_0^{\sqrt{a^2-x^2}} x\,dxdy$, as a single integral and then evaluate it. **(8)**

Q.4 a. Obtain the volume bounded by the surface $z = C\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)$ and a quadrant of the elliptic cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, z > 0 and where a, b > 0. **(8)**

b. Solve the following differential equations:

(i) $\sec x \frac{dy}{dx} = y + \sin x$.

(ii) $\left(\frac{y}{x}\sec y - \tan y\right)dx + (\sec y \ \log x - x)\,dy = 0$. **(8)**

Q.5 a. Solve the following differential equation by the method of variation of parameters.

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$$. **(9)**

b. Solve $(D^2 - 4D + 1)y = e^{2x} \sin 2x$. **(7)**

Q.6 a. Show that non-trivial solutions of the boundary value problem $y^{(iv)} - w^4 y = 0, y(0) = 0 = y''(0), \quad y(L) = 0 = y''(L) = 0$ are $y(x) = \sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi x}{L}\right)$, where D_n are constants. **(9)**

b. Show that the matrices A and A^T have the same eigenvalues. Further if λ, μ are two distinct eigenvalues, then show that the eigenvector corresponding to λ for A is orthogonal to eigenvector corresponding to μ for A^T. **(7)**

Q.7 a. Let T be a linear transformation defined by

$$T\left[\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \qquad T\left[\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$$

$$T\left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \qquad T\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right] = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}.$$

Find $T\left[\begin{pmatrix} 4 & 5 \\ 3 & 8 \end{pmatrix}\right]$. **(7)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(9)**

Q.8 a. Solve the following system of equations:

$$x_1 + 2x_2 - x_3 = 3$$
$$3x_1 - x_2 + 2x_3 = 1$$
$$2x_1 - 2x_2 + 3x_3 = 2$$
$$x_1 - x_2 + x_3 = -1$$

(6)

b. Find the series solution about the origin of the differential equation $x^2 y'' + 6xy' + (6 + x^2)y = 0$. **(10)**

Q.9 a. Express $f(x) = x^4 + 2x^3 - 6x^2 + 5x - 3$ in terms of Legendre polynomials. **(8)**

b. Evaluate $\int x^{-1} J_4(x)\, dx$, where $J_n(x)$ denotes Bessel function of order n. **(8)**