DISCRETE MATHEMATICS

Time : Three hours   Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1.   Explain any two relations with examples.

2.   Prove that the equality of numbers on a set of real numbers is an equivalence relation.

3.   Draw the truth table and circuit diagram for the OR and AND gates.

4.   Define tautology and contradiction.

5.   Explain the problem of Tower of Hanoi.

6.   What is a recurrance relation? Explain with example.

7.   Explain any two types of graphs with suitable example.

8. Define isomorphism of two graphs with examples.

9. Explain duality in lattices with example.

10. Define modular lattice. Write an example.

PART B — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Let $R = \{(x, 2x)/x \in I\}$ and $S = \{(x, 7x)/x \in I\}$ find $R \circ S, S \circ R, R \circ R, R \circ S \circ R, S \circ R \circ S$.

12. Construct the truth table for

$$(p \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R).$$

13. Prove that $(\exists x) M(x)$ follows logically from the premises $(x) (H(x) \to M(x))$ and $(\exists x) H(x)$.

14. Prove that a tree with n vertices has n-1 edges.

15. Prove that the maximum number of edges in a simple graph with n vertices is $n(n-1)/2$.

16. State and prove the modular inequality of a lattice.

PART C — (2 × 15 = 30 marks)

Answer any TWO questions.

17. (a) Let $X = \{1, 2, 3, 4, \cdots 7\}$ and $R = \{(x, y)/x - y \text{ is divisible by } 3\}$ show that R is an equivalence relation. Draw the graph of R.

(b) If $f : A \to B$ and $g : B \to C$ be bijective and prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

18. (a) Prove

$$P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R.$$

(b) Explain Travelling sales man problem.

19. (a) Explain the applications of Boolean algebra to switching theory.

(b) Prove that every finite Boolean algebra is atomic.