

5. Bauxite ore is made up of $\text{Al}_2\text{O}_3 + \text{SiO}_2 + \text{TiO}_2 + \text{Fe}_2\text{O}_3$. This ore is treated with conc. NaOH solution at 500K and 35 bar pressure for few hours and filtered hot. In the filtrate the species present are:

- (1) NaAl(OH)_4 only (2) $\text{Na}_2\text{Ti(OH)}_6$ only
 (3) NaAl(OH)_4 and Na_2SiO_3 both (4) Na_2SiO_3 only

Sol: Ans [1]

6. When copper pyrites is roasted in excess of air, a mixture of $\text{CuO} + \text{FeO}$ is formed. FeO is present as impurities. This can be removed as slag during reduction of CuO. The flux added to form slag is:

- (1) SiO_2 , which is an acid flux (2) Lime stone, which is a basic flux
 (3) SiO_2 , which is basic flux (4) CaO; which is basic flux

Sol: Ans [1]



7. If liquid is dispersed in solid medium, then this is called as:

- (1) Sol (2) Emulsion (3) Liquid aerosol (4) Gel

Sol: Ans [4]

8. Freundlich equation for adsorption of gases (in amount of X g) on a solid (in amount of m g) at constant temperature can be expressed as:

- (1) $\log \frac{X}{m} = \log p + \frac{1}{n} \log k$ (2) $\log \frac{X}{m} = \log k + \frac{1}{n} \log p$
 (3) $\frac{X}{m} \propto p^n$ (4) $\frac{X}{m} = \log p + \frac{1}{n} \log k$

Sol: Ans [2]

$$\frac{X}{m} \propto P^{1/n} \text{ or } \frac{X}{m} = KP^{1/n}$$

taking log on both the side

$$\log \frac{X}{m} = \log K + \frac{1}{n} \log P$$

9. If k_1 = Rate constant at temperature T_1 and k_2 = Rate constant at temperature T_2 for a first order reaction, then which of the following relations is *correct*? (E_a : activation energy)

- (1) $\log \frac{k_1}{k_2} = \frac{2.303E_a}{RT} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$ (2) $\log \frac{k_2}{k_1} = \frac{E_a}{2.303RT} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$
 (3) $\log \frac{k_2}{k_1} = \frac{E_a}{2.303RT} \left(\frac{T_1 T_2}{T_2 + T_1} \right)$ (4) $\log \frac{k_1}{k_2} = \frac{E_a}{2.303RT} \left(\frac{T_1 T_2}{T_2 - T_1} \right)$

Sol: Ans []

Question is Wrong. If T is removed then answer will be 2

13. A 5% solution of sugarcane (Mol wt = 342) is isotonic with 1% solution of X under similar conditions. The mol. wt of X is:

(1) 136.2 (2) 68.4 (3) 34.2 (4) 171.2

Sol: Ans [2]

$$\frac{W_1}{M_1} = \frac{W_2}{M_2}$$

14. Which is **not** the correct statement for ionic solids in which positive and negative ions are held by strong electrostatic attractive forces?

(1) The radius r^+/r^- increases as coordination number increases
 (2) As the difference in size of ions increases coordination number increases
 (3) When coordination number is eight, the r^+ / r^- ratio lies between 0.225 to 0.414
 (4) In ionic solid of the type AX(ZnS, Wurtzite) the coordination number of Zn^{2+} and S^{2-} respectively are 4 and 4

Sol: Ans [3]

15. The relative penetrating power of α , β , γ and neutron (n) follows the order:

(1) $\alpha > \beta > \gamma > n$ (2) $n > \gamma > \beta > \alpha$ (3) $\beta > \alpha > n > \gamma$ (4) None of these

Sol: Ans [4]

Penetrating power is highest for γ particle.

16. According hard and soft acid base principle. A hard acid :

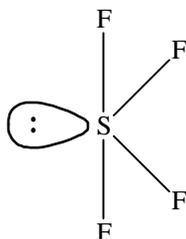
(1) has low charge density
 (2) shows preference for soft bases
 (3) shows preference for donor atoms of low electronegativity
 (4) is not polarizable

Sol: Ans [2]

17. Two types FXF angles are present in which of the following molecule (X = S, Xe, C)

(1) SF_4 (2) XeF_4 (3) SF_6 (4) CF_4

Sol: Ans [1]



SF_4 is sp^3d hybridised and here two different FXF angles are present.

18. Which is *correct* statement about σ - and π - molecular orbitals? Statements are:

- (a) π -bonding orbitals are ungerade
 (b) π -antibonding orbitals are ungerade
 (c) σ -antibonding orbitals are gerade
 (1) (a) only (2) (b) and (c) only (3) (c) only (4) (b) only

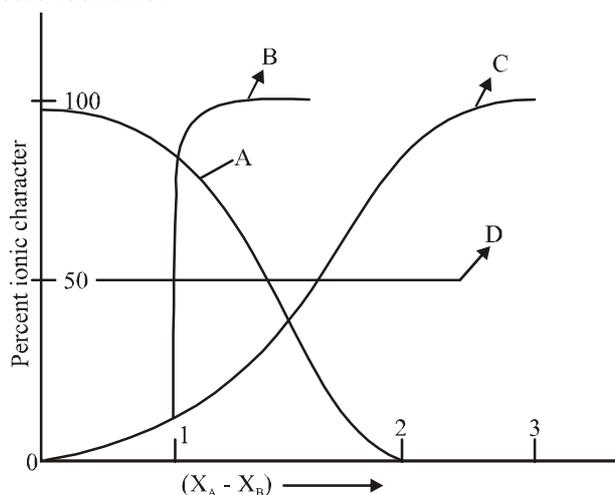
Sol: Ans [1]

19. In forming (i) $N_2 \rightarrow N_2^+$ and (ii) $O_2 \rightarrow O_2^+$; the electrons respectively are removed from

- (1) ($\pi^* 2py$ or $\pi^* 2px$) and ($\pi^* 2py$ or $\pi^* 2px$)
 (2) ($\pi 2py$ or $\pi 2px$) and ($\pi 2py$ or $\pi 2px$)
 (3) ($\pi 2py$ or $\pi 2px$) and ($\pi^* 2py$ or $\pi^* 2px$)
 (4) ($\pi^* 2py$ or $\pi^* 2px$) and ($\pi 2py$ or $\pi 2px$)

Sol: Ans [3]

20. For AB bond if percent ionic character is plotted against electronegativity difference ($X_A - X_B$). The shape of the curve would look like:

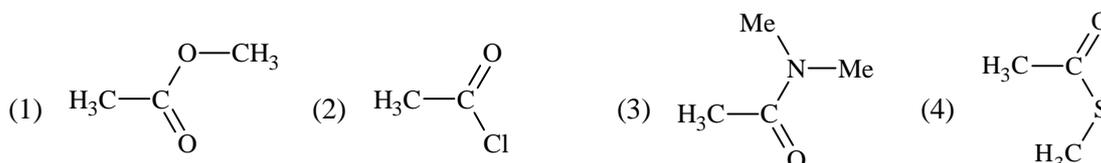


The *correct* curve is:

- (1) (A) (2) (B) (3) (C) (4) (D)

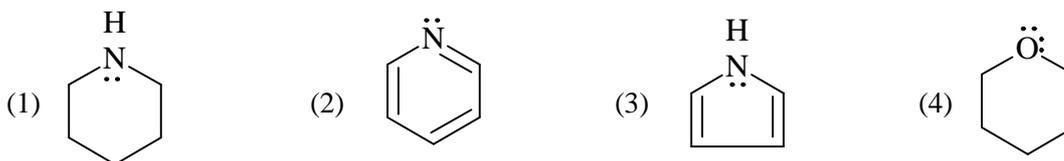
Sol: Ans [3]

21. Least active electrophile is:



Sol: Ans [3]

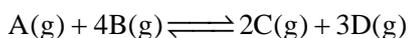
22. Strongest base is:



Sol: Ans [1]

2° amine and lone pair of e⁻ is in sp³ hybridised orbital.

23. 3 moles of A and 4 moles of B are mixed together and allowed to come into equilibrium according to the following reaction:



when equilibrium is reached, there is 1 mole of C. The equilibrium extent of the reaction is:

- (1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) 1

Sol: Ans [3]

24. For the reaction $A \rightarrow B$; $\Delta H = +24\text{kJ/mole}$. For the reaction $B \rightarrow C$; $\Delta H = -18\text{kJ/mole}$. The decreasing order of enthalpy of A, B, C follows the order:

- (1) A, B, C (2) B, C, A (3) C, B, A (4) C, A, B

Sol: Ans [2]



$$H_B - H_A = +24\text{kJ/mol} \quad H_C - H_B = 19\text{kJ/mole}$$

$$H_B > H_A \quad H_B > H_C$$

$$H_B > H_C > H_A$$

25. Given the hypothetical reaction mechanism



and the data as:

Species formed	Rate of its formation
B	0.002 mole/h . per mole of A
C	0.030 mole/h . per mole of B
D	0.011 mole/h . per mole of C
E	0.420 mole/h. per mole of D

The rate determining step is:

- (1) Step I (2) Step II (3) Step III (4) Step IV

Sol: Ans [1]

Slowest step is the rate determining step.

26. For a chemical reaction, ΔG will always be negative if :

- (1) ΔH and $T\Delta S$ both are positive (2) ΔH and $T\Delta S$ both are negative
 (3) ΔH is negative and $T\Delta S$ is positive (4) ΔH is positive and $T\Delta S$ is negative

Sol: Ans [3]

$$\Delta G = \Delta H - T\Delta S$$

27. In which reaction there will be increase in entropy?

- (1) $\text{Na(s)} + \text{H}_2\text{O(l)} \longrightarrow \text{NaOH(l)} + \frac{1}{2}\text{H}_{2(\text{g})}$ (2) $\text{Ag}^+(\text{aq}) + \text{Cl}^-(\text{aq}) \longrightarrow \text{AgCl(s)}$
 (3) $\text{H}_{2(\text{g})} + \frac{1}{2}\text{O}_{2(\text{g})} \longrightarrow \text{H}_2\text{O(l)}$ (4) $\text{Cu}^{2+}(\text{aq}) + 4\text{NH}_3(\text{g}) \longrightarrow [\text{Cu}(\text{NH}_3)_4]^{2+}(\text{aq})$

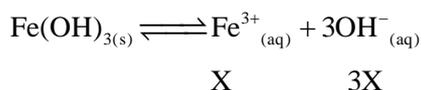
Sol: Ans [1]

$$\Delta S = +ve$$

28. The solubility product of iron (III) hydroxide is 1.6×10^{-39} . If X is the solubility of iron (III) hydroxide, then which one of the following expressions can be used to calculate X:

- (1) $K_{\text{sp}} = X^4$ (2) $K_{\text{sp}} = 9X^4$ (3) $K_{\text{sp}} = 27X^3$ (4) $K_{\text{sp}} = 27X^4$

Sol: Ans [4]



$$K_{\text{sp}} = [\text{Fe}^{3+}][\text{OH}^{-}]^3$$

$$K_{\text{sp}} = 27X^4$$

29. Small quantities of solution of compounds TX, TY and TZ are put into separate test tubes containing X, Y and Z solution. TX does not react with any of these. TY reacts with both X and Z. TZ reacts with X. The decreasing order of ease of oxidation of the anions X^- , Y^- , Z^- is:

- (1) Y^-, Z^-, X^- (2) Z^-, X^-, Y^- (3) Y^-, X^-, Z^- (4) X^-, Z^-, Y^-

Sol: Ans [1]

$$\text{Electrode potential} \propto \frac{1}{\text{oxidising tendency}}$$

30. In which of the following pairs, the constants/quantities are **not** mathematically related to each other?

- (1) Gibbs' free energy and standard cell potential
 (2) Equilibrium constant and standard cell potential
 (3) Rate constant and activation energy
 (4) Rate constant and standard cell potential

Sol: Ans [4]

31. In which case, the order of acidic strength is **not correct** ?

- (1) $\text{HI} > \text{HBr} > \text{HCl}$
- (2) $\text{HIO}_4 > \text{HBrO}_4 > \text{HClO}_4$
- (3) $\text{HClO}_4 > \text{HClO}_3 > \text{HClO}_2$
- (4) $\text{HF} > \text{H}_2\text{O} > \text{NH}_3$

Sol: Ans [2]

32. Which is **not the correct** order for the stated property?

- (1) $\text{Ba} > \text{Sr} > \text{Mg}$; Atomic radius
- (2) $\text{F} > \text{O} > \text{N}$; first ionization enthalpy
- (3) $\text{Cl} > \text{F} > \text{I}$; electron affinity
- (4) $\text{O} > \text{Se} > \text{Te}$; electronegativity

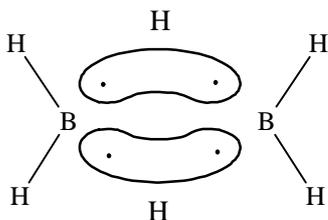
Sol: Ans [2]

I.E. for N is higher than O due to half filled subshell.

33. Which is **correct** statement about diborane structure?

- (1) All HBH bond angles are equal
- (2) All H–B bond lengths are equal
- (3) It has two three-centre-2 electron bonds
- (4) All hydrogens and boron atoms are in one plane

Sol: Ans [3]



34. Match the chemicals in column I with their uses in column II :

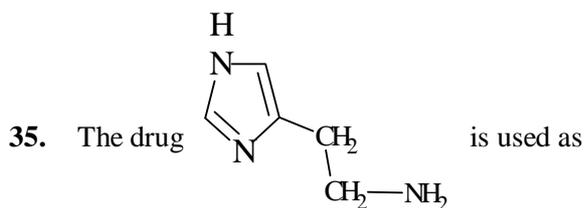
Column I

- (a) Sodium perborate
 - (b) Chlorine
 - (c) Bithional
 - (d) Potassium stearate
- (1) $a = \text{I}, b = \text{II}, c = \text{III}, d = \text{IV}$
 - (3) $a = \text{III}, b = \text{I}, c = \text{II}, d = \text{IV}$

Column II

- (I) Disinfectant
 - (II) Antiseptic
 - (III) Milk bleaching agent
 - (IV) Soap
- (2) $a = \text{II}, b = \text{III}, c = \text{IV}, d = \text{I}$
 - (4) $a = \text{IV}, b = \text{I}, c = \text{II}, d = \text{III}$

Sol: Ans [3]



- (1) Antacid (2) Analgesics (3) Antimicrobial (4) Antiseptic

Sol: Ans [1]

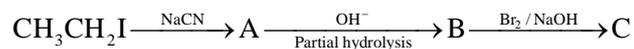
36. Given the polymers :

A = Nylon 6, 6; B = Buna-S; C = Polythene. Arrange these in increasing order of their inter-molecular forces (lower to higher) :

- (1) $A > B > C$ (2) $B > C > A$ (3) $B < C < A$ (4) $C < A < B$

Sol: Ans [1]

37. Given the following sequence of reaction



The major product 'C' is :

- (1) $\text{CH}_3\text{CH}_2\text{NH}_2$ (2) $\text{CH}_3\text{CH}_2\text{C}(=\text{O})\text{NHBBr}$
 (3) $\text{CH}_3\text{CH}_2\text{COONH}_4$ (4) $\text{CH}_3\text{CH}_2\text{C}(=\text{O})\text{NBr}_2$

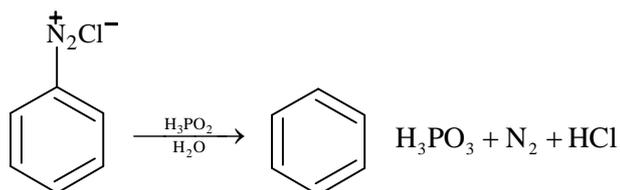
Sol: Ans [1]



38. Which one of the following is **not** the correct reaction of aryl diazonium salts ?

- (1) $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- + \text{Cu}_2\text{Cl}_2 \rightarrow \text{C}_6\text{H}_5\text{Cl}$ (2) $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- + \text{HBF}_4 \xrightarrow{\text{Heat}} \text{C}_6\text{H}_5\text{F}$
 (3) $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- + \text{H}_3\text{PO}_2 \rightarrow \text{C}_6\text{H}_5\text{PO}_4$ (4) $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- + \text{SnCl}_2/\text{HCl} \rightarrow \text{C}_6\text{H}_5\text{NHNH}_2$

Sol: Ans [3]



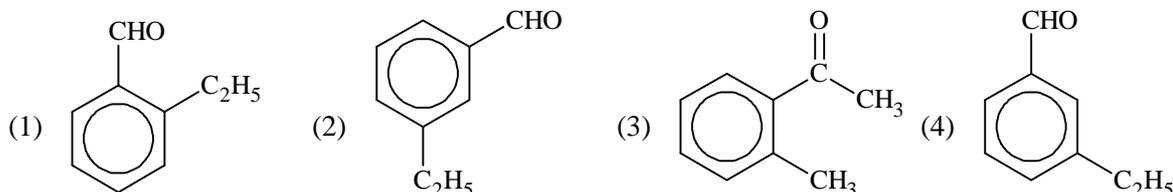
39. Amino group is ortho-, para- directing for aromatic electrophilic substitution. On nitration of aniline good amount of *m*-nitroaniline is obtained. This is due to :

- (1) In nitration mixture, ortho-, para- activity of NH_2 group is completely lost
- (2) $-\text{NH}_2$ becomes $-\text{NH}_3^+$, which is *m*-directing
- (3) $-\text{NH}_2$ becomes $-\text{NH}^+\text{SO}_4^-$, which is *m*-directing
- (4) $-\text{NH}_2$ becomes $-\text{NH}^+\text{NO}_2$, which is *m*-directing

Sol: Ans [2]

40. An aromatic compound 'X' with molecular formula $\text{C}_9\text{H}_{10}\text{O}$ gives the following chemical tests :

- (i) Forms 2, 4-DNP derivative
- (ii) Reduces Tollen's reagent;
- (iii) Undergoes Cannizzaro reaction and
- (iv) On vigorous oxidation 1, 2-benzenedicarboxylic acid is obtained X is



Sol: Ans [1]

41. The correct order of acidic strength of carboxylic acids is :

- (1) Formic acid < benzoic acid < acetic acid
- (2) Formic acid < acetic acid < benzoic acid
- (3) Acetic acid < formic acid < benzoic acid
- (4) Acetic acid < benzoic acid < formic acid

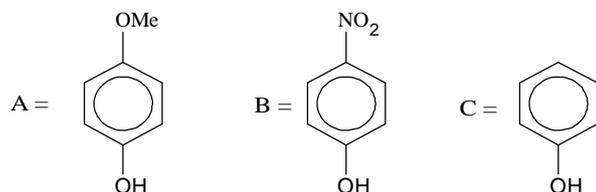
Sol: Ans [4]

42. In a reacting RCHO is reduced to RCH_3 using amalgamated zinc and concentrated HCl and warming the solution. The reaction is known as :

- (1) Meerwein-Ponndorf reaction
- (2) Clemmensen's reduction
- (3) Wolff-kishner reduction
- (4) Schiff's reaction

Sol: Ans [2]

43. Given



The decreasing order of their acidic character is :

- (1) $A > B > C$
- (2) $B > A > C$
- (3) $B > C > A$
- (4) $C > B > A$

47. The function of ZnCl_2 in Lucas test for alcohols is :

- (1) to act as acid catalyst and react with HCl to form H_2ZnCl_4
- (2) to act as base catalyst and react with NaOH to form $\text{Na}_2\text{Zn(OH)}_4$
- (3) to act as amphoteric catalyst
- (4) to act as neutral catalyst

Sol: Ans [1]

48. The number of isomeric pentyl alcohols possible are :

- (1) Two
- (2) Four
- (3) Six
- (4) Eight

Sol: Ans [4]

49. Mesodibromobutane on debromination gives :

- (1) Trans-2-butene
- (2) Cis-2-butene
- (3) 1-butene
- (4) 1-butyne

Sol: Ans [2]

50. Ceric ammonium sulphate and potassium permanganate are used as oxidising agents in acidic medium for oxidation of ferrous ammonium sulphate to ferric sulphate. The ratio, of number of moles of ceric ammonium sulphate required per mole of ferrous ammonium sulphate to the number of moles of KMnO_4 required per mole of ferrous ammonium sulphate, is :

- (1) 5.0
- (2) 0.2
- (3) 0.6
- (4) 2.0

Sol: Ans [1]

51. In spectrochemical series chlorine is above than water i.e. $\text{Cl} > \text{H}_2\text{O}$, this is due to :

- (1) good π -acceptor properties of Cl
- (2) strong σ -donor and good π -acceptor properties of Cl
- (3) good π -donor properties of Cl
- (4) larger size of Cl than H_2O

Sol: Ans [2]

52. For the given complex $[\text{CoCl}_2(\text{en})(\text{NH}_3)_2]^+$, the number of geometrical isomers, the number of optical isomers and total number of isomers of all type possible respectively are:

- (1) 2, 2 and 4
- (2) 2, 2 and 3
- (3) 2, 0 and 2
- (4) 0, 2 and 2

Sol: Ans [2]

53. Which sequence of reactions shows correct chemical relation between sodium and its compounds ?

- (1) $\text{Na} + \text{O}_2 \rightarrow \text{Na}_2\text{O} \xrightarrow{\text{HCl(aq)}} \text{NaCl} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\Delta} \text{Na}$
- (2) $\text{Na} \xrightarrow{\text{O}_2} \text{Na}_2\text{O} \xrightarrow{\text{H}_2\text{O}} \text{NaOH} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\Delta} \text{Na}$
- (3) $\text{Na} + \text{H}_2\text{O} \rightarrow \text{NaOH} \xrightarrow{\text{HCl}} \text{NaCl} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\Delta} \text{Na}$
- (4) $\text{Na} + \text{H}_2\text{O} \rightarrow \text{NaOH} \xrightarrow{\text{CO}_2} \text{Na}_2\text{CO}_3 \xrightarrow{\text{HCl}} \text{NaCl} \xrightarrow{\text{Electrolysis}} \text{Na}$
(Molten)

Sol: Ans [4]

54. For square planar complex of platinum (II), $[\text{Pt}(\text{NH}_3)(\text{Br})(\text{Cl})\text{Py}]^0$, how many isomeric forms are possible ?

- (1) Two (2) Three (3) Four (4) Six

Sol: Ans [2]

55. When EDTA solution is added to Mg^{2+} ion solution, then which of the following statements is **not true** ?

- (1) four coordinate sites of Mg^{2+} are occupied by EDTA and remaining two sites are occupied by water molecules
- (2) All six coordinate sites of Mg^{2+} are occupied
- (3) pH of the solution is decreased
- (4) Colourless $[\text{Mg-EDTA}]^{2-}$ chelate is formed

Sol: Ans [1]

56. In which pair of species, both species do have the similar geometry ?

- (1) CO_2, SO_2 (2) NH_3, BH_3 (3) $\text{CO}_3^{2-}, \text{SO}_3^{2-}$ (4) $\text{SO}_4^{2-}, \text{ClO}_4^-$

Sol: Ans [4]

Hybridization of S is sp^3 in SO_4^{2-}

Hybridisation of Cl is sp^3 in ClO_4^-

57. The number of unpaired electrons in gaseous species of Mn^{3+} , Cr^{3+} and V^{3+} respectively are.....and most stable species is

- (1) 4, 3 and 2 and V^{3+} is most stable (2) 3, 3 and 2 and Cr^{3+} is most stable
- (3) 4, 3 and 2 and Cr^{3+} is most stable (4) 3, 3 and 3 and Mn^{3+} is most stable

Sol: Ans [3]

$\text{Mn}^{3+} - [\text{Ar}]3\text{d}^4$, $\text{Cr}^{3+} - [\text{Ar}]3\text{d}^3$, $\text{V}^{3+} - [\text{Ar}]3\text{d}^2$

Cr^{3+} is most stable state.

58. Which is **not** correct statement about the chemistry of 3d and 4f series elements ?

- (1) 3d elements show more oxidation states than 4f-series elements
- (2) The energy difference between 3d and 4s orbitals is very little
- (3) Europium (II) is more stable than Ce(II)
- (4) The paramagnetic character in 3d-series elements increases from scandium to copper

Sol: Ans [4]

Paramagnetic character increases from Sc to Mn and then decrease from Fe to Zn due to increase of no. of unpaired electron in 3d-orbital from Sc to Mn and then decrease from Fe to Zn.

59. Perxenate ion is :

- (1) XeO_6^{4-} (2) HXeO_4^- (3) XeO_4^{2-} (4) XeO_4^-

Sol: Ans [1]

60. The stability of interhalogen compounds follows the order :

- (1) $\text{IF}_3 > \text{BrF}_3 > \text{ClF}_3$ (2) $\text{BrF}_3 > \text{IF}_3 > \text{ClF}_3$ (3) $\text{ClF}_3 > \text{BrF}_3 > \text{IF}_3$ (4) $\text{ClF}_3 > \text{IF}_3 > \text{BrF}_3$

Sol: Ans [1]



61. Two trains, each moving with a velocity of 30 ms^{-1} , cross each other. One of the trains gives a whistle whose frequency is 600 Hz . If the speed of sound is 330 ms^{-1} , the apparent frequency for passengers sitting in the other train before crossing would be

- (1) 600 Hz (2) 630 Hz (3) 920 Hz (4) 720 Hz

Sol: Ans [4] $f' = f \left(\frac{v - v_0}{v - v_s} \right)$
 $= 600 \left(\frac{330 + 30}{330 - 30} \right) = 720 \text{ Hz}$

62. An electric dipole of dipole moment \vec{p} is placed in a uniform electric field \vec{E} . The maximum torque experienced by the dipole is

- (1) pE (2) p/E (3) E/p (4) $\vec{p} \cdot \vec{E}$

Sol: Ans [1] $\vec{\tau} = \vec{P} \times \vec{E}$
 $\tau = PE \sin \theta$
 $(\tau)_{\max} = PE$

63. The electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 20 V.m . The flux over a concentric sphere of radius 20 cm will be

- (1) 20 V.m. (2) 25 V.m. (3) 40 V.m. (4) 200 V.m.

Sol: Ans [1] According to Gauss's theorem,

$$\phi_E = \frac{\Sigma Q}{\epsilon_0}$$

since the charge inside the spherical surface remains the same, so the flux will be same.

64. A spherical shell of radius R has a charge $+q$ units. The electric field due to the shell at a point

- (1) inside is zero and varies as r^{-1} outside it (2) inside is constant and varies as r^{-2} outside it
(3) inside is zero and varies as r^{-2} outside it (4) inside is constant and varies as r^{-1} outside it

Sol: Ans [3] $E_{\text{inside}} = 0$

$$E_{\text{outside}} = \frac{kQ}{r^2}$$

65. An air capacitor is charged with an amount of charge q and dipped into an oil tank. If the oil is pumped out, the electric field between the plates of capacitor will

- (1) increase (2) decrease (3) remain the same (4) become zero

Sol: Ans [2] $E = \frac{q}{\epsilon_0 KA}$ as K decreases to 1, E increases.

66. A parallel plate capacitor is charged. If the plates are pulled apart

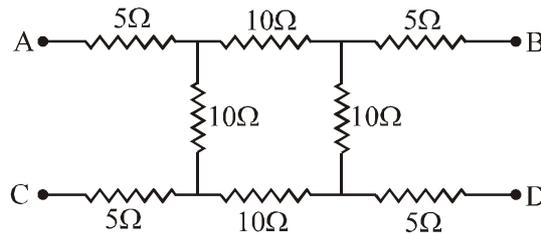
- (1) the capacitance increases
 (2) the potential difference increases
 (3) the total charge increases
 (4) the charge and potential difference remain the same

Sol: Ans [2] $C = \frac{\epsilon_0 A}{d}$

$$v = \frac{q}{c} = \frac{q}{\left(\frac{\epsilon_0 A}{d}\right)}$$

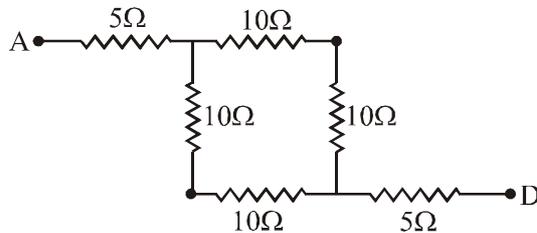
$$\therefore V \propto d$$

67. The equivalent resistance between the terminals A and D in the following circuit is



- (1) 10 Ω (2) 20 Ω (3) 5 Ω (4) 30 Ω

Sol: Ans [2]



68. A metallic wire of resistance of 12 Ω is bent to form a square. The resistance between two diagonal points would be

- (1) 12 Ω (2) 24 Ω (3) 6 Ω (4) 3 Ω

Sol: Ans [4] We know $R = \rho \frac{l}{A}$.

$$\therefore \text{Resistance of each side of the square} = \frac{12}{4} = 3\Omega.$$

$$\therefore \frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{6}$$

$$R_{\text{eq}} = 3\Omega$$

69. A lead-acid battery of a car has an e.m.f. of 12 V. If the internal resistance of the battery is 0.5 ohm, the maximum current that can be drawn from the battery will be

- (1) 30 A (2) 20 A (3) 6 A (4) 24 A

Sol: Ans [4] $I = \frac{E}{R+r}$

for I to maximum, $R = 0$

$$\therefore I = \frac{E}{r} = \frac{12}{0.5} = 24 \text{ A}$$

70. In a potentiometer arrangement, a cell of e.m.f. 1.5 V gives a balance point at 27 cm length of wire. If the cell is replaced by another cell and balance point shifts to 54 cm, the e.m.f. of the second cell is

- (1) 3 V (2) 1.5 V (3) 0.75 V (4) 2.25 V

Sol: Ans [1] $\frac{E_2}{E_1} = \frac{l_2}{l_1}$

$$\frac{E_2}{1.5} = \frac{54}{27}$$

$$E_2 = 3\text{V}$$

71. In a thermocouple, the neutral temperature is 270°C and the temperature of inversion is 525°C . The temperature of cold junction would be

- (1) 30°C (2) 255°C (3) 15°C (4) 25°C

Sol: Ans [3] $\frac{\theta_i + \theta_c}{2} = \theta_n \Rightarrow \frac{525 + \theta_c}{2} = 270$

$$\text{Solving, } \theta_c = 15^\circ\text{C}$$

72. A steady electric current is flowing through a cylindrical conductor.

- (1) The magnetic field in the vicinity of the conductor is zero
 (2) The electric field in the vicinity of the conductor is non-zero
 (3) The magnetic field at the axis of the conductor is zero
 (4) The electric field at the axis of the conductor is zero

Sol: Ans [3] Factual

73. If two parallel wires carry current in opposite directions

- (1) the wires attract each other
- (2) the wires repel each other
- (3) the wires experience neither attraction nor repulsion
- (4) the forces of attraction or repulsion do not depend on current direction

Sol: Ans [2] Factual

74. A charged particle is moving along a magnetic field line. The magnetic force on the particle is

- (1) along its velocity
- (2) opposite to its velocity
- (3) perpendicular to its velocity
- (4) zero

Sol: Ans [4] $F = qvB \sin \theta$, given, $\theta = 0$

75. A current carrying straight wire is kept along the axis of a circular loop carrying a current. The straight wire

- (1) will exert an inward force on the circular loop
- (2) will exert an outward force on the circular loop
- (3) will exert a force on the circular loop parallel to itself
- (4) will not exert any force on the circular loop

Sol: Ans [4] Magnetic field due to circular coil is along its axis i.e. along length of straight wire.

$$\Rightarrow F = IB \sin 0$$

$$\therefore F = 0$$

76. R , L and C represent the physical quantities resistance, inductance and capacitance respectively. Which one of the following combinations has dimension of frequency ?

- (1) $\frac{1}{\sqrt{RC}}$
- (2) $\frac{R}{L}$
- (3) $\frac{1}{LC}$
- (4) $\frac{C}{L}$

Sol: Ans [2] Dimension of $\frac{L}{R}$ is that of time.

77. In an AC series circuit, the instantaneous current is maximum when the instantaneous voltage is maximum. The circuit element connected to the source will be

- (1) pure inductor
- (2) pure capacitor
- (3) pure resistor
- (4) combination of a capacitor and an inductor

Sol: Ans [3] In AC circuit, there is no phase difference between the current and the voltage if the circuit element is pure resistor.

78. A plane e.m. wave of frequency 30 MHz travels in free space along the x -direction. The electric field component of the wave at a particular point of space and time $E = 6$ V/m along y -direction. Its magnetic field component B at this point would be

- (1) 2×10^{-8} T along z -direction
 (2) 6×10^{-8} T along x -direction
 (3) 2×10^{-8} T along y -direction
 (4) 6×10^{-8} T along z -direction

Sol: Ans [1] $\frac{E}{B} = C \Rightarrow B = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}$ T

Direction of propagation is along $\vec{E} \times \vec{B}$

79. A thin lens of glass ($\mu = 1.5$) of focal length + 10 cm is immersed in water ($\mu = 1.33$). The new focal length is

- (1) 20 cm
 (2) 40 cm
 (3) 48 cm
 (4) 12 cm

Sol: Ans [2] $\frac{1}{10} = \left(\frac{3}{2} - 1\right) \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$$\frac{1}{10} = \left(\frac{1}{2}\right) \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

Again $\frac{1}{f_w} = \left(\frac{3 \times 3}{2 \times 4} - 1\right) \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$$\frac{1}{f_w} = \frac{1}{8} \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore \frac{f_w}{10} = 4 \Rightarrow f_w = 40 \text{ cm}$$

80. If Young's double slit experiment of light, interference is performed in water, which one of the following is correct ?

- (1) fringe width will decrease
 (2) fringe width will increase
 (3) there will be no fringe
 (4) fringe width will remain unchanged

Sol: Ans [1] We know

$$\beta' = \frac{\beta}{\mu}$$

\therefore the fringe-width (β) will decrease as $\mu > 1$

81. In order to increase the angular magnification of a simple microscope, one should increase

- (1) the object size
 (2) the aperture of the lens
 (3) the focal length of the lens
 (4) the power of the lens

Sol: Ans [4] We know Angular magnification = $\left(1 + \frac{D}{f}\right)$. If power of the lens will increase, its focal length will decrease, so its angular magnification will increase.

82. Which one of the following property of light *does not* support wave theory of light ?

- (1) Light obeys laws of reflection and refraction (2) Light waves get polarised
 (3) Light shows photoelectric effect (4) Light shows interference

Sol: Ans [3]

83. A ray of light strikes a material's slab at an angle of incidence 60° . If the reflected and refracted rays are perpendicular to each other, the refractive index of the material is

- (1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\sqrt{2}$ (4) $\sqrt{3}$

Sol: Ans [4] By Snell's law

$$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

84. Light of wavelength λ falls on a metal having work function $\frac{hc}{\lambda_0}$. Photoelectric effect will take place only if

- (1) $\lambda < \lambda_0$ (2) $\lambda \geq 2\lambda_0$ (3) $\lambda \leq \lambda_0$ (4) $\lambda = 4\lambda_0$

Sol: Ans [3] For photoelectric effect to take place

$$h\nu \geq w$$

$$\frac{hc}{\lambda} \geq \frac{hc}{\lambda_0}$$

$$\therefore \lambda \leq \lambda_0$$

85. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because

- (1) carbon nucleus is an unstable particle
 (2) it is not energetically favourable
 (3) nuclei do not come very close due to Coulombic repulsion
 (4) lithium nucleus is more tightly bound than a carbon nucleus

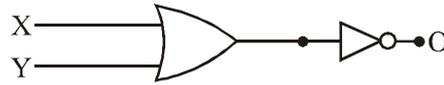
Sol: Ans [3] It needs very high temperature to get nucleus closer.

86. During the β -decay

- (1) an atomic electron is ejected
 (2) an electron, already present within the nucleus, is ejected
 (3) a proton in the nucleus decays emitting an electron
 (4) a neutron in the nucleus decays emitting an electron

Sol: Ans [4] $n = p^+ + \beta^{-1} + \bar{\nu} + \text{energy}$

87. The following logic circuit represents

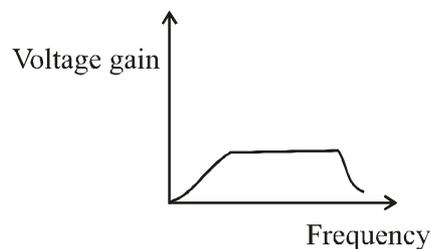


- (1) NAND gate with output $O = \overline{X + Y}$ (2) NOR gate with output $O = \overline{X + Y}$
 (3) NAND gate with output $O = \overline{X \cdot Y}$ (4) NOR gate with output $O = \overline{X \cdot Y}$

Sol: Ans [2] $O = \overline{x + y}$

88. For a transistor amplifier, the voltage gain

- (1) is high at high and low frequencies and constant in the middle frequency range
 (2) is low at high and low frequencies and constant in the middle frequency range
 (3) remains constant for all frequencies
 (4) is high at high frequencies and low at low frequencies and constant in middle frequency range



Sol: Ans [2] Factual

89. In the communication systems, AM is used for broadcasting because

- (1) its use avoids receiver complexity
 (2) it is more noise immune than other modulation system
 (3) it requires less transmitting power
 (4) no other modulation system can give the necessary bandwidth for faithful transmission

Sol: Ans [1] Factual

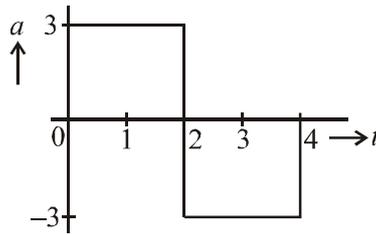
90. A typical optical fibre consists of a fine core of a material of refractive index μ_1 , surrounded by a glass or plastic cladding with refractive index μ_2

- (1) μ_2 is slightly less than μ_1
 (2) μ_2 is slightly higher than μ_1
 (3) μ_2 should be equal to μ_1
 (4) the difference $\mu_2 - \mu_1$ should be strictly equal to 1

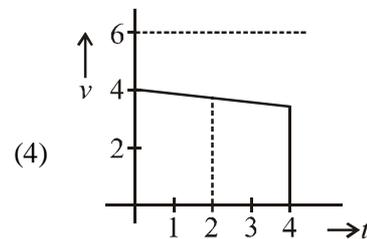
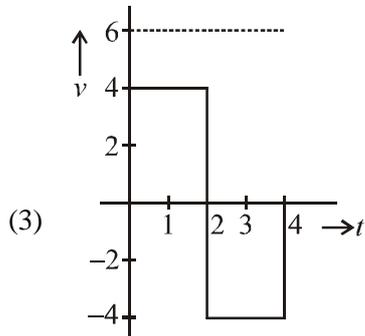
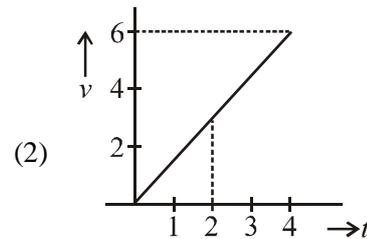
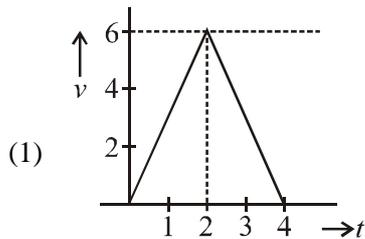
Sol: Ans [1] For total internal reflection to take place,

$$\mu_1 > \mu_2$$

91. A particle starts from rest at $t = 0$ and undergoes an acceleration a in ms^{-2} with time t in seconds which is as shown



Which one of the following plot represents velocity v in ms^{-1} versus time t in seconds ?



Sol: Ans [1] We know

$$v = u + at$$

$$\therefore v = 3t$$

also v when $t = 2 \text{ s}$ is

$$v = 6 \text{ m/s}$$

for the second stage motion.,

$$u = 6 \text{ m/s}$$

$$a = -3 \text{ m/s}^2$$

$$t = 2 \text{ s}$$

$$v = 6 - (3 \times 2) = 0 \text{ m/s}$$

92. A body is tied with a string and is given a circular motion with velocity v in radius r . The magnitude of the acceleration is

- (1) v/r (2) v^2/r (3) v/r^2 (4) v^2/r^2

Sol: Ans [2] Uniform circular motion.

93. A block of mass M at the end of the string is whirled round a vertical circle of radius R . The critical speed of the block at the top of the swing is

- (1) $(R/g)^{1/2}$ (2) g/R (3) M/Rg (4) $(Rg)^{1/2}$

Sol: Ans [4] at highest point

$$Mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{Rg}$$

94. The acceleration a of a particle starting from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time t will be

- (1) $\frac{\alpha t^2}{2} + \beta$ (2) $\frac{\alpha t^2}{2} + \beta t$ (3) $\alpha t^2 + \frac{1}{2}\beta t$ (4) $\frac{(\alpha t^2 + \beta)}{2}$

Sol: Ans [2] $\frac{dv}{dt} = (\alpha t + \beta)$

$$\Rightarrow \int_0^v dv = \int_0^t (\alpha t + \beta) dt$$

$$\Rightarrow v = \frac{\alpha t^2}{2} + \beta t$$

95. A particle is projected with certain velocity at two different angles of projections with respect to horizontal plane so as to have same range R on a horizontal plane. If t_1 and t_2 are the time taken for the two paths, then which one of the following relations are correct ?

- (1) $t_1 t_2 = 2R/g$ (2) $t_1 t_2 = R/g$ (3) $t_1 t_2 = R/2g$ (4) $t_1 t_2 = 4R/g$

Sol: Ans [1] $t_1 = \frac{2u \sin \theta}{g}$, $t_2 = \frac{2u \sin(90 - \theta)}{g}$

$$\Rightarrow t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2} = \left(\frac{2R}{g} \right)$$

96. A bullet hits and gets embedded in a solid block resting on a frictionless surface. In this process which one of the following is correct ?

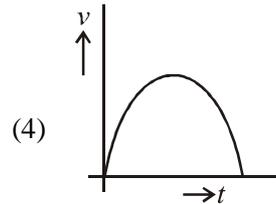
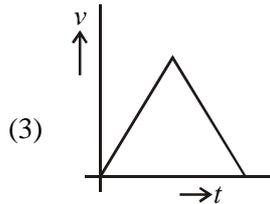
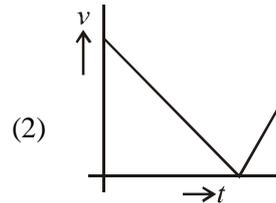
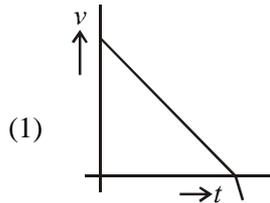
- (1) only momentum is conserved
 (2) only kinetic energy is conserved
 (3) neither momentum nor kinetic energy is conserved
 (4) both momentum and kinetic energy are conserved

Sol: Ans [1] It is inelastic collision and no external horizontal force is present.

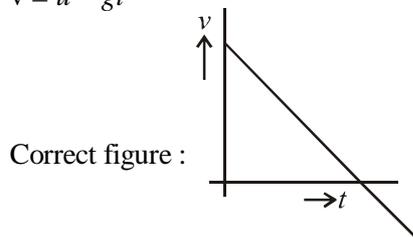
97. For vectors \vec{A} and \vec{B} making an angle θ which one of the following relations is correct ?

- (1) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ (2) $\vec{A} \times \vec{B} = AB \sin \theta$ (3) $\vec{A} \times \vec{B} = AB \cos \theta$ (4) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

103. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plot represent the velocity time plot for the period ball remains in air ?



Sol: Ans [1] $v = u - gt$



104. For an object thrown at 45° to horizontal, the maximum height (H) and horizontal range (R) are related as

- (1) $R = 16 H$ (2) $R = 8 H$ (3) $R = 4 H$ (4) $R = 2 H$

Sol: Ans [3] $h = R_{\max} = \frac{v^2}{g}$

$$H = H_{\max} = \frac{v^2}{4g}$$

105. A lift is moving upward with increasing speed with acceleration a . The apparent weight will be

- (1) less than the actual weight
 (2) more than the actual weight and have a fixed value
 (3) more than the actual weight which increases as long as velocity increases
 (4) zero

Sol: Ans [2] Apparent weight = $m(g + a)$

106. When a body is taken from poles to equator on the earth, its weight

- (1) increases
 (2) decreases
 (3) remains the same
 (4) increases at south pole and decreases at north pole

Sol: Ans [2] Due to

1. Rotation of earth 2. Shape of earth

107. The escape velocity of 10 g body from the earth is 11.2 kms^{-1} . Ignoring the air resistance, the escape velocity of 10 kg of the iron ball from the earth will be

- (1) 0.0112 kms^{-1} (2) 0.112 kms^{-1} (3) 11.2 kms^{-1} (4) 0.56 kms^{-1}

Sol: Ans [3] Escape velocity = $\sqrt{2gR}$ is independent of mass of body.

108. A metallic rod of length l and cross-sectional area A is made of a material of Young modulus Y . If the rod is elongated by an amount y , then the work done is proportional to

- (1) y (2) $1/y$ (3) y^2 (4) $1/y^2$

Sol: Ans [3] Factual

109. The rate of flow of water in a capillary tube of length l and radius r is V . The rate of flow in another capillary tube of length $2l$ and radius $2r$ for same pressure difference would be

- (1) $16V$ (2) $9V$ (3) $8V$ (4) $2V$

Sol: Ans [3] Rate of flow = $\left(\frac{\pi r^4 \Delta p}{8\eta l} \right)$

110. The water flows from a tap of diameter 1.25 cm with a rate of $5 \times 10^{-5} \text{ m}^3\text{s}^{-1}$. The density and coefficient of viscosity of water are 10^3 kgm^{-3} and 10^{-3} Pas , respectively. The flow of water is

- (1) steady with Reynolds number 5100 (2) turbulent with Reynolds number 5100
(3) steady with Reynolds number 3900 (4) turbulent with Reynolds number 3900

Sol: Ans [2] $R = \frac{\rho v d}{\eta} = \frac{\rho \cdot (V) d}{\left(\frac{\pi d^2}{4} \right) \times \eta}$ (where $V = 5 \times 10^{-5} \text{ m}^3/\text{sec}$)

$$= \frac{10^3 \times 5 \times 10^{-5} \times 4}{3.14 \times (1.25 \times 10^{-2}) \times 10^{-3}} \approx 5100$$

111. The excess pressure inside a spherical drop of radius r of a liquid of surface tension T is

- (1) directly proportional to r and inversely proportional to T
(2) directly proportional to T and inversely proportional to r
(3) directly proportional to the product of T and r
(4) inversely proportional to the product of T and r

Sol: Ans [2] $\Delta p = \frac{2T}{r}$

112. Soap bubbles can be formed floating in air by blowing soap solution in air, with the help of a glass tube, but not water bubbles. It is because

- (1) the excess pressure inside water bubble being more due to large surface tension
- (2) the excess pressure inside water bubble being less due to large surface tension
- (3) the excess pressure inside water bubble being more due to large viscosity
- (4) the excess pressure inside water bubble being less due to less viscosity

Sol: Ans [1] $\Delta p = \frac{4T}{R}$

for larger T, pressure difference is more

113. The ratio of the vapour densities of two gases at a given temperature is 9 : 8. The ratio of the r.m.s. velocities of their molecules is

- (1) $3:2\sqrt{2}$
- (2) $2\sqrt{2}:3$
- (3) 9 : 8
- (4) 8 : 9

Sol: Ans [2] $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

114. The mean kinetic energy of one mole of gas per degree of freedom (on the basis of theory of gases) is

- (1) $\frac{1}{2}kT$
- (2) $\frac{3}{2}kT$
- (3) $\frac{3}{2}RT$
- (4) $\frac{1}{2}RT$

Sol: Ans [4] Factual

115. A Carnot engine working between 450 K and 600 K has a work output of 300 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

- (1) 400 J
- (2) 800 J
- (3) 1600 J
- (4) 3200 J

Sol: Ans [There is no matching answer]

$$\frac{W}{Q} = 1 - \frac{450}{600}$$

$$\Rightarrow \frac{300}{Q} = \frac{150}{600} \Rightarrow Q = 1200 \text{ J}$$

116. According to Newton's law of cooling, the rate of cooling is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the temperature difference between the body and the surroundings and n is equal to

- (1) three
- (2) two
- (3) one
- (4) four

Sol: Ans [3] $-\frac{dT}{dt} = k(\Delta\theta)$

117. A black body at a high temperature T radiates energy at the rate of U (in W/m^2). When the temperature falls to half (i.e. $T/2$), the radiated energy (in W/m^2) will be

- (1) $\frac{U}{8}$ (2) $\frac{U}{16}$ (3) $\frac{U}{4}$ (4) $\frac{U}{2}$

Sol: Ans [2] Energy radiated $\propto T^4$

118. Two simple harmonic motions are represented by

$$y_1 = 5(\sin 2\pi t + \sqrt{3} \cos 2\pi t)$$

$$y_2 = 5 \sin (2\pi t + \pi/4)$$

The ratio of the amplitudes of two SHM's is

- (1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 1 : $\sqrt{3}$

Sol: Ans [3] $y_1 = 10 \sin (2\pi t + \pi/3)$

$$y_2 = 5 \sin (2\pi t + \pi/4)$$

$$\text{Ratio of amplitudes} = \frac{10}{5} = 2$$

119. If a simple pendulum of length L has maximum angular displacement α , then the maximum kinetic energy of bob of mass M is

- (1) $\frac{1}{2} \frac{ML}{g}$ (2) $\frac{Mg}{2L}$ (3) $MgL(1 - \cos \alpha)$ (4) $MgL \sin \alpha/2$

Sol: Ans [3] $\Delta K + \Delta U = 0$

$$\Rightarrow K_{\max} = mgL(1 - \cos \alpha), \text{ kinetic energy at lowest point.}$$

120. An open organ pipe of length l vibrates in its fundamental mode. The pressure variation is maximum

- (1) at the two ends (2) at the distance $l/2$ inside the ends
 (3) at the distance $l/4$ inside the ends (4) at the distance $l/8$ inside the ends

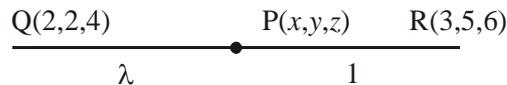
Sol: Ans [2] Displacement of particle from mean position and pressure variation have phase difference $\pi/2$



121. If $P(x, y, z)$ is a point on the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ such that the projections of OP on the axis are $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ respectively, then P divides QR in the ratio.

- (1) 1 : 2 (2) 3 : 2 (3) 2 : 3 (4) 1 : 3

Sol: Ans [2]



$$\frac{3\lambda + 2}{\lambda + 1} = \frac{13}{5}$$

$$15\lambda + 10 = 13\lambda + 13$$

$$2\lambda = 3$$

$$\lambda = 3/2$$

122. The orthocentre of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is

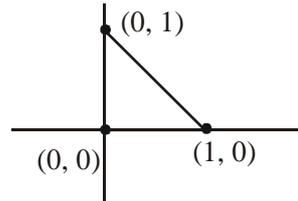
- (1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (3) $\left(\frac{1}{4}, \frac{1}{4}\right)$ (4) $(0, 0)$

Sol: Ans [4]

$$xy = 0 \Rightarrow x = 0, y = 0$$

$$x + y = 1$$

So point of triangle are $(0, 0), (0, 1), (1, 0)$



\therefore Orthocentre is the point of intersection of altitudes so orthocentre is $(0, 0)$

123. If the sum of the distances from two perpendicular lines in a plane is 1, then its locus is

- (1) a square (2) a circle
(3) a straight line (4) two intersecting lines

Sol: Ans [1] Let the lines are $x + y = 0$ & $x - y = 0$

Locus of the points is (h, k)

$$\text{Then } \left| \frac{h+k}{\sqrt{2}} \right| + \left| \frac{h-k}{\sqrt{2}} \right| = 1$$

If is of the type $|x| + |y| = 1$
So, it is square.

124. Point Q is symmetric to $P(4, -1)$ with respect to the bisector of the first quadrant. The length of PQ is

- (1) $3\sqrt{2}$ (2) $5\sqrt{2}$ (3) $7\sqrt{2}$ (4) $9\sqrt{2}$

Sol: Ans [2]

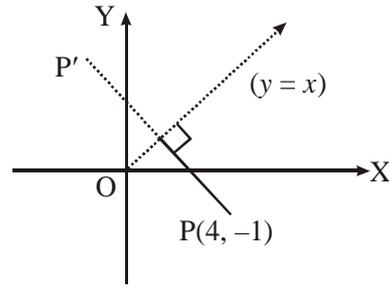
Bisector of first quadratn is the st. line $y = x$.

Let $P \equiv (C_1, -1)$ and P' is its symmetric point about $x = y$.

So, $PP' = 2 PQ$

$$= 2 \frac{4 - (-1)}{\sqrt{2}} \quad (\text{As } PQ \perp (x = y))$$

$$= 5\sqrt{2}$$



125. The radius of the circle, which is touched by the line $y = x$ and has its centre on the positive direction of x-axis and also cuts-off a chord of length 2 units along the line $\sqrt{3}y - x = 0$, is

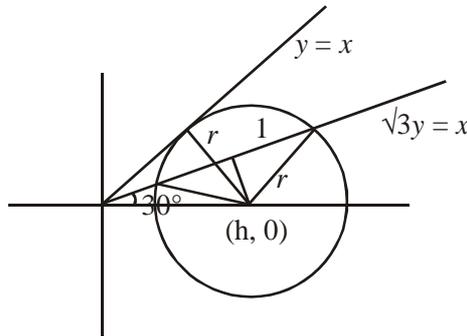
- (1) $\sqrt{5}$ (2) $\sqrt{3}$ (3) $\sqrt{2}$ (4) 1

Sol: Ans [3]

$$h^2 = 2r \quad \dots(i)$$

$$r^2 + 1 + \frac{h^2}{4} \quad \dots(ii)$$

From (i) and (ii) $h = 2$ and $r = \sqrt{2}$.



126. Tangents drawn from the point $(4, 3)$ to the circle $x^2 + y^2 - 2x - 4y = 0$ are inclined at an angle

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Sol: Ans [4]

$$SG_1 = T^2$$

$$\Rightarrow (x^2 + y^2 - 2x - 4y) (25 - 8 - 12) = \{4x + 3y - (x + 4) - 2(y + 3)\}^2$$

$$\Rightarrow 5x^2 + 5y^2 - 10x - 20y = 9x^2 + y^2 + 6xy$$

$$\Rightarrow 4x^2 - 4y^2 + 6xy + ()x + ()y + () = 0$$

$\therefore a + b = 0, (4 - 4) = 0 \Rightarrow$ Tangents are \perp each other.

127. If l denotes the semi-latus rectum of the parabola $y^2 = 4ax$ and SP and SQ denote the segments of any focal chord PQ , S being the focus, then SP, l and SQ are in the relation.

- (1) A.P (2) G.P (3) H.P (4) $l^2 = SP^2 + SQ^2$

Sol: Ans [3]

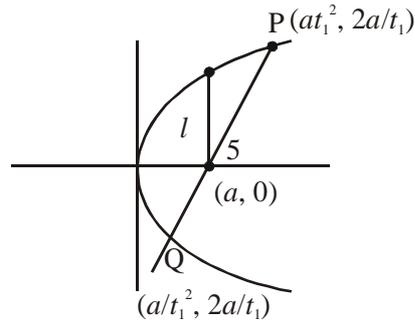
$$l = 2a$$

$$\begin{aligned} SQ &= \sqrt{\left(a - \frac{a}{t_1^2}\right)^2 + \frac{4a^2}{t_1^2}} \\ &= \sqrt{a^2 + \frac{a^2}{t_1^4} + \frac{2a^2}{t_1^2}} = \left(a + \frac{a}{t_1^2}\right) \end{aligned}$$

$$SP = \sqrt{(a - at_1^2)^2 + 4at_1^2} = (a + at_1^2)$$

$$\text{Now } \frac{2 \times SP \times SQ}{SP + SQ} = \frac{2 \times a(1 + t_1^2) \times a \left(1 + \frac{1}{t_1^2}\right)}{2a + a \left(\frac{1}{t_1^2} + t_1^2\right)}$$

$$= \frac{2a \left(1 + 1 + \frac{1}{t_1^2} + t_1^2\right)}{\left(2 + \frac{1}{t_1^2} + t_1^2\right)} = 2a$$

**128.** The eccentricity of the ellipse $x^2 + 4y^2 + 8y - 2x + 1 = 0$ is

(1) $\sqrt{3}/2$

(2) $\sqrt{5}/2$

(3) $1/2$

(4) $1/4$

Sol: Ans [1]

$$x^2 + 4y^2 + 8y - 2x + 1 = 0$$

$$(x-1)^2 + 4(y+1)^2 = 4$$

$$\frac{(x-1)^2}{2} + (y+1)^2 = 1 \quad \Rightarrow \quad a^2 = 4$$

$$\Rightarrow a = 2, b = 1$$

$$\text{Now, } b^2 = a^2(1 - e^2) \quad \Rightarrow \quad 1 = 4(a - e^2)$$

$$\Rightarrow 3 = 4e^2 = \left[e = \left(\frac{\sqrt{3}}{2} \right) \right]$$

129. The equation of the tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

(1) $x - y + 1 = 0$

(2) $x - y + 2 = 0$

(3) $x - y + 3 = 0$

(4) $x - y - 2 = 0$

Sol: Ans [1]

$$y = x; \frac{x^2}{3} - \frac{y^2}{2} = 1$$

Equation of tangent is $y = mx \pm \sqrt{a^2m^2 - b^2}$

$$\Rightarrow y = 1 - x \pm \sqrt{3-2} \quad \Rightarrow y = x \pm 1$$

$$\Rightarrow x - y = \pm 1 \quad \Rightarrow x - y \pm 1 = 0$$

130. Which of the following statements is a tautology?

(1) $(\sim q \wedge p) \wedge q$

(2) $(\sim q \wedge p) \wedge (p \wedge \sim p)$

(3) $(\sim q \wedge p) \vee (p \vee \sim p)$

(4) $(p \wedge q) \wedge (\sim (q \wedge q))$

Sol: Ans [3] $(p \vee \sim p)$ is a tautology.

Hence $(\sim q \wedge p) \vee (p \vee \sim p)$ is a tautology

131. If the variable takes the values 0, 1, 2, ..., n with frequencies proportional to the binomial coefficients $C(n, 0), C(n, 1), C(n, 2), \dots, C(n, n)$ respectively, then the variance of the distribution is

(1) n

(2) $\sqrt{n/2}$

(3) $n/2$

(4) $n/4$

Sol: Ans [4] $p = q = 1/2$

$$\Rightarrow \text{Variance} = npq = \frac{n}{4}$$

132. Let A, B and C be the three events such that $P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A \cap B) = 0.08, P(A \cap C) = 0.28, P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ satisfies.

(1) $P(B \cap C) \leq 0.23$

(2) $P(B \cap C) \leq 0.48$

(3) $0.23 \leq P(B \cap C) \leq 0.48$

(4) $0.23 \leq P(B \cap C) \geq 0.48$

Sol: Ans [2]

$P(A) = 0.3; P(B) = 0.4$

$P(C) = 0.8; P(A \cap B) = 0.08; P(A \cap C) = 0.28$

$P(A \cap B \cap C) = 0.09$

If $P(A \cup B \cup C) \geq 0.75$ then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$\Rightarrow 0.3 + 0.4 + 0.8 - 0.36 - P(B \cap C) + 0.09 \geq 0.75$$

$$\Rightarrow P(B \cap C) \leq 1.5 + 0.09 - 0.36 - 0.75 \leq 0.48$$

133. Out of $3n$ consecutive natural numbers, 3 natural numbers are chosen at random without replacement. The Probability that the sum of the chosen numbers is divisible by 3 is

(1) $n(3n^2 - 3n + 2)/2$

(2) $(3n^2 - 3n + 2)/(2(3n - 1)(3n - 2))$

(3) $(3n^2 - 3n + 2)/((3n - 1)(3n - 2))$

(4) $n(3n - 1)(3n - 2)/3(n - 1)$

Sol: Ans [3] Favourable cases = $3 \cdot {}^n C_3 + n^3$
 Total cases = ${}^{3n} C_3$
 Probability = $\frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$

134. If X and Y are independent binomial variates $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then $P(X + Y = 3)$ is

- (1) $\frac{35}{47}$ (2) $\frac{55}{1024}$ (3) $\frac{220}{512}$ (4) $\frac{11}{204}$

Sol: Ans [2] $P(x + y = 3)$ is

$$= \left(\frac{1}{2}\right)^{12} \left[{}^5 C_0 {}^7 C_3 + {}^5 C_1 {}^7 C_2 + {}^5 C_2 {}^7 C_1 + {}^5 C_3 {}^7 C_0 \right]$$

$$= \frac{{}^{12} C_3}{2^{12}} = \frac{55}{1024}$$

135. $\int_0^{\pi/4} \log(1 + \tan \theta) d\theta =$

- (1) $\pi \log 2$ (2) $(\pi \log 2)/2$ (3) $(\pi \log 2)/8$ (4) $\log 2$

Sol: Ans [3] $I = \int_0^{\pi/4} \log(1 + \tan \theta) d\theta \quad \dots(i)$

$$= \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - \theta\right)\right] d\theta$$

$$I = \int_0^{\pi/4} \log\left[\frac{2}{1 + \tan \theta}\right] d\theta \quad \dots(ii)$$

From (i) and (ii)

$$2I = \int_0^{\pi/4} \log 2 d\theta = \log 2 \cdot \frac{\pi}{4}$$

$$I = \frac{\pi}{8} \log 2$$

136. $\int_a^b \sqrt{(x-a)(b-x)} dx (b > a)$ is equal to

- (1) $\pi(b-a)^2/8$ (2) $\pi(b+a)^2/8$ (3) $(b-a)^2$ (4) $(b+a)^2$

Sol: Ans [1] $I = \int_a^b \sqrt{(x-a)(b-x)} dx (b > a)$

put $x = a \cos^2 \theta + b \sin^2 \theta$.

$$\begin{aligned} \text{Then } I &= \int_0^{\pi/2} \frac{(b-a)^2}{2} \sin^2 2\theta d\theta \\ &= \frac{\pi}{8} (b-a)^2 \end{aligned}$$

137. The area bounded by the curves $y = \sqrt{5-x^2}$ and $y = |x-1|$ is

- (1) $\left(\frac{5\pi}{4} - 2\right)$ square units (2) $(5\pi - 2)/4$ square units
 (3) $(5\pi - 2)/2$ square units (4) $\left(\frac{\pi}{2} - 5\right)$ square units

Sol: Ans [2] Area = $\int_{-1}^2 \sqrt{5-x^2} dx - \int_{-1}^1 (1-x) dx - \int_1^2 (x+1) dx = \frac{5\pi}{4} - \frac{1}{2}$

138. $\lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{3n} \right) =$

- (1) $\log 2$ (2) $\log 3$ (3) $\log 5$ (4) 0

Sol: Ans [2] $\sum_{r=0}^{2n} \frac{1}{n+r} = \int_0^2 \frac{1}{1+x} dx = \log(1+x) \Big|_0^2 = \log 3$

139. The degree and order respectively of the differential equation of all the parabolas whose axis is x-axis, are

- (1) 2, 1 (2) 1, 2 (3) 2, 2 (4) 1, 1

Sol: Ans [2] The equation of family of parabolas is $y^2 = 4a(x-h)$

$$\Rightarrow 2yy' = 4a$$

$$\Rightarrow 2yy'' + 2(y')^2 = 0$$

$$\Rightarrow \text{degree '1' \& order '2'}$$

140. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x+3y)$ with x-axis and which passes through $(1, 2)$ is

- (1) $6x + 9y + 2 = 26e^{3(x-1)}$ (2) $6x - 9y + 2 = 26e^{3(x-1)}$
 (3) $6x + 9y - 2 = 26e^{3(x-1)}$ (4) $6x - 9y - 2 = 26e^{3(x-1)}$

Sol: Ans [1] Given that $\frac{dy}{dx} = 2x + 3y$

Putting $2x + 3y = t$, we get $6x + 9y + 2 = 26e^{3(x-1)}$

141. If $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx = x[f(x) - g(x)] + c$, then

(1) $f(x) = \log(\log x); g(x) = \frac{1}{\log x}$

(2) $f(x) = \log x; g(x) = \frac{1}{\log x}$

(3) $f(x) = \frac{1}{\log x}; g(x) = \log(\log x)$

(4) $f(x) = \frac{1}{x \log x}; g(x) = \frac{1}{\log x}$

Sol: Ans [1] $\int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$

$$= x \log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x}$$

142. If $f\left(\frac{3x-4}{3x+4}\right) = x+2$, then $\int f(x) dx$ is

(1) $e^{x+2} \log \left| \frac{3x-4}{3x+4} \right| + c$

(2) $-\frac{8}{3} \log |1-x| + \frac{2}{3}x + c$

(3) $\frac{8}{3} \log |1-x| + \frac{x}{3} + c$

(4) $e^{[(3x-4)(3x+4)]} - \frac{x^2}{2} - 2x + c$

Sol: Ans [2] $f\left(\frac{3x-4}{3x+4}\right) = x+2$

$$\Rightarrow f(x) = \frac{4x+4}{3-3x} + 2 = \frac{10-2x}{3(1-x)}$$

$$\Rightarrow \int f(x) dx = -\frac{8}{3} \log(1-x) + \frac{2}{3}x + c$$

143. The derivative of $f(\tan x)$ with respect to $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$, is

(1) $\frac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) 1

(4) 0

Sol: Ans [1] $\frac{df(\tan x)}{dg(\sec x)} = \frac{f'(\tan x) \cdot \sec^2 x}{g'(\sec x) \sec x \tan x}$

$$\Rightarrow \frac{f'(1) \cdot \sqrt{2}}{g'(\sqrt{2}) \cdot 1} = \frac{1}{\sqrt{2}}$$

144. If $y = (\log_{\cos x} \sin x)(\log_{\sin x} \cos x) + \sin^{-1} \frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is equal to

- (1) $\frac{8}{(4+\pi^2)}$ (2) 0 (3) $-\frac{8}{(4+\pi^2)}$ (4) 1

Sol: Ans [1] $y = \frac{\log \sin x}{\log \cos x} \cdot \frac{\log \cos x}{\log \sin x} + 2 \tan^{-1} x$

$$y = 1 + 2 \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{2}{1+x^2} = \frac{8}{4+\pi^2}$$

145. If $y = \sin^{-1} \left(\frac{5x + 12\sqrt{1-x^2}}{13} \right)$, then $\frac{dy}{dx}$ is equal to

- (1) $-\frac{1}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{3}{\sqrt{1-x^2}}$ (4) $\frac{x}{\sqrt{1-x^2}}$

Sol: Ans [2] Let $x = \sin \theta$ and $\cos \alpha = \frac{5}{13}$

$$y = \sin^{-1} \sin(\theta + \alpha)$$

$$y = \theta + \alpha = \sin^{-1} x + \alpha$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

146. The value of a in order that $f(x) = \sin x - \cos x - ax + b$ decreases for all real values of x is given by

- (1) $a \geq \sqrt{2}$ (2) $a < \sqrt{2}$ (3) $a \geq 1$ (4) $a < 1$

Sol: Ans [1] $f'(x) = \cos x + \sin x - a \leq 0$

$$\Rightarrow a \geq \cos x + \sin x$$

$$\Rightarrow a \geq \sqrt{2}$$

147. If $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$, then the maximum value of xy , for all $x \geq 0, y \geq 0$ is

- (1) 2500 (2) 3000 (3) 1200 (4) 3500

Sol: Ans [1] $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$

$$\log_{10}(x + y) \leq 2$$

$$(x + y) \leq 100$$

$$\Rightarrow \sqrt{xy} \leq \frac{x+y}{2} \leq 50$$

$$\Rightarrow xy \leq 2500$$

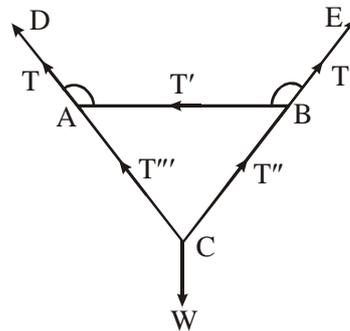
148. Let ABC be an equilateral triangle formed by weightless inextensible strings, the side AB is horizontal. A and B are tied to fixed points D and E by equal weightless inextensible strings AD, BE, a weight of W gm is attached at C. The angle DAB, ABE are each 150° . Then which of the following statements is true?

- (1) The tensions in the strings AB, BC and CA are equal
- (2) The tensions in the strings BE, AD are inversely proportional
- (3) The tension in BE is less than the tension in AB
- (4) The tension in AD is twice the tension in BC

Sol: Ans [1] Considering equilibrium at 'B'

$$\text{we get } T = W \text{ and } T' = \frac{\sqrt{3}}{2} W$$

$$\text{also } T'' = T''' = \frac{\sqrt{3}}{2} W.$$



149. Two parabola paths of angles α and β of projection aimed at a target on the horizontal plane through 0, fall p units short and the other p units far from the target. If θ is the correct angle of projection so as to hit the target, then $\sin 2\theta$ is equal to

(1) $\frac{1}{2}(\tan \alpha + \tan \beta)$

(2) $\frac{1}{2}(\tan \alpha - \tan \beta)$

(3) $\frac{1}{2}(\sin 2\alpha + \sin 2\beta)$

(4) $\frac{1}{2}(\sin 2\alpha - \sin 2\beta)$

Sol: Ans [3] Let velocity be v and angle of projection be α

$$\text{then } v_x = v \cos \alpha, v_y(0) = v \sin \alpha$$

$$\text{when the particle hits ground, } s_y = 0$$

$$s_y = v \sin \alpha t - \frac{1}{2} g t^2 = 0$$

$$\Rightarrow v \sin \alpha t = \frac{1}{2} g t^2$$

$$\Rightarrow t = \frac{2v \sin \alpha}{g}$$

$$s_x = v \cos \alpha t = \frac{v^2 \sin 2\alpha}{g}$$

let target be at R units from 0.

$$\text{Then } \frac{v^2 \sin 2\alpha}{g} = R - p \quad \dots(i)$$

$$\frac{v^2 \sin 2\beta}{g} = R + p \quad \dots(ii)$$

$$\frac{v^2 \sin 2\theta}{g} = R$$

(i) + (ii) gives

$$\frac{v^2}{g} (\sin 2\alpha + \sin 2\beta) = 2R = \frac{2v^2 \sin 2\theta}{g}$$

$$\Rightarrow \sin 2\theta = \frac{1}{2} (\sin 2\alpha + \sin 2\beta)$$

150. The linear programming problem:

Maximize $z = x_1 + x_2$

Subject to constraints

$$x_1 + 2x_2 \leq 2000$$

$$x_1 + x_2 \leq 1500$$

$$x_2 \leq 600$$

$$x_1 \leq 0 \text{ has}$$

- | | |
|--|--|
| (1) no feasible solution | (2) unique optimal solution |
| (3) a finite number of optimal solutions | (4) infinite number of optimal solutions |

Sol: Ans [4] Shaded portion represents feasible region.

at A, $z = x_1 + x_2 = 600$

at B, $z = x_1 + x_2 = 800 + 600 = 1400$

at C, $z = x_1 + x_2 = 1500$

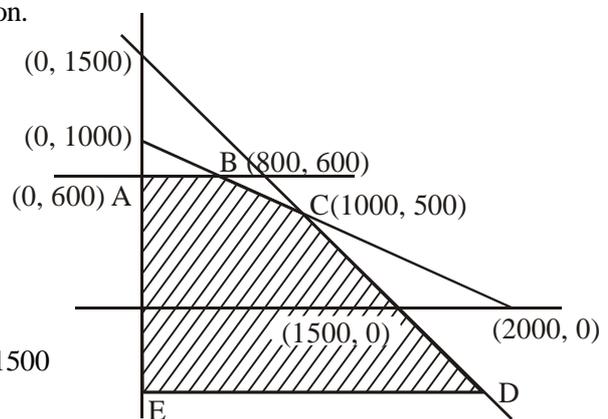
going along AE, z decreases

from 600 $\Rightarrow z < 600$

going along CD, z remains constant at 1500

(since CD is graph of $x_1 + x_2 = 1500$)

so there are infinite optional solutions.



151. A_1, A_2, \dots, A_n are thirty sets, each with five elements and B_1, B_2, \dots, B_n are n sets, each with three elements. Let:

$$\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^n B_j = S$$

If each element of S belongs to exactly ten of A_i 's and exactly nine of the B_j 's then n is

- (1) 45 (2) 35 (3) 40 (4) 30

Sol: Ans [1] Total no. of elements in all A_i 's = $30 \times 5 = 150$

Let no. of elements in $S = x$

since each element of S , belongs to exactly A_i 's, no. of elements is A_i 's = $10x$

$$\Rightarrow 10x = 150$$

$$\Rightarrow x = 15$$

since each element of S belongs to exactly 9 B_j 's, no. of elements in all B_j 's = $15 \times 9 = 135$

$$\text{no. of } B_j\text{'s} = n = \frac{135}{3} = 45.$$

152. Let R and S be two non-void relations on a set A . Which of the following statements is false?

- (1) R and S are transitive implies $R \cap S$ is transitive
 (2) R and S are transitive implies $R \cup S$ is transitive
 (3) R and S are symmetric implies $R \cup S$ is symmetric
 (4) R and S are reflexive implies $R \cap S$ is reflexive

Sol: Ans [2] Let $R = \{(a, b)\}$ is transitive

and $S = \{(b, c)\}$ is transitive

$R \cup S = \{(a, b), (b, c)\}$ is not transitive.

153. Let $f : [4, \infty[\rightarrow [4, \infty[$ be defined by

$$f(x) = 5^{x(x-4)}$$

Then $f^{-1}(x)$

- (1) $2 - \sqrt{4 + \log_5 x}$ (2) $2 + \sqrt{4 + \log_5 x}$ (3) $\left(\frac{1}{5}\right)^{x(x-4)}$ (4) Not defined

Sol: Ans [2] $y = 5^{x^2-4x}$

$$\log_5 y = x^2 - 4x \Rightarrow x = 2 \pm \sqrt{4 + \log_5 y}$$

$$\text{hence } f^{-1}(x) = 2 + \sqrt{4 + \log_5 x}$$

154. The number of solutions of the equation

$$z^2 + \bar{z} = 0 \text{ is}$$

- (1) 2 (2) 4 (3) 6 (4) 8

Sol: Ans [2] $z^2 + \bar{z} = 0$

$$x^2 - y^2 + x + i(2xy - y) = 0$$

$$\Rightarrow x^2 - y^2 + x = 0 \text{ \& } 2xy - y = 0$$

155. If a, b, c are sides of a triangle, then $\frac{(a+b+c)^2}{(ab+bc+ca)}$ always belongs to:

- (1) [1, 2] (2) [2, 3] (3) [3, 4] (4) [4, 5]

Sol: Ans [3] Since $a^2 + b^2 + c^2 \geq ab + bc + ac$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ac} + 2 \geq 3$$

$$\Rightarrow [3, 4]$$

156. If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$ and if λ_1 and λ_2 are two values of λ obtained

from $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then $\frac{\lambda_1}{\lambda_2^2} + \frac{\lambda_2}{\lambda_1^2}$ equals:

- (1) 4192 (2) 4144 (3) 4096 (4) 4048

Sol: Ans [4] $\alpha + \beta = \frac{\lambda - 1}{\lambda}, \alpha\beta = \frac{5}{\lambda}$

$$\Rightarrow (\lambda - 1)^2 = 14\lambda \Rightarrow \lambda_1 + \lambda_2 = 16 \text{ \& } \lambda_1\lambda_2 = 1$$

$$\Rightarrow \frac{\lambda_1^3 + \lambda_2^3}{(\lambda_1\lambda_2)^2} = \lambda_1^3 + \lambda_2^3 = 16^3 - 3 \times 16 = 4048.$$

157. If $a, a_1, a_2, \dots, a_{2n}, b$ are in arithmetic progression and $a, g_1, g_2, \dots, g_{2n}, b$ are in geometric progression and h is the harmonic mean of a and b , then

$$\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \dots + \frac{a_n + a_{n+1}}{g_n g_{n+1}} \text{ is equal to}$$

- (1) $2nh$ (2) n/h (3) nh (4) $\frac{2n}{h}$

Sol: Ans [4] According to the given series

$$a_1 + a_{2n} = a_2 + a_{2n-1} = \dots = a_n + a_{n+1} = a + b$$

$$\text{and } g_1 g_{2n} = g_2 g_{2n-1} = \dots = g_n g_{n+1} = ab$$

$$\text{Hence value is } \frac{n(a+b)}{ab} = \frac{2n}{h}$$

158. The sum of the products of the numbers $\pm 1, \pm 2, \dots, \pm n$, taken two at a time is

(1) $\frac{-n(n+1)}{2}$

(2) $\frac{n(n+1)(2n+1)}{6}$

(3) $\frac{-n(n+1)(2n+1)}{6}$

(4) 0

Sol: Ans [4] Clearly the answer is (4)

159. $\sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1$ is equal to

(1) $\frac{n(n+1)(2n+1)}{6}$

(2) $\left[\frac{n(n+1)}{2}\right]^2$

(3) $\frac{n(n+1)}{2}$

(4) $\frac{n(n+1)(n+2)}{6}$

Sol: Ans [4] $\sum_{i=1}^n \frac{i^2+i}{2} = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} = \frac{n(n+1)(n+2)}{6}$

160. The sum of the series $\log_4 2 - \log_8 2 + \log_{16} 2 - \log_{32} 2 + \dots$ is

(1) e^2

(2) $\log_e 2 + 1$

(3) $\log_e 3 - 2$

(4) $1 - \log_e 2$

Sol: Ans [4] The given series is

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots = 1 - \log_e 2$$

161. The sum of the series $\frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + \dots$ is

(1) $e - 1$

(2) $\sqrt{e} - 1$

(3) $\sqrt{e} - 2$

(4) $\sqrt{e} + e$

Sol: Ans [2] $T_n = \frac{1.3.5 \dots (2n-1)}{1.2.3 \dots 2n} = \frac{1}{2^n (n)!}$

$$\text{Again } e^{1/2} = 1 + \frac{1}{2} + \frac{1}{2^2 \cdot 2!} + \frac{1}{2^3 \cdot 3!} + \dots$$

$$\Rightarrow \sqrt{e} - 1 = \frac{1}{2} + \frac{1}{2^2 \cdot 2!} + \dots$$

162. Range of the function $f(x) = \frac{x}{1+x^2}$ is

(1) $(-\infty, \infty)$

(2) $[-1, 1]$

(3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(4) $[-\sqrt{2}, \sqrt{2}]$

Sol: Ans [3] $f(x) = \frac{x}{1+x^2}$

since $1+x^2 \geq 2x \Rightarrow \frac{x}{1+x^2} \leq \frac{1}{2}$

163. $\lim_{x \rightarrow \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

(1) $\frac{1}{16}$

(2) $\frac{1}{8}$

(3) $\frac{1}{4}$

(4) $\frac{\pi}{2}$

Sol: Ans [1] Put $x = \frac{\pi}{2} + h$

$$\begin{aligned} \text{then } \lim_{h \rightarrow 0} \frac{\sin h - \tan h}{-8h^3} \\ &= \lim_{h \rightarrow 0} \frac{\cos h - \sec^2 h}{-24h^2} \\ &= \frac{-1 - 2\sec^3 h}{-48} = \frac{1}{16} \end{aligned}$$

164. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$$
 becomes continuous for all x , is given by

(1) $a^{3/2}$

(2) $a^{1/2}$

(3) $-a^{1/2}$

(4) $-a^{3/2}$

Sol: Ans [3] $f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$

rationalising Nr & Dr we get

$$\begin{aligned} &= \frac{-a(\sqrt{a+x} + \sqrt{a-x})}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}} \\ &= \frac{-a(2\sqrt{a})}{a+a} = -\sqrt{a} \end{aligned}$$

165. The system of equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$x + 2y + \lambda z = \mu$ has no solution if:

(1) $\lambda = 3, \mu = 10$

(2) $\lambda \neq 3, \mu = 10$

(3) $\lambda \neq 3, \mu \neq 10$

(4) $\lambda = 3, \mu \neq 10$

Sol: Ans [4] For No solution $D = 0$ & $D_1 \neq 0$

$$\Rightarrow \lambda = 3 \text{ and } \mu \neq 10.$$

166. If a_1, a_2, a_3, \dots form a geometric progression and $a_i > 0$ for all $i \geq 1$, then

$$\begin{vmatrix} \log a_m & \log a_{m+1} & \log a_{m+2} \\ \log a_{m+3} & \log a_{m+4} & \log a_{m+5} \\ \log a_{m+6} & \log a_{m+7} & \log a_{m+8} \end{vmatrix} \text{ is equal to}$$

(1) $\log a_{m+8} - \log a_m$ (2) $\log a_m$ (3) $2\log a_{m+1}$ (4) 0

Sol: Ans [4] Let $a_{m+1} = a_m R$, $a_{m+2} = a_m R^2 \dots$

$$\log a_{m+1} = \log a_m + \log R, \log a_{m+2} = \log a_m + 2\log R$$

\Rightarrow Given determinant is

$$\begin{vmatrix} \log a_m & \log a_m + \log R & \log a_m + 2\log R \\ \log a_m + 3\log R & \log a_m + 4\log R & \log a_m + 5\log R \\ \log a_m + 6\log R & \log a_m + 7\log R & \log a_m + 8\log R \end{vmatrix} = 0$$

167. If x is positive integer, then

$$\begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix} \text{ is equal to}$$

(1) $2x!(x+1)!$ (2) $2x!(x+1)!(x+2)!$
 (3) $2x!(x+3)!$ (4) $2(x+1)!(x+2)!(x+3)!$

Sol: Ans [2] Given determinant is

$$\begin{vmatrix} x! & (x+1)x! & (x+2)(x+1)x! \\ (x+1)! & (x+2)(x+1)! & (x+3)(x+2)(x+1)! \\ (x+2)! & (x+3)(x+2)! & (x+4)(x+3)(x+2)! \end{vmatrix} \\ = x!(x+1)!(x+2)! \begin{vmatrix} 1 & (x+1) & (x+2)(x+1) \\ 1 & (x+2) & (x+3)(x+2) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix} = 2x!(x+1)!(x+2)!$$

168. If $\tan\left(\frac{x}{2}\right) = \operatorname{cosec} x - \sin x$, then the value of $\tan^2\left(\frac{x}{2}\right)$ is

(1) $2 - \sqrt{5}$ (2) $2 + \sqrt{5}$ (3) $-2 - \sqrt{5}$ (4) $-2 + \sqrt{5}$

Sol: Ans [4] $\tan \frac{x}{2} = \operatorname{cosec} x - \sin x$

$$= \frac{\cos x}{\tan x} = \frac{(1 - \tan^2 x/2)^2}{2 \tan x/2(1 + \tan^2 x/2)}$$

$$\Rightarrow 2 \tan^2 x/2 (1 + \tan^2 x/2) = (1 - \tan^2 x/2)^2$$

$$\Rightarrow \tan^2 x/2 = -2 + \sqrt{5}.$$

169. Consider the system of equations in x, y, z as

$$x \sin 3\theta - y + z = 0$$

$$x \cos 2\theta + 4y + 3z = 0$$

$$2x + 7y + 7z = 0$$

If this system has a non-trivial solution, then for integer n , values of θ are given by

$$(1) \pi \left(n + \frac{(-1)^n}{3} \right) \quad (2) \pi \left(n + \frac{(-1)^n}{4} \right) \quad (3) \pi \left(n + \frac{(-1)^n}{6} \right) \quad (4) n\pi/2$$

Sol: Ans [3] For nontrivial solution

$$D = 0$$

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 7 \sin 3\theta + 14 \cos 2\theta - 14 = 0$$

$$\Rightarrow \sin \theta = 0 \text{ \& \; } \sin \theta = 1/2$$

$$\Rightarrow \theta = n\pi \text{ \& \; } \theta = n\pi + (-1)^n \pi/6.$$

170. The value of $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$ is

$$(1) 0 \quad (2) \frac{1}{2} \quad (3) \frac{3}{2} \quad (4) 1$$

Sol: Ans [3] $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$= 2 \left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} \right] = 2 \left[1 - \frac{1}{4} \right] = \frac{3}{2}$$

171. The sides BC, CA and AB of a triangle ABC are of lengths a , b and c respectively. If D is the mid point of BC and AD is perpendicular to AC, then the value of $\cos A \cos C$ is

- (1) $3(a^2 - c^2)/2ac$ (2) $2(a^2 - c^2)/3bc$ (3) $(a^2 - c^2)/3ac$ (4) $2(c^2 - a^2)/3ac$

Sol: Ans [4] Using sine rule in $\triangle ACD$,

$$\frac{\sin(90 - c)}{b} = \frac{1}{a/2} \Rightarrow \cos c = \frac{2b}{a}$$

using sine rule in $\triangle ABD$,

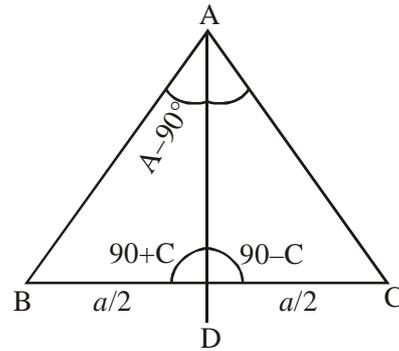
$$\frac{\sin(A - 90)}{a/2} = \frac{\sin(90 + c)}{c}$$

$$\Rightarrow -\cos A = \frac{b}{c} \Rightarrow \cos A = -\frac{b}{c}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{b}{c}$$

$$\Rightarrow c^2 - a^2 = -3b^2$$

$$\cos A \cos C = -\frac{2b^2}{ac} = \frac{2(c^2 - a^2)}{3ac}$$



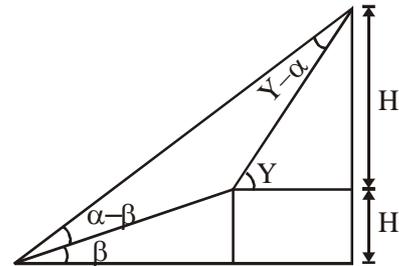
172. The angle of elevation of the top of a hill from a point is α . After walking b meters towards the top up a slope inclined at an angle β to the horizon, the angle of elevation of the top becomes γ . Then the height of the hill is

- (1) $b \sin \alpha \sin (\gamma - \beta) / \sin (\gamma - \alpha)$ (2) $b \sin \alpha \sin (\gamma - \alpha) / \sin (\gamma - \beta)$
 (3) $b \sin (\gamma - \beta) / \sin (\gamma - \alpha)$ (4) $\sin (\gamma - \beta) / b \sin \alpha \sin (\gamma - \alpha)$

Sol: Ans [1] Height = $H_2 + H_1$

$$= b \sin \beta + \frac{b \sin (\alpha - \beta) \sin \gamma}{\sin (\gamma - \alpha)}$$

$$= \frac{b \sin \alpha \sin (\gamma - \beta)}{\sin (\gamma - \alpha)}$$



173. The coefficient of the term independent of x is the expansion of

$$\left[\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}} \right]^{10} \text{ is}$$

- (1) 210 (2) 105 (3) 70 (4) 112

Sol: Ans [1]
$$\left[\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}} \right]^{10}$$

$$= \left[(x^{1/3} + 1) - \frac{(x^{1/2} + 1)}{x^{1/2}} \right]^{10}$$

$$= \left[x^{1/3} - \frac{1}{x^{1/2}} \right]^{10}$$

$$T_{r+1} = {}^{10}C_r \cdot x^{\frac{10-r}{3}} (x^{-1/2})^r$$

$$\Rightarrow \frac{20-5r}{6} = 0 \Rightarrow r = 4. \Rightarrow \text{coefficient} = {}^{10}C_4 = 210.$$

174. $7^9 + 9^7$ is divisible by

- (1) 128 (2) 24 (3) 64 (4) 72

Sol: Ans [3] $7^9 = (8-1)^9 = -1 + {}^9C_1 \cdot 8^1 - {}^9C_3 \cdot 8^3 + \dots + {}^9C_9 \cdot 8^9$

$9^7 = (8+1)^7 = 1 + {}^7C_1 \cdot 8 + {}^7C_2 \cdot 8^2 + {}^7C_3 \cdot 8^3 + \dots$

$7^9 + 9^7 = 16 \times 8 + 8^2({}^7C_2 - {}^9C_2) + 8^3(\dots)$

$= 64 + 8^3 \cdot A$

Which is divisible by 64.

175. The value of $C(47, 4) + \sum_{r=1}^5 C(52-r, 3)$ is

- (1) $C(52, 4)$ (2) $C(51, 4)$ (3) $C(52, 3)$ (4) $C(51, 3)$

Sol: Ans [1] $C(47, 4) + \sum_{r=1}^5 C(52-r, 3)$

${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{52}C_4.$

176. Let $\vec{a} = 2\vec{i} + \vec{j} - 2\vec{k}$, $\vec{b} = \vec{i} + \vec{j}$. If \vec{c} is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$ and $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and angle between $\vec{a} \times \vec{b}$ and \vec{c} is 30° , then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ equals:

- (1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) 2 (4) $\frac{\sqrt{3}}{2}$

Sol: Ans [1] $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c} \cdot \vec{a} = 8$

$\Rightarrow |\vec{c}| = 1$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \times 3 = \frac{3}{2}$

177. Let $\vec{a} = \vec{i} + \vec{j} - \vec{k}$; $\vec{i} - \vec{j} + \vec{k}$ and \vec{c} be a unit vector perpendicular to \vec{a} and coplanar with \vec{a} and \vec{b} , then \vec{c} is

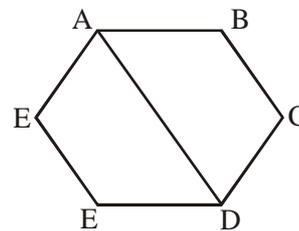
- (1) $\frac{1}{\sqrt{2}}(\vec{j} + \vec{k})$ (2) $\frac{1}{\sqrt{2}}(\vec{j} - \vec{k})$ (3) $\frac{1}{\sqrt{6}}(\vec{i} - 2\vec{j} + \vec{k})$ (4) $\frac{1}{\sqrt{6}}(2\vec{i} - \vec{j} + \vec{k})$

Sol: Ans [4] $\vec{c} = \vec{a} + \lambda\vec{b}$
 $= (1 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (\lambda - 1)\hat{k}$
 $\vec{c} \cdot \vec{a} = 0 \Rightarrow \lambda = 3$
 $\Rightarrow \vec{c} = \frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{6}}$

178. ABCDEF is a regular hexagon with centre at the origin such that $\vec{AD} + \vec{EB} + \vec{FC} = \lambda\vec{ED}$. Then λ equals

- (1) 2 (2) 4 (3) 6 (4) 3

Sol: Ans [2] $\vec{AD} = \vec{AE} + \vec{ED}$
 $\vec{EB} = \vec{ED} + \vec{DB}$
 $\vec{FC} = 2\vec{ED}$
 $\vec{AD} + \vec{EB} + \vec{FC} = 4\vec{ED}$



179. A parallelepiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelepiped is

- (1) 7 (2) 9 (3) 11 (4) $\sqrt{155}$

Sol: Ans [1] Length = $\sqrt{(5-2)^2 + (9-3)^2 + (7-5)^2}$
 $= \sqrt{9+36+4} = 7$

180. A non-zero vector \vec{a} is parallel to the line of intersection of the plane determined by the vectors \vec{i} , $\vec{i} + \vec{j}$ and the plane determined by the vectors $\vec{i} - \vec{j}$, $\vec{i} + \vec{k}$. The angle between \vec{a} and $\vec{i} - 2\vec{j} + 2\vec{k}$ is

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Sol: Ans [4] A vector normal to first plane is $\hat{i} \times (\hat{i} + \hat{j}) = \hat{k}$
 A vector normal to second plane is $(\hat{i} - \hat{j}) \times (\hat{i} + \hat{k}) = -\hat{j} + \hat{k} - \hat{i}$
 then $\vec{a} = \hat{k} \times (-\hat{j} + \hat{k} - \hat{i}) = \hat{i} - \hat{j}$
 The angle between \vec{a} and given vector is $\frac{\pi}{4}$.



