THEORY OF EQUATIONS:
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(1) If an equation (i:e $f(x)=0$) contains all positive co-efficients of any powers of x , it has no positive roots then.
eg: $x^4+3x^2+2x+6=0$ has no positive roots .

(2) For an equation , if all the even powers of x have some sign coefficients and all the odd powers of x have the opposite sign coefficients , then it has no negative roots .

(3)Summarising DESCARTES RULE OF SIGNS:

For an equation $f(x)=0$, the maximum number of positive roots it can have is the number of sign changes in $f(x)$; and the maximum number of negative roots it can have is the number of sign changes in $f(-x)$.
Hence the remaining are the minimum number of imaginary roots of the equation(Since we also know that the index of the maximum power of x is the number of roots of an equation.)

(4) Complex roots occur in pairs, hence if one of the roots of an equation is $2+3i$, another has to be $2-3i$ and if there are three possible roots of the equation , we can conclude that the last root is real . This real roots could be found out by finding the sum of the roots of the equation and subtracting $(2+3i)+(2-3i)=4$ from that sum. (More about finding sum and products of roots next time)

07/10/2002 THEORY OF EQUATIONS
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(1) For a cubic equation $ax^3+bx^2+cx+d=o$

sum of the roots = $-\ b/a$
sum of the product of the roots taken two at a time = c/a
product of the roots = $-d/a$

(2) For a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$

sum of the roots = $-\ b/a$
sum of the product of the roots taken three at a time = c/a
sum of the product of the roots taken two at a time = $-d/a$
product of the roots = e/a

(3) If an equation $f(x)= 0$ has only odd powers of x and all these have the same sign coefficients or if $f(x) = 0$ has only odd powers of x and all these have the same sign
coefficients then the equation has no real roots in each case(except for $x=0$ in the second case.

(4) Besides Complex roots , even irrational roots occur in pairs. Hence if 2+root(3) is a root , then even 2-root(3) is a root .
(All these are very useful in finding number of positive , negative , real ,complex etc roots of an equation)

Today's Section:

08/10/2002
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(1) If for two numbers x+y=k(=constant), then their PRODUCT is MAXIMUM if
x=y(=k/2). The maximum product is then (k^2)/4 .

(2) If for two numbers x*y=k(=constant), then their SUM is MINIMUM if
x=y(=root(k)). The minimum sum is then 2*root(k) .

(3) |x| + |y| >= |x+y| (|| stands for absolute value or modulus)
(Useful in solving some inequations)

(4) Product of any two numbers = Product of their HCF and LCM .
Hence product of two numbers = LCM of the numbers if they are prime to each other .

1) For any regular polygon , the sum of the exterior angles is equal to 360 degrees
hence measure of any external angle is equal to 360/n. (where n is the number of sides)

(2) If any parallelogram can be inscribed in a circle , it must be a rectangle.

(3) If a trapezium can be inscribed in a circle it must be an isosceles trapezium (i:e oblique sies equal).

(4) For an isosceles trapezium , sum of a pair of opposite sides is equal in length to the sum of the other pair of opposite sides .(i:e AB+CD = AD+BC , taken in order) .

(5) Area of a regular hexagon : root(3)*3/2*(side)*(side)

1) For any 2 numbers a>b

a>AM>GM>HM>b (where AM, GM ,HM stand for arithmetic, geometric , harmonic menasa respectively)

(2) (GM)^2 = AM * HM

(3) For three positive numbers a, b ,c

(a+b+c) * (1/a+1/b+1/c)>=9

(4) For any positive integer n

2<= (1+1/n)^n <=3

(5) a^2+b^2+c^2 >= ab+bc+ca
If a=b=c , then the equality holds in the above.

(6) a^4+b^4+c^4+d^4 >=4abcd

(7) (n!)^2 > n^n (! for factorial)

This is for 21/10/2002
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(1) If a+b+c+d=constant , then the product a^p * b^q * c^r * d^s will be maximum

if a/p = b/q = c/r = d/s .

(2)Consider the two equations

a1x+b1y=c1
a2x+b2y=c2

Then ,
If a1/a2 = b1/b2 = c1/c2 , then we have infinite solutions for these equations.
If a1/a2 = b1/b2 <> c1/c2 , then we have no solution for these equations.(<> means not equal to)
If a1/a2 <> b1/b2 , then we have a unique solutions for these equations..

(3) For any quadrilateral whose diagonals intersect at right angles , the area of the quadrilateral is 0.5*d1*d2, where d1,d2 are the lenghts of the diagonals.

(4) Problems on clocks can be tackled as assuming two runners going round a circle , one 12 times as fast as the other . That is ,
the minute hand describes 6 degrees /minute
the hour hand describes 1/2 degrees /minute .

Thus the minute hand describes 5(1/2) degrees more than the hour hand per minute .

(5) The hour and the minute hand meet each other after every 65(5/11) minutes after being together at midnight.
(This can be derived from the above) .

1)If n is even , n(n+1)(n+2) is divisible by 24

(2)If n is any integer , n^2 + 4 is not divisible by 4

(3)Given the coordinates (a,b) (c,d) (e,f) (g,h) of a parallelogram , the coordinates of the meeting point of the diagonals can be found out by solving for
[(a+e)/2,(b+f)/2] =[(c+g)/2 , (d+h)/2]

(4)Area of a triangle
1/2*base*altitude = 1/2*a*b*sinC = 1/2*b*c*sinA = 1/2*c*a*sinB = root(s*(s-a)*(s-b)*(s-c)) where s=a+b+c/2
=a*b*c/(4*R) where R is the CIRCUMRADIUS of the triangle = r*s ,where r is the inradius of the triangle .

(5)In any triangle
a=b*CosC + c*CosB
b=c*CosA + a*CosC
c=a*CosB + b*CosA

(6)If a1/b1 = a2/b2 = a3/b3 = ... , then each ratio is equal to
(k1*a1+ k2*a2+k3*a3+..............) / (k1*b1+ k2*b2+k3*b3+..............) , which is also equal to
(a1+a2+a3+............./b1+b2+b3+..........)

(7)In any triangle
a/SinA = b/SinB =c/SinC=2R , where R is the circumradius

cosC = (a^2 + b^2 - c^2)/2ab

$\sin 2A = 2 \sin A * \cos A$
$\cos 2A = \cos^2(A) - \sin^2(A)$

1) $x^n - a^n = (x-a)(x^{(n-1)} + x^{(n-2)} + \ldots + a^{(n-1)})$ ......Very useful for finding multiples .For example (17-14=3 will be a multiple of $17^3 - 14^3$)

(2) $e^x = 1 + (x)/1! + (x^2)/2! + (x^3)/3! + \ldots$to infinity
(2a) $2 < e < 3$

(3) $\log(1+x) = x - (x^2)/2 + (x^3)/3 - (x^4)/4 \ldots$to infinity [Note the alternating sign . .Also note that the ogarithm is with respect to base e]

(4) In a GP the product of any two terms equidistant from a term is always constant .

(5) For a cyclic quadrilateral , area = $\text{root}((s-a) * (s-b) * (s-c) * (s-d))$, where $s=(a+b+c+d)/2$

(6) For a cyclic quadrilateral , the measure of an external angle is equal to the measure of the internal opposite angle.

(7) $(m+n)!$ is divisible by $m! * n!$.

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"I have miles to go before I sleep

02/11/2002
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(1) If a quadrilateral circumscribes a circle , the sum of a pair of opposite sides is equal to the sum of the other pair .

(2)The sum of an infinite GP = $a/(1-r)$, where a and r are resp. the first term and common ratio of the GP .

(3)The equation whose roots are the reciprocal of the roots of the equation ax^2+bx+c is cx^2+bx+a

(4) The coordinates of the centroid of a triangle with vertices (a,b) (c,d) (e,f) is$((a+c+e)/3 , (b+d+f)/3)$.

(5) The ratio of the radii of the circumcircle and incircle of an equilateral triangle is 2:1 .

(6) Area of a parallelogram = base * height

(7)APPOLLONIUS THEOREM:

In a triangle , if AD be the median to the side BC , then
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ or $2(AD^2 + DC^2)$.

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1) for similar cones , ratio of radii = ratio of their bases.

(2) The HCF and LCM of two nos. are equal when they are equal .

(3) Volume of a pyramid = $1/3 * \text{base area} * \text{height}$

(4) In an isosceles triangle , the perpendicular from the vertex to the base or the angular bisector from vertex to base bisects the base.

(5) In any triangle the angular bisector of an angle bisects the base in the ratio of the other two sides.

(6) the quadrilateral formed by joining the angular bisectors of another quadrilateral is always a rectangle.

(7) Roots of $x^2+x+1=0$ are $1,w,w^2$ where $1+w+w^2=0$ and $w^3=1$

($|a|+|b| = |a+b|$ if $a*b>=0$
else $|a|+|b| >= |a+b|$

(9) $2<= (1+1/n)^n <=3$

(10) WINE and WATER formula:

If Q be the volume of a vessel
q qty of a mixture of water and wine be removed each time from a mixture
n be the number of times this operation be done
and A be the final qty of wine in the mixture

then ,
$A/Q = (1-q/Q)^n$

(11) Area of a hexagon = $\text{root}(3) * 3 * (\text{side})^2$

(12) $(1+x)^n \sim (1+nx)$ if $x<<1$

(13) Some pythagorean triplets:

3,4,5 ($3^2=4+5$)
5,12,13 ($5^2=12+13$)
7,24,25 ($7^2=24+25$)
8,15,17 ($8^2 / 2 = 15+17$)
9,40,41 ($9^2=40+41$)
11,60,61 ($11^2=60+61$)
12,35,37 ($12^2 / 2 = 35+37$)
16,63,65 ($16^2 /2 = 63+65$)
20,21,29(EXCEPTION)

(14) Appolonius theorem could be applied to the 4 triangles formed in a parallelogram.

(15) Area of a trapezium = $1/2 * (\text{sum of parallel sids}) * \text{height} = \text{median} * \text{height}$
where median is the line joining the midpoints of the oblique sides.

(16) when a three digit number is reversed and the difference of these two numbers is taken , the middle number is always 9 and the sum of the other two numbers is always 9 .

(17) ANy function of the type $y=f(x)=(ax-b)/(bx-a)$ is always of the form $x=f(y)$.

(1 Let W be any point inside a rectangle ABCD .
Then
WD^2 + WB^2 = WC^2 + WA^2

(19) Let a be the side of an equilateral triangle . then if three circles be drawn inside this triangle touching each other then each's radius = a/(2*(root(3)+1))

(20) Let 'x' be certain base in which the representation of a number is 'abcd' , then the decimal value of this number is a*x^3 + b*x^2 + c*x + d

5) For a cyclic quadrilateral , area = root(s* (s-a) * (s-b) * (s-c) * (s-d)) , where s=(a+b+c+d)/2

Here are some neat shortcuts on Simple/Compound Interest.

Shortcut #1:

We all know the traditional formula to compute interest...
CI = P*(1+R/100)^N - P

The calculation get very tedious when N>2 (more than 2 years). The method suggested below is elegant way to get CI/Amount after 'N' years.

You need to recall the good ol' Pascal's Triange in following way:
Code:

```
Number of Years (N)
  -------------------
      1              1
      2            1  2  1
      3          1  3  3  1
      4        1  4  6  4  1
      .      1 .... .... ... ...  1
```

Example: P = 1000, R=10 %, and N=3 years. What is CI & Amount?

Step 1: 10% of 1000 = 100, Again 10% of 100 = 10 and 10% of 10 = 1
We did this three times b'cos N=3.

Step 2:
Now Amount after 3 years = 1 * 1000 + 3 * 100 + 3 * 10 + 1 * 1 = Rs.1331/-
The coefficents - 1,3,3,1 are lifted from the pascal's triangle above.

Step 3:
CI after 3 years = 3*100 + 3*10 + 3*1 = Rs.331/- (leaving out first term in step 2)

If N =2, we would have had, Amt = 1 * 1000 + 2 * 100 + 1 * 10 = Rs. 1210/-
CI = 2 * 100 + 1* 10 = Rs. 210/-

This method is extendable for any 'N' and it avoids calculations involving higher powers on 'N' altogether!

A variant to this short cut can be applied to find depreciating value of some property. (Example, A property worth 100,000 depreciates by 10% every year, find its value after 'N' years).

Shortcut #2:

(i) When interest is calculated as CI, the number of years for the Amount to double (two times the principal) can be found with this following formula:
P * N ~ 72 (approximately equal to).
Exampe, if R=6% p.a. then it takes roughly 12 years for the Principal to double itself.

Note: This is just a approximate formula (when R takes large values, the error % in formula increases).

(ii) When interest is calculated as SI, number of years for amt to double can be found as:
N * R = 100 . BTW this formula is exact!

Adding to what 'Peebs' said, this shortcut does work for any P/N/R.

Basically if you look closely at this method, what I had posted is actually derived from the Binomial expansion of the polynomial -- (1+r/100)^n but in a more "edible" format digestable by us!

BTW herez one shortcut on recurring decimals to fractions ...Its more easier to explain with an example..

Eg: 2.384384384 ....

Step 1: since the 3 digits '384' is recurring part, multiply 2.384 by 1000 = so we get 2384.

Next '2' is the non recurring part in the recurring decimal so subtract 2 from 2384 = 2382.

If it had been 2.3848484.., we would have had 2384 - 23 = 2361. Had it been 2.384444.. NR would be 2384 - 238 = 2146 and so on.

We now find denominator part ....

Step 3: In step 1 we multiplied 2.384384... by 1000 to get 2384, so put that first.

Step 4: next since all digits of the decimal part of recurring decimal is recurring, subtract 1 from step 3. Had the recurring decimal been 2.3848484, we need to subtract 10. If it had been 2.3844444, we needed to have subtracted 100 ..and so on...

Hence here, DR = 1000 - 1 = 999

Hence fraction of the Recurring decimal is 2382/999!!

Some more examples ....

1.56787878 ... = (15678 - 156) / (10000 - 100) = 15522/9900

23.67898989... = (236789 - 2367) / (10000 - 100) = 234422/9900

124.454545... = (12445 - 124) / (100 - 1) = 12321/99