BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2008

CS-60 : FOUNDATION COURSE IN MATHEMATICS IN COMPUTING

Time : 3 hours *Maximum Marks : 75*

Note : *Question No. 1 is* ***compulsory****. Attempt any* ***three*** *questions from Questions No. 2 to 6. Use of calculator is permitted.*

1. (a) Fill in the blanks in the following questions :

(i) If a line makes angles α, β, γ with the coordinate axes, then

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = \ldots\ldots\ldots\ldots$$

(ii) The length of the line whose projections on the axes are 2, 3, 6 is ....

(iii) Volume of the sphere

$x^2 + y^2 + z^2 + 2x - 4y + 8z - 2 = 0$ is ....

(b) Find the roots of the equation

$$(x^2 - 5x)^2 - 30(x^2 - 5x) - 216 = 0$$

(c) Find the equations of the lines which pass through (4, 5) and make an angle of 45° with the line $2x + y + 1 = 0$.

(d) Find the equation of the circle which is concentric with $x^2 + y^2 - 8x + 12y + 43 = 0$ and passes through (6, 2).

(e) Evaluate

$$\lim_{x \to 0} \frac{x + \sin x}{x^2 + x}$$

(f) State whether it is even or odd for the following functions :

(i) $f(x) = 7x^2 - 11$

(ii) $f(x) = e^{3x} - e^{-3x}$

(g) Verify $(A \cup B)^C = A^C \cap B^C$, where

$A = \{1, 3, 4, 5, 9\}$, $B = \{2, 4, 6, 9, 10\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(h) The power transmitted by a belt is proportional to

$$T v - \frac{W v^3}{g}$$

where v = speed of the belt, T = tension on the driving side, and W = weight per unit length of belt. Find the speed at which the transmitted power is maximum.

(i) Evaluate

$$\int (\log x^3 + 9 \sin^3 x)\,(27 \sin^2 x \cos x + \frac{3}{x})\, dx$$

(j) If x and y are real, solve the equation

$$\frac{ix}{1 + iy} = \frac{3x + 4i}{x + 3y}.$$

(k) Determine the equation of a circle if its centre is (8, –6) and which passes through the point (5, –2).

(l) Find the equation of a line perpendicular to the line $3x - 4y + 7 = 0$ and which passes through the point (–3, 2).

(m) Find the value of the determinant :

$$\begin{vmatrix} x & x+4y & 2y \\ 7y & 13y & 3y \\ 3z & 3z+16x & 8x \end{vmatrix}$$

(n) Solve the following equations by Cramer's rule :

$$x + y + z = 1$$
$$x + 2y = 3$$
$$x + 2y + z = 7$$

(o) Can Rolle's theorem be applied to the function

$$f(x) = \sin^2 x$$

on the interval $[0, \pi]$? Find 'c' in case it can be applied. *15×3=45*

2. (a) Evaluate any ***one*** of the following :

(i) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(ii) $\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$

(b) If $\sin y = x \sin (a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2 (a+y)}{\sin a}$$

(c) Find $\frac{dy}{dx}$ for each of the following, where

(i) $y = \cos^{-1} (4x^3 - 3x)$

(ii) $y = x^{(x^x)}$ 3+3+4

3. (a) Integrate any ***one*** of the following :

(i) $\int e^{3x} \sin x \, dx$

(ii) $\int \frac{1}{e^x - 1} \, dx$

(b) Evaluate

$$\int_0^4 e^{2x} \, dx$$

(c) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. 3+3+4

4. (a) Express $\frac{(1+i)(2+i)}{3+i}$ in the form $a + ib$.

(b) Find the value of 'k' for which the function

$$f(x) = \begin{cases} \frac{\sin 5x}{3x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

is continuous at $x = 0$.

(c) A curve is drawn to pass through the points given by the following table :

x	y
1	2
1·5	2·4
2	2·7
2·5	2·8
3	3
3·5	2·6
4	2·1

Estimate the area bounded by the curve, the x-axis and the lines $x = 1$, $x = 4$. *3+3+4*

5. (a) Find the equation of the circle with centre (1, 1) and which touches the line $x + y = 1$.

(b) Find the focus, vertex, length of latus rectum, equation of the directrix of the parabola $y^2 = -4x$.

(c) Find the eccentricity, foci, length of the latus rectum of the ellipse

$$3x^2 + 4y^2 - 12x - 8y + 4 = 0.$$ *3+3+4*

6. (a) Find the equation of a sphere with centre (–1, 4, –5) and radius as 5 units.

(b) Find the equation of a right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, –3, 6.

(c) Find the equation of a cone whose vertex is at the origin and the guiding curve is

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1, \quad x + y + z = 1.$$

3+3+4