Engg. Mathematics - III 9/11/07 (16)

903–08. **(REVISED COURSE)** **RC–8702**

(3 Hours) **[Total Marks : 100**

1) Question No. **1** is **compulsory**.
2) Attempt any **four** questions from remaining **six** questions.
3) **Figures** to the **right** indicate **full** marks.

Use Laplace transform to evaluate **5**

$$\int_0^{\infty} e^{-t}\ \frac{(1-\cos 2t)}{2t}\ dt$$

S.T. $v = e^{2x}(y\cos 2y + x\sin 2y)$ is harmonic. Find the harmonic conjugate function u and the analytic function $f(z)$. **5**

Find complex form of Fourier series for $f(x) = e^{-ax}$ in $(-\pi, \pi)$ **5**

Evaluate $\displaystyle\int_c \frac{\cos(\pi z^2)}{(z^2 - 3z + 2)}\, dz$ where c is the circle $|z| = 3$ **5**

Find (i) $L\left\{e^{-2t}\displaystyle\int_0^t t\cos 3t\, dt\right\}$ **4**

(ii) $L\left\{\dfrac{\sinh 2t\ \sin 3t}{t}\right\}$ **4**

Define orthogonal and orthonormal set of functions S.T. $\{\cos n x\}$ n = 1, 2 ..... is orthogonal set of functions over $[-\pi, \pi]$ **6**

Find the image of the strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Draw the rough sketches. **6**

Obtain Half-range sine series for $f(x) = \pi x - \pi^2$ in $(0, \pi)$ and hence deduce that **6**

$\dfrac{1}{1^6} + \dfrac{1}{3^6} + \dfrac{1}{5^6} + \ldots\ldots\ldots = \dfrac{\pi^6}{960}$ (use Parseval's identity)

Evaluate (i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{13 - 5\cos\theta}$ **4**

(ii) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2-4)^2}$ **4**

Solve using Laplace Transform **6**

$(D^2 + D)y = t^2 + 2t$ at $y(0) = 4$ and $y'(0) = 2$

4. (a) Find (i) $L^{-1}\left[\dfrac{e^{-\pi s}}{s^2(s^2+1)}\right]$

(ii) $L^{-1}\left[\dfrac{(s-2)^2}{(s^2-4s+8)^2}\right]$

(b) Obtain Laurent's series for $f(z) = \dfrac{1}{z^2+4z+3}$

when (i) $1 < |z| < 3$
(ii) $|z| > 3$

(c) Obtain Fourier series of $x\cos x$ in $(-\pi, \pi)$.

5. (a) Using Cauchy's residue theorem evaluate –

(i) $\oint_c \dfrac{z+3}{2z^2+3z-2}\,dz$, where c is $|z-i| = 2$

(ii) $\oint e^{\frac{1}{z}}\sin\left(\dfrac{1}{z}\right)dz$, where c is $|z| = 1$

(b) Find Laplace Transform of

$f(t) = \left(\dfrac{\pi - t}{2}\right)^2$; $0 < t < 2\pi$ and $f(t) = f(t+2\pi)$

(c) Find the Fourier sine integral for
$f(x) = e^{-ax}$, $(a > 0)$

6. (a) (i) Evaluate $L\left[t^2 H(t-2) + t^3\delta(t-3)\right]$

(ii) Find L^{-1}, By convolution theorem $L^{-1}\left[\dfrac{s}{(s+a)^2}\right]$

(b) Find the Fourier cosine transform of the function –
$f(x) = \cos x$; $0 < x < a$
$= 0$; $x \geq a$

(c) If f(z) is an analytic function, prove that –

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^n = n^2|f(z)|^{n-2}\,|f'(z)|^2$$

7. (a) Expand $f(x) = 4 - x$; $3 < x < 4$
$= x - 4$; $4 < x < 5$
has no sine terms

(b) Find the bilinear transformation which maps the points $z = 1, -1, \infty$ onto the po
$w = 1 + i,\ 1 - i,\ 1,$ •

(c) Evaluate $\int_c |z|^2\,dz$ where 'c' is the boundary of the square 'c' with the ver
(0, 0), (1, 0), (1, 1), (0, 1).