Paper - II

151.a. We have $\alpha^2 = 5\alpha - 3$

$$\Rightarrow \alpha^2 - 5\alpha + 3 = 0 \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$$

$$\text{Similary, } \beta^2 = 5\beta - 3 : \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$$

$$\therefore \alpha = \frac{5+\sqrt{13}}{2} \text{ and } \beta = \frac{5-\sqrt{13}}{2} \text{ or vice - versa}$$

$$\alpha^2 + \beta^2 = \frac{50+26}{4} = 19 \;\&\; \alpha\beta = \frac{1}{4}(25-13) = 3$$

Thus, the equation having $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ as its roots is

$$x^2 - x\left[\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right] + \frac{\alpha\beta}{\alpha\beta} = 0 \quad \Rightarrow x^2 - x\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) + 10$$

or $3x^2 - 19x + 1 = 0$

152.**a** $y = (x + \sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2}.2x\right);$$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\;\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}} = \frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$$

$$\text{or} \sqrt{1+x^2}\,\frac{dy}{dx} = ny \;\; \text{or} \sqrt{1+x^2}\;\; y_1 = ny \;\; (y_1 = \frac{dy}{dx})$$

$$\text{Squaring, } (1+x^2)y_1^2 = n^2y^2$$

$$\text{Differentiating}, (1+x^2)2y_1y_2 + y_1^2.2x = n^2.2yy_1$$

$$(\text{Here, } y_2 = \frac{d^2y}{dx^2}) \text{ or } (1+x^2)y_2 + xy_1 = x^2y$$

153.c. $1, \log_9(3^{1-x}+2), \log_3(4.3^x - 1)$ are in A.P.

$$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4.3^x - 1)$$

$$\log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$$

$$\log_3(3^{1-x}+2) = \log_3[3(4.3^x - 1)]$$

$$3^{1-x} + 2 = 3(4.3^x - 1)(\text{Put } 3^x = t)$$

$3.3^{-x} + 2 = 12.3^x - 3$

$$\frac{3}{t} + 2 = 12t - 3 \;\; \text{or} \;\; 12t^2 - 5t - 3 = 0:$$

$$\text{Hence } t = -\frac{1}{3}, \frac{3}{4} \Rightarrow 3^x = \frac{3}{4}$$

$$\Rightarrow x = \log_3\left(\frac{3}{4}\right) \text{or } x = \log_3 3 - \log_3 4 \Rightarrow x = 1 - \log_3 4$$

154.a. $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$;

$$P(E_1 U E_2 U E_3) = 1 - P(\bar{E}_1)\,P(\bar{E}_2)\,P(\bar{E}_3)$$

$$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{3}{4}$$

155.b. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; Period $= \frac{2\pi}{2} = \pi$

156.d. $l = AR^{p-1} \Rightarrow \log l = \log A + (p-1)\log R$

$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1)\log R$

$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$

Now,

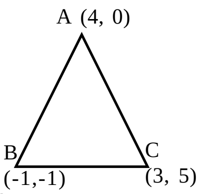
$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & P & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (q-1)\log R & r & 1 \end{vmatrix} = 0$$

157.a. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2x}} \Rightarrow \lim_{x\to 0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2x}}$;

$$\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2x}} \Rightarrow \lim_{x\to 0}\frac{|\sin x|}{x}$$

the function does not exist of LHS $\neq$ RHS

158.a. $AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$



$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$;

So. In isosceles triangle side AB = CA

For right angled triangle. $BC^2 = AB^2 + AC^2$

So, here BC = $\sqrt{52}$ or $BC^2 = 52$ or

$$(\sqrt{26})^2 + (\sqrt{26})^2 = 52$$

So, given triangle is right angled and also isosceles

159.b. Total student = 100; for 70 stds. $75 \times 70 = 5250$

$\Rightarrow 7200 - 5250 = 1950$

Average of girls $= \frac{1950}{30} = 65$

160.a. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}}-\sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}}=x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}=x \Rightarrow \tan x\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

OR $\cot x=\frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$ or $\operatorname{cosec} x=\frac{1+\cos\alpha}{1-\cos\alpha}$

$$\sin x=\frac{1-\cos\alpha}{1+\cos\alpha}=\frac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1} \text{ or } \sin x=\tan^2\frac{\alpha}{2}$$

161.c. Order = 3, degree = 3

162.a. $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ .....(i)

a (x - 4) + b (y - 7) + c (z-4) = 0.....(ii)
Line passing through point (3, 2, 0)
a + 5x + 4c +0 ....(iii)
Solving the equation we get by eqn (ii)
x - y + z = 1.........(iv)

163.b. $\frac{d^2y}{dx^2}=e^{-2x};\frac{dy}{dx}=\frac{e^{-2x}}{-2}+c;\quad y=\frac{e^{-2x}}{4}+cx+d$.

164.d. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}}=\lim_{x\to\infty}\left(\frac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{\frac{1}{x}}=1$

165.a. $f(x)=\sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$ exists if $-1\le\log_3\left(\frac{x}{3}\right)\le1$

$$\Leftrightarrow 3^{-1}\le\frac{x}{3}\le3^1 \Leftrightarrow 1\le x\le 9 \text{ or } x\in[1,9]$$

166.b.

167.b. $ar^4=2$

$$a\times ar\times ar^2\times ar^3\times ar^4\times ar^5\times ar^6\times ar^7\times ar^8$$

$$=a^9r^{36}=(ar^4)^9=2^9=512.$$

168.d. $\int_0^{10\pi}|\sin x|\,dx=10\left[\int_0^{\pi/2}\sin x\,dx+\int_{\pi/2}^{\pi}\right]\sin x\,dx$

$$=10[\cos x]_0^{\pi/2}+[\cos x]_{\pi/2}^{\pi};\quad 10[1+1]=10\times2=20$$

169.b. $\int_0^{\pi/4}\tan^n x(1+\tan^2x)\,dx=$

$$\int_0^{\pi/4}\tan^n x\sec^2 x\,dx;=\int_0^1 t^n dt \quad \text{where } t=\tan x$$

$$I_n+I_{n+2}=\frac{1}{n+1};\ \Rightarrow \lim_{x\to\infty} n[I_n+I_{n+2}]$$

$$=\lim_{x\to\infty} n.\frac{1}{n+1}=\frac{n}{n+1}=\frac{n}{n\left(1+\frac{1}{n}\right)}=1$$

170.c. $\int_1^0 [x^2]\,dx+\int_1^{\sqrt2}[x^2]dx=0+\int_1^{\sqrt2}dx=\sqrt2-1$

171.b. $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2x}dx=\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2x}+2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2x}$

$$=0+4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2x};\ 1=4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}$$

$$1=4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2x}\Rightarrow 1=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2x}$$

$$-4\pi\int\frac{x\sin x}{1+\cos^2x};\Rightarrow 2I=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2x}dx$$

put cos x = t and solve it.

172.c. we have, $\lim_{x\to2}\frac{xf(2)-2f(x)}{x-2}$

$$\left(\frac{0}{0}\right)=\lim_{x\to2}f(2)-2f'(x)=f(2)-2f'(2)=4-2\times4=-4$$

173.b. Let $|z|=|\omega|=r$

$\therefore z=re^{i\theta}\ \omega=re^{i\phi}$ where $\theta+\phi=\pi$.

$\therefore \overline{\omega}=re^{-i\phi}$

$\therefore z=re^{i(\pi-\phi)}=re^{i\pi}.e^{-i\phi}=-re^{-i\phi}=-\overline{\omega}$

174.c. Given $|z-4|<|z-2|$ Let $z=x+iy$

$$\Rightarrow|(x-4)+iy)|<|(x-2)+iy|$$

$$\Rightarrow(x-4)^2+y^2<(x-2)^2+y^2$$

$$\Rightarrow x^2-8x+16<x^2-4x+4$$

$$\Rightarrow 12<4x\Rightarrow x>3\Rightarrow \operatorname{Re}(z)>3$$

175.b.

176.b. Let a = first term of G.P.
r = common ratio of G.P. Then G.P. is a, ar, ar^2

Given $S_\infty=20\Rightarrow\frac{a}{1-r}=20\Rightarrow a=20(1-r)$ ...(i)

Also $a^2+a^2r^2+a^2r^4+.....\text{to }\infty=100$

$$\Rightarrow\frac{a^2}{1-r^2}=100\Rightarrow a^2=100(1-r)(1+r)\text{ ......(ii)}$$

From (i), $a^2=400(1-r)^2$;From (ii) and (iii), we get

$100(1-r)(1+r)=400(1-r)^2$

$$\Rightarrow 1+r=4-4r\Rightarrow 5r-3\Rightarrow r=3/5.$$

177.a. $1^3-2^3+3^3-4^3+....+9^3=1^3+3^3+5^3$

$$+...9^3-(2^3+4^3+...+8^3)=S_1-S_2.$$

For $S_1, t_n=(2n-1)^3=8n^3-12n^2+6n-1$

$$S_1=\sum t_n=8\sum n^3-12\sum n^2+6\sum n-\sum 1$$

$$=\frac{8n^2(n+1)^2}{4}-\frac{12n(n+1)(2n+1)}{6}+\frac{6n(n+1)}{2}-n$$

Here n = 5. Hence,

$S_1 = 2\times 25\times 36 - 2\times 5\times 6\times 11+3\times 30-5$

= 1800 - 660 + 90 - 5 =1890 - 665 = 1225.

For $S_2, t_n = 8n^3$; $S_2 = \sum t_n = 8\sum n^3$

$= \frac{8n^2(n+1)^2}{4} = 2\times 16\times 25 = 800.$(for n = 4)

$\therefore$ Required sum = 1225 -800 = 425.

178.a.Let α, β and y, δ are the roots of the equations.

$x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ ·

$\therefore \alpha+\beta = -a,\ \alpha\beta = b$ and $y+\delta = -b, y\,\delta = a.$

Given $\alpha - \beta = y - \delta \Rightarrow (\alpha-\beta)^2 = (y-\delta)^2$

$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = (y+\delta)^2 - 4y\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a \Rightarrow (a^2 - b^2) + 4(a-b) = 0$

$\Rightarrow a + b + 4 = 0 (\because a \neq b)$

179.c.

180.a.p + q = -p and pq = q $\Rightarrow$ q (p -1) = 0 $\Rightarrow$ q = 0 or p =1.

If q = 0, then p = 0. i.e., p = q. $\therefore$ p = 1 and q = -2

181.a.ab + bc +ca = $\frac{(a+b+c)^2 - 1}{2} < 1$

182.d. Required number of numbers

= $5\times 6\times 6\times 4 = 36\times 20 = 720$.

183.c.Required number of numbers = $3\times 5\times 5\times 5 = 375$

184.d.Required number are =5! + 5! + - 4! = 216.

185.b. Required sum = (2 + 4 + 6 +.....+ 100) +

(5 + 10 + 15+.....+ 100) - (10 + 20 +....+100)

= 2550 + 1050 -530 = 3050.

186.a.we have $t_{p+1} = {}^{p+q}C_p x^p$ and $t_{q+1} = {}^{p+q}C_q x^q$

${}^{p+q}C_p = {}^{p+q}C_q$.

187.c.we have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;

so middle term = t_7;

$t_7 = t_{6+1} = {}^{12}C_6 = \frac{12!}{6!6!} = 924$.

188.c.

189.c. $t_{r+2} = {}^{2n}C_{r+1} x^{r+1}$; $t_{3r} = {}^{2n}C_{3r-1} x^{3r-1}$

Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$; $\Rightarrow {}^{2n}C_{2n-(r+1)}$

$= {}^{2n}C_{3r-1} \Rightarrow 2n - r - 1 = 3r - 1 \Rightarrow 2n - 4r$

190.c.we have $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ By

$R_3 \to R_3 - (xR_1 + R_2)$; $= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+x) \end{vmatrix}$

$= (ax^2 + 2bx + c)(b^2 - ac) = (+)(-) = -ve.$

191.b. $a_1 = \sqrt{7} < 7$. Let $a_m < 7$ Then $a_m + 1 = \sqrt{7 + a_m}$

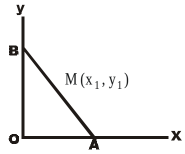
$\Rightarrow a_{m+1}^2 = 7 + a_m < 7 + 7 < 14.$

$\Rightarrow a_{m+1} < \sqrt{14} < 7.$; So $a_n < 7\ \forall$ n. $\therefore a_n > 3$

192.a.

193.d.Equation of AB is x cos α +y sin α = p;

$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$; $\Rightarrow \frac{1}{p/\cos\alpha} + \frac{1}{p/\sin\alpha} = 1$



So co- rdinates of A and B are

$\left(\frac{p}{\cos\alpha}, o\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$;

So coordinates of midpoint of AB are

$\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right) = (x_1, y_1)$ (let); x_1

$= \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = P/2_{y1}$; $\cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow \frac{p^2}{4}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2}\right) = 1$ Locus of (x_1, y_1) is

$\frac{1}{x^2} + \frac{1}{y^2} + = \frac{4}{p^2}$.

194.a.

195.a. $3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$;

$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

196.c.Equation of circles $x^2 + y^2 = 1 = (1)^2$

$\Rightarrow x^2 + y^2 = (y - mx)^2 \Rightarrow x^2 = m^2x^2 - 2mxy$;

$\Rightarrow x^2(1 - m^2) + 2mxy = 0$

$\tan 45 = \pm\frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2}$; $\Rightarrow 1 - m^2 = \pm\ 2m$

$\Rightarrow m^2 \pm 2m - 1 = 0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}.$

197.a.Let (h, k)be the centre of any such circle. Equation of such circle is $(x - h)^2 + (y - k)^2 = 3^2$ Since (h, k) lies on $x^2 + y^2 = 25$, $\therefore h^2 + k^2 = 25$.

$x^2 + y^2 - (2xh + 2yk) + 25 = 9$; Locus of (h, k) is

$x^2 + y^2 = 16$, which clearly satisfies (a).

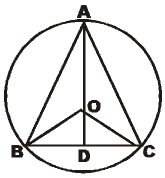
198.b.

199.c.Let ABC be an equilateral triangle, whose median is AD.

Given AD =3a.

In ΔABD, $AB^2 = AD^2 + BD^2$;

$\Rightarrow x^2 = 9a^2 + (x^2/4)$ where AB = BC = AC = x.



$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2$.

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = (3a-r)^2 + \frac{x^4}{4} \Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2$;

$\Rightarrow 6ar = 12a^2 \Rightarrow r = 2a$

So equation of circle is $x^2 + y^2 = 4a^2$

200.b. Any tangent to the parabola $y^2 = 8ax$ is

$y = mx + \frac{2a}{m}$.........(i)

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then,

$\sqrt{2a} = \pm\frac{2a}{m\sqrt{m^2+1}} \quad \Rightarrow m^2(1+m^2) = 2$

$\Rightarrow (m^2+2)(m^2-1) = 0; \quad \Rightarrow m = \pm 1.$

So, from (i), y = $\pm(\lambda + 2a)$.

201.a. $r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$;

$\Rightarrow s-a < s-b < s-c \Rightarrow -a < -b < -c; \Rightarrow a > b > c$

202.b. The given equation is tanx + secx = 2cos;

$\Rightarrow$ sinx + 1 = $2\cos^2 x$

$\Rightarrow$ sinx + 1 = $2(1 - \sin^2 x)$; $\Rightarrow 2\sin^2 x + \sin x - 1 = 0$;

$\Rightarrow$ (2sinx-1)(sinx +1) = 0 $\Rightarrow$ sinx = $\frac{1}{2}, -1.$;

$\Rightarrow x = 30^0,\ 150^0,\ 270^0$.

203.b.

204.a.we have $\lim_{n\to\infty}\frac{1^p + 2^p + ... + n^p}{n^{p+1}}$;

$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^p}{n^p.n} = \int_0^1 x^p dx = \left(\frac{x^{p+1}}{p+1}\right)_0^1 = \frac{1}{p+1}$

205.d.Since $\lim_{x\to O}|x|$ does not exist, hence the required limit does not exist.

206.a. $\lim_{n\to 1}\frac{\sqrt{f(x)-1}}{\sqrt{x-1}} \qquad \left(\frac{0}{0}\right)$ form

Using L' Hospital's rule =

$\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{f(x)}}f'(x)}{1/2\sqrt{x}} = \frac{f'(1)}{\sqrt{f(1)}} = \frac{2}{1} = 2.$

207.b.

208.d. $\therefore$ f' (x) - g''(x) = 0 Integrating, f'(x) -g (x) = c;

$\Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2$

$\therefore$ f'(x) - g'(x) = 2; Integrating, f (x) -g (x) = 2x + c_1

$\Rightarrow f(2) - g(2) = 4 + c_1 \quad \Rightarrow 9 - 3 = 4 + c_1$;

$\Rightarrow c_1 = 2 \quad \therefore f(x) - g(x) = 2x + 2$

At x = 3/2, f (x) - g(x) = 3 + 2 = 5.

209.c.f (x + y) = f (x) $\times$ f(y)

Differentiate with respect to x, treating y as constant f'(x + y) = f' (x) f (y)

Putting x = 0 any = x, we get f'(x) = f'(0) f(x);

$\Rightarrow$ f'(5) = 3 f'(5) = 3 $\times$ 2 = 6

210.a.Distance of origin from (x, y) = $\sqrt{x^2 + y^2}$

$= \sqrt{a^2 + b^2 - 2ab\cos\left(t - \frac{at}{b}\right)} = \sqrt{a^2 + b^2 - 2ab}$

$\left(\because \max.\cos\left(t - \frac{at}{b}\right) = 1\right) = a - b$

211.a.Let f(x) = $\frac{ax^3}{3} + \frac{bx^2}{2} + cx \Rightarrow$ f(0) = 0 and

f(1) = $\frac{a}{3} + \frac{b}{2} + c = \frac{2a + 3b + 6c}{6} = 0.$

Also f (x) is continuous and differentiable in [0,1] and [0, 1]. So by Rolle's theorem, f'(x) = 0 i.e $ax^2 + bx + c = 0$ has at least one root in [0,1].

212.d. we have $\int_0^2 f(x)dx = \frac{3}{4}$;

Now, $\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$

$= [x\ f(x)]_0^2 - \frac{3}{4} = 2f\ (2) - \frac{3}{4};\ = 0 - \frac{3}{4}(\because f(2) = 0) = -\frac{3}{4}.$

213.a.

214.b.we have, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{6} = 4\times 2\times\frac{\sqrt{3}}{2} = 4\sqrt{3}.$

Now, $(\vec{a}\times\vec{b})^2 + (\vec{a}.\vec{b})^2 = a^2b^2$;

$\Rightarrow \left(\vec{a}\times\vec{b}\right)^2 + 48 = 16\times 4 \quad \Rightarrow \left(\vec{a}\times\vec{b}\right)^2 = 16$

215.a.we have,

$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = \left(\vec{a}\times\vec{b}\right)\left\{\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right\}$

$= (\vec{a}\times b).\{(\vec{m}.\vec{a})\vec{c} - (\vec{m}.\vec{c})\vec{a})\}$(Where $\vec{m} = \vec{b}\times\vec{c}$)

$= \left\{(\vec{a}\times\vec{b}).\vec{c}\right\}.\left\{(\vec{a}.(\vec{b}\times\vec{c})\right\} = [\vec{a}\ \vec{b}\ \vec{c}]^2 = 4^2 = 16.$

216.a. $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a} \quad \Rightarrow (\vec{b} + \vec{c})^2 = (\vec{a})^2$

$= 5^2 + 3^2 + 2\vec{b}.\vec{c} = 7^2$

$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta = 49 - 34 = 15;$

$\Rightarrow 2\times 5\times 3\cos\ \theta = 15;$

$\Rightarrow \cos\theta = 1/2; \quad \Rightarrow \theta = \frac{\pi}{3} = 60^0$

217.a.we have, $\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow (\vec{a}+\vec{b}+\vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow 25+16+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -25 \therefore |\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}| = 25$

218.b.

219.b. we have $\vec{a}\times\vec{b} = 39\,\vec{k} = \vec{c}$

Also $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}, |\vec{c}| = 39;$

$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$

220.c.

221.a. $P(A\cup B) = P(A)+P(B)-P(A\cap B);$

$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$

$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3};$

Now, $P(\overline{A}\cap B) = P(B) - P(A\cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$.

222.d. The event follows binomial distribution with n = 5,
p= 3/6 = 1/2. q = 1 - p= 1/2,; variance = npq = 5/4.

223.b. Equation of plane through (1, 0, 0) is
a (x - 1) +by +cz = 0 ....(i)
(i) Passes through (0, 1, 0).

-a + b = 0 $\Rightarrow$ b = a ; Also, cos $45^0 = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$

$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a.$

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

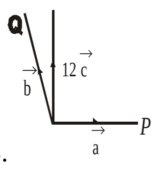
224.a.Let two forces be p and Q. Given P + Q = 18 and

$P\,\hat{a} + Q\,\hat{b} = 12\,\hat{c}; \Rightarrow P\,\hat{a} - 12\,\hat{c} = -Q\,\hat{b}$

$\Rightarrow P^2 + 144 = Q^2 = (18-P)^2;$

$\Rightarrow P^2 + 144 = 324 - 36P + p^2$

$\Rightarrow 36P = 180 \Rightarrow P = 5 \text{ and } Q = 13.$



(where $\vec{a}$ and $\vec{b}$ are unit vectors along P and Q).

225.a.