MATHEMATICS

1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(1) $\frac{1}{2}(1-\sqrt{5})$ (2) $\frac{1}{2}\sqrt{5}$

(3) $\sqrt{5}$ (4) $\frac{1}{2}(\sqrt{5}-1)$

Ans. (4)

Sol: Given $ar^{n-1} = ar^n + ar^{n+1}$
$\Rightarrow 1 = r + r^2$
$\therefore r = \frac{\sqrt{5}-1}{2}$.

2. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is

(1) 1 (2) 3
(3) 4 (4) 5

Ans. (2)

Sol: $\sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5} \Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\therefore x = 3$.

3. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then $\frac{a}{b}$ equals

(1) $\frac{5}{n-4}$ (2) $\frac{6}{n-5}$

(3) $\frac{n-5}{6}$ (4) $\frac{n-4}{5}$

Ans. (4)

Sol: ${}^nC_4\, a^{n-4}(-b)^4 + {}^nC_5\, a^{n-5}(-b)^5 = 0$

$\Rightarrow \left(\frac{a}{b}\right) = \frac{n-5+1}{5}$.

4. The set S = {1, 2, 3, …, 12) is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(1) $\frac{12!}{3!\,(4!)^3}$ (2) $\frac{12!}{3!\,(3!)^4}$

(3) $\frac{12!}{(4!)^3}$ (4) $\frac{12!}{(3!)^4}$

Ans. (3)

Sol: Number of ways is $^{12}C_4 \times {}^8C_4 \times {}^4C_4$
$= \frac{12!}{(4!)^3}$.

5. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ is defined, is

(1) $[0, \pi]$
(2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$
(4) $\left[0, \frac{\pi}{2}\right)$

Ans. (4)

Sol: f (x) is defined if $-1 \le \frac{x}{2} - 1 \le 1$ and $\cos x > 0$

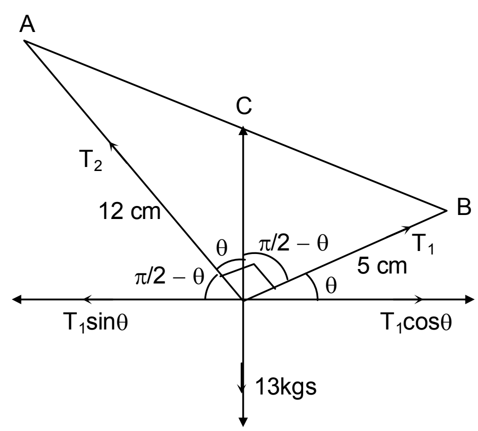
or $0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$.

6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are

(1) 12 kg and 13 kg
(2) 5 kg and 5 kg
(3) 5 kg and 12 kg
(4) 5 kg and 13 kg

Ans. (3)



Sol: $T_2 \cos\left(\frac{\pi}{2} - \theta\right) = T_1 \cos\theta \Rightarrow T_1 \cos\theta = T_2 \sin\theta$

$T_1 \sin\theta + T_2 \cos\theta = 13$.

$\because$ OC = CA = CB

$\Rightarrow \angle AOC = \angle OAC$ and $\angle COB = \angle OBC$

$\therefore \sin\theta = \sin A = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

$\Rightarrow \frac{T_1}{T_2} = \frac{5}{12} \Rightarrow T_1 = \frac{5}{12} T_2$

$T_2\left(\frac{5}{12} \cdot \frac{5}{13} + \frac{12}{13}\right) = 13$

$T_2\left(\frac{169}{12 \cdot 13}\right) = 13$

$T_2 = 12$ kgs. $\Rightarrow T_1 = 5$ kgs.

7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(1) 1/729 (2) 8/9
(3) 8/729 (4) 8/243

7. (4)

Sol: Probability of getting score 9 in a single throw = $\frac{4}{36} = \frac{1}{9}$

Probability of getting score 9 exactly twice = ${}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}$.

8. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval

(1) $0 < k < ½$ (2) $k \geq ½$
(3) $-½ \leq k \leq ½$ (4) $k \leq ½$

Ans. (2)

Sol: Equation of circle $(x - h)^2 + (y - k)^2 = k^2$
It is passing through (– 1, 1) then
$(-1 - h)^2 + (1 - k)^2 = k^2$
$h^2 + 2h - 2k + 2 = 0$
$D \geq 0$
$2k - 1 \geq 0 \Rightarrow k \geq 1/2$.

9. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angles α with the positive x-axis, then cosα equals

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{2}$
(3) 1 (4) $\frac{1}{\sqrt{2}}$

Ans. (1)

Sol: If direction cosines of L be l, m, n, then
$2l + 3m + n = 0$
$l + 3m + 2n = 0$
Solving, we get, $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$

$\therefore l : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$.

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$
(3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

Ans. (3)

Sol: General equation of all such circles is
$x^2 + y^2 + 2gx = 0$.
Differentiating, we get
$2x + 2y\frac{dy}{dx} + 2g = 0$
$\therefore$ Desired equation is
$x^2 + y^2 + \left(-2x - 2y\frac{dy}{dx}\right)x = 0$
$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$.

11. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is
(1) 2
(2) 1/2
(3) $\frac{1}{\sqrt{2}}$
(4) $\sqrt{2}$

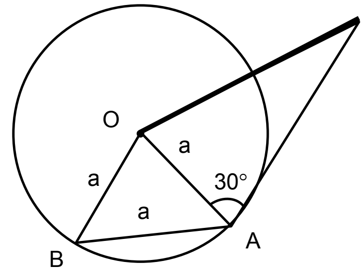
Ans. (4)

Sol: Using A.M. ≥ G.M.
$\frac{p^2 + q^2}{2} \ge pq$
$\Rightarrow pq \le \frac{1}{2}$
$(p + q)^2 = p^2 + q^2 + 2pq$
$\Rightarrow p + q \le \sqrt{2}$.

12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is
(1) $\frac{2a}{\sqrt{3}}$
(2) $2a\sqrt{3}$
(3) $\frac{a}{\sqrt{3}}$
(4) $a\sqrt{3}$

Ans. (3)

Sol: ΔOAB is equilateral
$\therefore$ OA = OB = AB = a
Now $\tan 30° = \frac{h}{a}$
$\therefore h = \frac{a}{\sqrt{3}}$.



13. The sum of the series
$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$ is
(1) $-{}^{20}C_{10}$
(2) $\frac{1}{2}\,{}^{20}C_{10}$
(3) 0
(4) $^{20}C_{10}$

Ans. (2)

Sol: $(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \ldots + {}^{20}C_{10}x^{10} + \ldots + {}^{20}C_{20}x^{20}$
put $x = -1$,
$0 = {}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \ldots + {}^{20}C_{20}$
$0 = 2\,({}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9) + {}^{20}C_{10}$
$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + \ldots + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$.

14. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
(1) ellipse
(2) parabola
(3) circle
(4) hyperbola

Ans. (1), (4)

Sol: Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow G \equiv \left(x + y\frac{dy}{dx},\ 0\right)$

$\left|x + y\frac{dy}{dx}\right| = |2x|$

$\Rightarrow\ y\frac{dy}{dx} = x$ or $y\frac{dy}{dx} = -3x$

$y\,dy = x\,dx$ or $y\,dy = -3x\,dx$

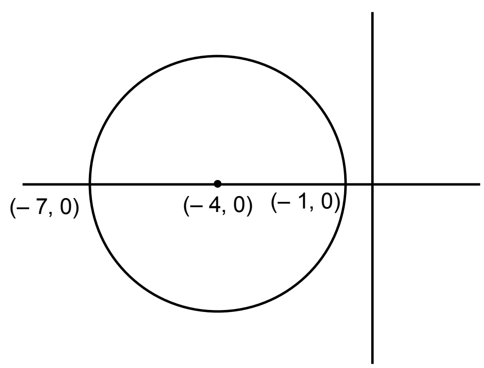
$\frac{y^2}{2} = \frac{x^2}{2} + c$ or $\frac{y^2}{2} = -\frac{3x^2}{2} + c$

$x^2 - y^2 = -2c$ or $3x^2 + y^2 = 2c$.

15. If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is
(1) 4
(B) 10
(3) 6
(4) 0

Ans. (3)

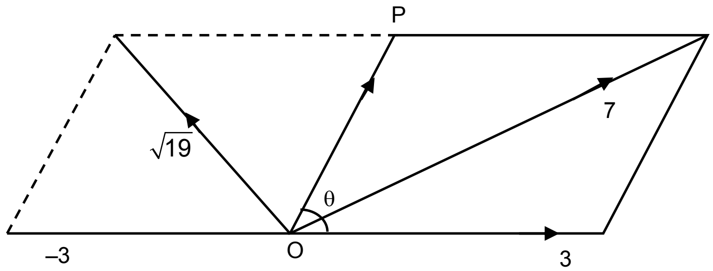
Sol: From the Argand diagram maximum value of $|z + 1|$ is 6.
Alternative:
$|z + 1| = |z + 4 - 3|$
$\le |z + 4| + |-3| = 6$.



16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is
(1) 5 N
(2) 6 N
(3) 3N
(4) 4N

Ans. (1)

Sol:



$7^2 = P^2 + 3^2 + 2 \times 3 \times P\cos\theta$ …(1)

$\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2 \times (-3) \times P\cos\theta$ …(2)

adding we get

$68 = 2P^2 + 18 \Rightarrow P = 5.$

17. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(1) 0.06 (2) 0.14
(3) 0.2 (3) 0.7

Ans. ( )

Sol: The desired probability

$= 0.7 \times 0.2 + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \ldots$

$= 0.14\,[\,1 + (0.56) + (0.56)^2 + \ldots\,]$

$= 0.14\left[\dfrac{1}{1-0.56}\right] = \dfrac{0.14}{0.44} = \dfrac{7}{22}.$

18. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0,\ y \neq 0$ then D is

(1) divisible by neither x nor y (2) divisible by both x and y
(3) divisible by x but not y (4) divisible by y but not x

Ans. (2)

Sol: $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy.$

Hence D is divisible by both x and y.

19. For the hyperbola $\dfrac{x^2}{\cos^2\alpha} - \dfrac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies?

(1) eccentricity (2) directrix
(3) abscissae of vertices (4) abscissae of foci

Ans. (4)

Sol: $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$
coordinates of focii are $(\pm ae, 0)$
$\therefore b^2 = a^2(e^2 - 1) \Rightarrow e = \sec\alpha$.
Hence abscissae of foci remain constant when α varies.

20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol: $l = \cos\frac{\pi}{4}$, $m = \cos\frac{\pi}{4}$
we know $l^2 + m^2 + n^2 = 1$
$\frac{1}{2} + \frac{1}{2} + n^2 = 1$
$\Rightarrow n = 0$
Hence angle with positive direction of z-axis is $\frac{\pi}{2}$.

21. A value of C for which the conclusion of Mean Value Theorem holds for the function f(x) = $\log_e x$ on the interval [1, 3] is

(1) $2\log_3 e$ (2) $\frac{1}{2}\log_e 3$

(3) $\log_3 e$ (4) $\log_e 3$

Ans. (1)

Sol: Using mean value theorem
$f'(c) = \frac{f(3) - f(1)}{3 - 1}$
$\Rightarrow \frac{1}{c} = \frac{\log 3 - \log 1}{2}$
$\Rightarrow c = \frac{2}{\log_e 3} = 2\log_3 e$.

22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ans. (2)

Sol: $f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}(\cos x - \sin x)$

$= \dfrac{\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)}{1+(\sin x+\cos x)^2}$

f(x) is increasing if $-\dfrac{\pi}{2} < x+\dfrac{\pi}{4} < \dfrac{\pi}{2}$

$-\dfrac{3\pi}{4} < x < \dfrac{\pi}{4}$

hence f(x) is increasing when $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{4}\right)$.

23. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

(1) 5^2 (2) 1
(3) 1/5 (4) 5

Ans. (3)

Sol: $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 25 & 25\alpha+5\alpha^2 & 5\alpha+25\alpha^2+5\alpha \\ 0 & \alpha^2 & 5\alpha^2+25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$625\alpha^2 = 25$

$\Rightarrow |\alpha| = \dfrac{1}{5}$.

24. The sum of the series $\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}-\dots$ upto infinity is

(1) e^{-2} (2) e^{-1}
(3) $e^{-1/2}$ (4) $e^{1/2}$

Ans. (2)

Sol: $e^{-x} = 1-x+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+\dfrac{x^4}{4!}-\cdots$

put x = 1

$\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}\cdots = e^{-1}$.

25. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

(1) exactly two values of θ (2) more than two values of θ
(3) no value of θ (4) exactly one value of θ

Ans. (4)

Sol: $|2\hat{u}\times 3\hat{v}| = 1$

$6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\sin\theta = \frac{1}{6}$

Hence there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

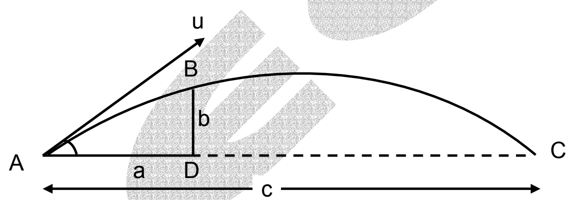
26. A particle just clears a wall of height b at distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

(1) $\tan^{-1}\frac{b}{ac}$ (2) $45°$

(3) $\tan^{-1}\frac{bc}{a(c-a)}$ (4) $\tan^{-1}\frac{bc}{a}$

Ans. (3)

Sol: $a = (u\cos\alpha)t$ and $b = (u\sin\alpha)t - \frac{1}{2}gt^2$



$b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$

also, $c = \frac{u^2\sin 2\alpha}{g}$

$b = a\tan\alpha - \frac{a^2 g}{2}\left(\frac{\sin 2\alpha}{cg}\right)\sec^2\alpha$

$b = a\tan\alpha - \frac{a^2}{2c}2\tan\alpha$

$\Rightarrow \left(a - \frac{a^2}{c}\right)\tan\alpha = b$

$\tan\alpha = \frac{bc}{a(c-a)}$.

27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(1) 40 (2) 20
(3) 80 (4) 60

Ans. (3)

Sol: $52x + 42y = 50(x + y)$
$2x = 8y$
$\Rightarrow \frac{x}{y} = \frac{4}{1}$ and $\frac{x}{x+y} = \frac{4}{5}$
$\therefore$ % of boys = 80.

28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

(1) (–1, 1) (2) (0, 2)
(3) (2, 4) (4) (–2, 0)

Ans. (4)

Sol: Point must be on the directrix of the parabola.
Hence the point is (–2, 0).

29. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

(1) (4, 9, –3) (2) (4, –3, 3)
(3) (4, 3, 5) (4) (4, 3, –3)

Ans. (1)

Sol: Coordinates of centre (3, 6, 1)
Let the coordinates of the other end of diameter are (α, β, γ)
then $\frac{\alpha+2}{2} = 3, \frac{\beta+3}{2} = 6, \frac{\gamma+5}{2} = 1$
Hence $\alpha = 4$, $\beta = 9$ and $\gamma = -3$.

30. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k},\ \bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

(1) 0 (2) 1
(3) –4 (4) –2

Ans. (4)

Sol: $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

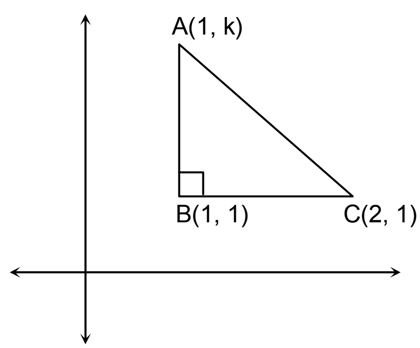
$$\begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

3x + 2 – x + 2 = 0
2x = –4
x = –2.

31. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(1) {1, 3} (2) {0, 2}
(3) {–1, 3} (4) {–3, –2}

Ans. (3)



Sol: $\frac{1}{2} \times 1(k-1) = \pm 1$
$k - 1 = \pm 2$
k = 3
k = –1

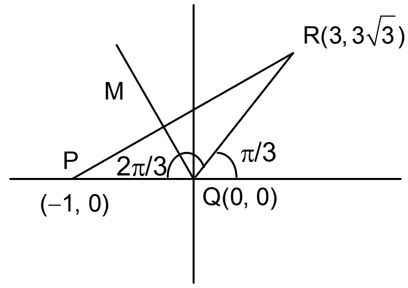
32. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR

(1) $\sqrt{3}x + y = 0$ (2) $x + \frac{\sqrt{3}}{2}y = 0$
(3) $\frac{\sqrt{3}}{2}x + y = 0$ (4) $x + \sqrt{3}y = 0$

Ans. (1)

Sol: Slope of the line QM is $\tan\frac{2\pi}{3} = -\sqrt{3}$

Hence equation is line QM is $y = -\sqrt{3}\,x$.



33. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is

(1) −1/2 (2) −2
(3) 1 (4) 2

Ans. (3)

Sol: Equation of bisectors of lines $xy = 0$ are $y = \pm x$
put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get $(1 - m^2)\,x^2 = 0$
$\Rightarrow m = \pm 1$.

34. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}dt$. Then F(e) equals

(1) $\frac{1}{2}$ (2) 0
(3) 1 (4) 2

Ans. (1)

Sol: $f(x) = \int_1^x \frac{\log t}{1+t}dt$

$F(e) = f(e) + f\left(\frac{1}{e}\right)$

$F(e) = \int_1^e \frac{\log t}{1+t}dt + \int_1^{1/e} \frac{\log t}{1+t}dt$

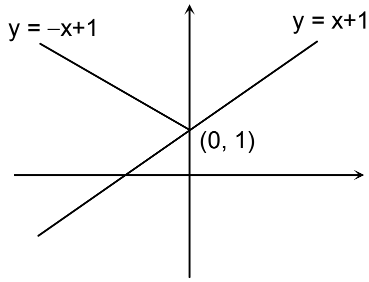
$= \int_1^e \frac{\log t}{1+t} + \int_1^e \frac{\log t}{t(1+t)}dt$

$= \int_1^e \frac{\log t}{t}dt = \frac{1}{2}$.

35. Let $f : R \to R$ be a function defined by $f(x) = \text{Min}\{x + 1, |x| + 1\}$. Then which of the following is true?

(1) $f(x) \geq 1$ for all $x \in R$ (2) f(x) is not differentiable at x = 1
(3) f(x) is differentiable everywhere (4) f(x) is not differentiable at x = 0

Ans. (3)



Sol: f(x) = min{x + 1, |x| + 1}
f(x) = x + 1 $\forall$ x $\in$ R.

36. The function f: R ~ {0} → R given by

$f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$

can be made continuous at x = 0 by defining f(0) as

(1) 2
(2) –1
(3) 0
(4) 1

Ans. (4)

Sol: $\lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$

$\lim_{x\to 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$\lim_{x\to 0} \frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$

$\lim_{x\to 0} \frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1.$

37. The solution for x of the equation

$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is

(1) 2
(2) π
(3) $\frac{\sqrt{3}}{2}$
(4) $2\sqrt{2}$

Ans. ( )

Sol: $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$\left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$

$\sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$

$\sec^{-1} x = \frac{3\pi}{4}$

$x = -\sqrt{2}$.

38. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

(1) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$
(2) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$
(3) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$
(4) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

Ans. (1)

Sol: $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$= \frac{1}{2}\int \sec\left(x-\frac{\pi}{3}\right)dx$

$= \frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{6}+\frac{\pi}{4}\right) + c$

$= \frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right) + c.$

39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

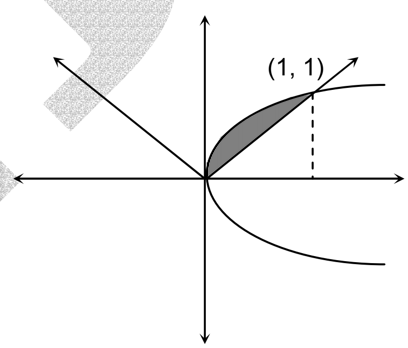
(1) 2/3
(2) 1
(3) 1/6
(4) 1/3

Ans. (3)

Sol: $A = \int_0^1 \left(\sqrt{x} - x\right)dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$



40. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(1) (–3, 3)
(2) (–3, ∞)
(3) (3, ∞)
(4) (–∞, –3)

Ans. (1)

Sol: $x^2 + ax + 1 = 0$

$\alpha + \beta = -a \qquad \alpha\beta = 1$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$|\alpha - \beta| = \sqrt{a^2 - 4}$

$\sqrt{a^2 - 4} < \sqrt{5}$

$a^2 - 4 < 5$

$a^2 - 9 < 0$

$a \in (-3, 3).$