B

Register Number | | | | | |

Part III — MATHEMATICS

(New Syllabus)

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. The particular integral of the differential equation $f(D)_y = e^{ax}$ where $f(D) = (D-a)\,g(D)$, $g(a) \neq 0$, is

a) me^{ax}

b) $\dfrac{e^{ax}}{g(a)}$

c) $g(a)\,e^{ax}$

d) $\dfrac{xe^{ax}}{g(a)}$.

2. The order and degree of the differential equation

$\sin x\,(dx + dy) = \cos x\,(dx - dy)$ are

a) 1, 1 b) 0, 0

c) 1, 2 d) 2, 1.

[Turn over

3. The number of rows in the truth table of $\sim [\, p \wedge (\sim q)\,]$ is

a) 2 b) 4

c) 6 d) 8.

4. In a set of integers with operation $*$ defined by $a * b = a + b - ab$, the value of $3 * (4 * 5)$ is

a) 25 b) 15

c) 10 d) 5.

5. In the multiplicative group of n^{th} roots of unity, the inverse of ω^k, where $k < n$, is

a) $\omega^{\frac{1}{k}}$

b) ω^{-1}

c) ω^{n-k}

d) $\omega^{\frac{n}{k}}$.

6. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$, then

a) $a + b = 0$ b) $a + 3b = 0$

c) $3a + b = 0$ d) $3a + b = 1$.

7. $\lim_{x \to 0} \dfrac{x}{\tan x}$ is

a) 1 b) -1

c) 0 d) ∞.

8. If $u = \log \left[\frac{x^2 + y^2}{xy} \right]$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is

a) 0 b) u

c) $2u$ d) u^{-1}.

9. An asymptote to the curve $y^2 (a + 2x) = x^2 (3a - x)$ is

a) $x = 3a$ b) $x = -\frac{a}{2}$

c) $x = \frac{a}{2}$ d) $x = 0$.

10. The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ is

a) $\sqrt{2} + 1$ b) $\sqrt{2} - 1$

c) $2\sqrt{2} - 2$ d) $2\sqrt{2} + 2$.

11. The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

a) i b) $-i$

c) 1 d) -1.

12. If p represents the variable complex number z and if $| 2z - 1 | = 2 | z |$, then the locus of p is

a) the straight line $x = \frac{1}{4}$

b) the straight line $y = \frac{1}{4}$

c) the straight line $z = \frac{1}{2}$

d) the circle $x^2 + y^2 - 4x - 1 = 0$.

13. If ω is a cube root of unity, then the value of

$$\left(1 - \omega + \omega^2\right)^4 + \left(1 + \omega - \omega^2\right)^4 \text{ is}$$

a) 0

b) 32

c) -16

d) -32.

14. The arguments of n^{th} roots of a complex number differ by

a) $\frac{2\pi}{n}$

b) $\frac{\pi}{n}$

c) $\frac{3\pi}{n}$

d) $\frac{4\pi}{n}$.

15. If B and B' are the ends of the minor axis, F_1 and F_2 are the foci of the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then the area of $F_1 BF_2 B'$ is

a) 16

b) 8

c) $16\sqrt{2}$

d) $32\sqrt{2}$.

16. If A is a square matrix of order n, then $|\ adj(A)\ |$ is

a) $|A|^2$

b) $|A|^n$

c) $|A|^{n-1}$

d) $|A|$.

17. In a system of 3 linear non-homogeneous equations with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$, then the system has

a) unique solution

b) two solutions

c) infinitely many solutions

d) no solution.

18. If the equations $-2x + y + z = l$, $x - 2y + z = m$ and $x + y - 2z = n$ are such that $l + m + n = 0$, then the system has

a) a non-zero unique solution

b) trivial solution

c) infinitely many solutions

d) no solution.

19. If $\rho(A) = \rho(A, B) =$ the number of unknowns, then the system is

a) consistent and has infinitely many solutions

b) consistent and has unique solution

c) consistent

d) inconsistent.

20. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$, then

a) $\vec{u}$ is a unit vector

b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$

c) $\vec{u} = \vec{0}$

d) $\vec{u} \neq \vec{0}$.

21. ' + ' is not a binary operation on

a) N

b) Z

c) $\mathbb{C}$

d) $Q - \{0\}$.

22. $Var(4x+3)$ is

a) 7

b) $16\ Var(X)$

c) 19

d) 0.

23. For a binomial distribution with mean 2 and variance $\frac{4}{3}$, p is equal to

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) $\frac{3}{4}$

d) $\frac{2}{\sqrt{3}}$.

24. The random variable X follows a normal distribution whose probability function is given by $f(x) = ce^{\frac{-\frac{1}{2}(x-100)^2}{25}}$. The value of c is

a) $\sqrt{2\pi}$

b) $\frac{1}{\sqrt{2\pi}}$

c) $5\sqrt{2\pi}$

d) $\frac{1}{5\sqrt{2\pi}}$.

25. In a Poisson distribution if $P[X = 2] = P[X = 3]$, then the value of its parameter λ is

a) 6 b) 2

c) 3 d) 0.

26. The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle is $(a > b)$

a) $\pi b(a - b)$

b) $2\pi a(a - b)$

c) $\pi a(a - b)$

d) $2\pi b(a - b)$.

27. $\int_0^\infty x^5 e^{-4x} dx$ is

a) $\frac{\lfloor 6}{4^6}$

b) $\frac{\lfloor 6}{4^5}$

c) $\frac{\lfloor 5}{4^6}$

d) $\frac{\lfloor 5}{4^5}$.

28. The volume of the solids obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its major and minor axes are in the ratio $(a > b)$

a) $b^2 : a^2$ b) $a^2 : b^2$

c) $a : b$ d) $b : a$.

29. If $y = ke^{\lambda x}$ then its differential equation is

a) $\frac{dy}{dx} = \lambda y$

b) $\frac{dy}{dx} = ky$

c) $\frac{dy}{dx} + ky = 0$

d) $\frac{dy}{dx} = e^{\lambda x}$.

30. On putting $y = vx$ the homogeneous differential equation

$x^2 dy + y(x+y)dx = 0$ becomes

a) $xdv + (2v + v^2)dx = 0$

b) $vdx + (2x + x^2)dv = 0$

c) $v^2 dx - (x + x^2)dv = 0$

d) $vdv + (2x + x^2)dx = 0$.

31. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis, is

a) $\frac{\sqrt{3}}{2}$

b) $\frac{5}{3}$

c) $\frac{3}{2}$

d) $\frac{\sqrt{5}}{2}$.

32. If P is any point on the hyperbola $\frac{x^2}{36} - \frac{y^2}{4} = 1$ and the ordinate at P meets the asymptotes at Q and Q' then $QP \cdot Q'P$ is

a) 36

b) 6

c) 4

') 2.

33. The equations of the major and minor axes of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ respectively are

a) $x = 3, \ y = 2$

b) $x = -3, \ y = -2$

c) $x = 0, \ y = 0$

d) $y = 0, \ x = 0$.

34. The surface area of a sphere, when the volume is increasing at the same rate as its radius, is

a) 1

b) $\frac{1}{2\pi}$

c) 4π

d) $\frac{4\pi}{3}$.

35. The angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is

a) $2 \tan^{-1}\left(\frac{3}{4}\right)$

b) $\tan^{-1}\left(\frac{4}{3}\right)$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{4}$.

36. If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $\left|\vec{a}\right| = \left|\vec{b}\right|$

d) $\vec{a}$ and $\vec{b}$ are unit vectors.

37. The projection of $\vec{OP}$ on a unit vector $\vec{OQ}$ equals thrice the area of parallelogram $OPRQ$. Then $\angle POQ$ is

a) $\tan^{-1}\left(\frac{1}{3}\right)$

b) $\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$

c) $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$

d) $\sin^{-1}\left(\frac{1}{3}\right)$.

38. The two lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$ are

a) parallel

b) intersecting

c) skew

d) perpendicular.

39. The projection of $\vec{i} - \vec{j}$ on Z-axis is

a) 0

b) 1

c) -1

d) ∞.

40. The unit normal vectors to the plane $2x - y + 2z = 5$ are

a) $2\vec{i} - \vec{j} + 2\vec{k}$

b) $\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$

c) $-\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$

d) $\pm\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$.

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. If $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

42. Solve the following system of linear equation by determinant method :

$$2x + 3y = 8, \quad 4x + 6y = 16.$$

43. Show that the points $(3, -1, -1)$, $(1, 0, -1)$ and $(5, -2, -1)$ are collinear.

44. Find the vector and Cartesian equations of a sphere with centre having position vector $2\vec{i} - \vec{j} + 3\vec{k}$ and radius 4 units.

45. Solve $x^4 + 4 = 0$, if $1 + i$ is one of the roots.

46. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

47. Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1, \quad -\frac{1}{2} \le x \le 4.$$

48. Verify Lagrange's law of mean for the function $f(x) = x^3 - 5x^2 - 3x$ on $[1, 3]$

49. Find an approximate value for $\sqrt[3]{65}$ by using differentials.

50. Find the area of the region bounded by $y = 2x + 4$, $y = 1$, $y = 3$ and y-axis.

51. Solve $x^2 \dfrac{dy}{dx} = y^2 + 2xy$ given that $y = 1$ when $x = 1$.

52. a) Construct the truth table for $(p \vee q) \wedge (\sim q)$.

b) Show that $p \wedge (\sim p)$ is a contradiction.

53. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

54. a) The difference between the mean and the variance of a binomial distribution is 1 and the difference between their squares is 11. Find n.

b) Show that the total probability is 1 (for a Poisson distribution).

55. a) Marks in an aptitude test given to 800 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90 marks.

OR

b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$ and $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Discuss the solutions of the system of equations for all values of λ (use rank method) :

$$x + y + z = 2, \; 2x + y - 2z = 2, \; \lambda x + y + 4z = 2.$$

57. Find the vector and Cartesian equations of the plane passing through the point $(-1, -2, 1)$ and perpendicular to two planes $x + 2y + 4z + 7 = 0$ and $2x - y + 3z + 3 = 0$.

58. Solve the equation : $x^9 + x^5 - x^4 - 1 = 0$.

59. Find the eccentricity, centre, foci, vertices of the ellipse

$36x^2 + 4y^2 - 72x + 32y - 44 = 0$ and sketch the graph.

60. A cable of a suspension bridge is in the form of a parabola whose span is 40 m. The roadway is 5 m below the lowest point of the cable. An extra support is provided across the cable 30 m above the ground level. Find the length of the support if the heights of the pillars are 55 m.

61. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere).

62. Find the intervals of concavity and the points of inflexion of the function $f(x) = x^4 - 6x^2$.

63. Verify Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$.

64. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$.

65. Find the perimeter of the circle with radius a, by using integration.

66. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the bacteria triple in 1 hour, show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

67. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$ when $x = \log 2$, $y = 0$ and when $x = 0$, $y = 0$.

68. Show that the set G of all positive rationals forms a group under the composition $*$, defined by $a * b = \frac{ab}{3}$ for all $a, b \in G$.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equals to 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accident in a year, (ii) more than 3 accidents in a year $\left[e^{-3} = 0{\cdot}0498 \right]$.

70. a) Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$ respectively, (2, 4) is the centre of the hyperbola and the hyperbola passes through (2, 0).

OR

b) Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find the point of intersection.