5. (a) (i) Find the solution of Poisson's equation $\nabla^2 \phi = -\rho(r)$ using Green's function. (12)

(ii) Find the Green's function for the boundary value problem $\frac{d^2 y}{dx^2} - k^2 y = f(x)$ with boundary conditions $y(\pm \infty) = 0$. (8)

Or

(b) Explain the stretched string wave equation. (20)

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4529/MP1 **MAY 2010**

Paper I — MATHEMATICAL PHYSICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

1. (a) (i) State and prove Green's theorem in vector analysis. (10)

(ii) Explain outer product and inner product of tensors. (10)

Or

(b) (i) State and prove Cayley-Hamilton theorem. (10)

(ii) Obtain an expressions for grad ϕ and div A in spherical polar co-ordinates. (10)

2. (a) (i) Find the Fourier Series for the periodic function $f(x)$ defined by

$$f(x) = -\pi \text{ if } -\pi \le x \le 0$$
$$= x \quad \text{if } 0 \le x \le \pi$$

Hence prove that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \qquad (12)$$

(ii) Prove that $\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$. (8)

Or

(b) (i) Prove that $\lim_{n \to \infty} \int_{-\infty}^{\infty} \phi_n(x) f(x)\, dx = f(0)$ where $\phi_n(x)(n = 1, 2, 3 \cdots)$ as a delta sequence. (12)

(ii) Prove that $\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)$ where $f(x)$ is a continuous function. (8)

3. (a) (i) State and prove Cauchy's Residue theorem. (10)

(ii) Prove that $u(x, y) = x^2 - y^2$ is a harmonic function. Find $v(x, y)$. So that $f(z) = u + iv$. (10)

Or

(b) (i) Show that Fourier transform of a Gaussian probability function is also a gaussian probability function. (10)

(ii) Find the Fourier sine and cosine transforms for the function $f(x) = 2x \; 0 < x < 4$. (10)

4. (a) (i) Prove that $P_n(x) = 2^{\frac{1}{n}} n! \frac{d^n}{dx^n}(x^2 - 1)^n$. (10)

(ii) Prove that $x J_n^1(x) = n J_n(x) - x J_{n+1}(x)$. (10)

Or

(b) (i) Solve the Laguerre's differential equation using power series technique. (10)

(ii) Prove that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

$$J_{\frac{3}{x}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]. \qquad (10)$$