

B3.2-R3: BASIC MATHEMATICS

NOTE:

1. Answer question 1 and any FOUR questions from 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

a) If $P = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1 \\ q & -1 \end{bmatrix}$ and $(P+Q)^2 = P^2+Q^2$, determine the value of q.

b) Find the smallest integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$.

c) If $f(9) = 9, f'(9) = 4$, evaluate $\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$.

d) Find the solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

It is given that $x = -9$ is one of the roots.

e) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{3n}{2(n+1)}$

f) Find the unit vector perpendicular to the plane of two vectors \vec{a} and \vec{b} , where

$$\vec{a} = i - j + 2k \text{ and } \vec{b} = 2i + 3j - k.$$

g) Find the vertex and latus rectum of the parabola $(y+3)^2 = 2(x+2)$.

(7x4)

2.

a) For what values of λ and μ , the simultaneous equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions.

b) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

(10+8)

3.

a) Find the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

b) Find the center, the length of the axes and the eccentricity of the ellipse

$$2x^2 + 3y^2 - 4x - 12y + 13 = 0$$

(9+9)

4.

a) If $y = e^t \cos t$, $x = e^t \sin t$, and $y''(x + y)^2 = K(xy' - y)$. Find K .

b) If x, y, z are all different and given that

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

Determine the value of $(1+xyz)$.

(9+9)

5.

a) Find the area between the parabola $y^2 = 4ax$ and the line $y = mx$.

b) If e and e' are the eccentricities of a hyperbola and its conjugate, prove that

$$\left(\frac{1}{e^2}\right) + \left(\frac{1}{e'^2}\right) = 1$$

(10+8)

6.

a) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$.

b) Show that $\sin x(1 + \cos x)$ has a maximum value when $x = \frac{\pi}{3}$.

c) Find the value of a and b in order that

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1.$$

(6+6+6)

7.

a) State Lagrange's mean value theorem. Hence show that

$$e^x > 1 + x, \quad x > 0$$

b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A - 5I = 0$. Use this result to find A^{-1} .

c) Find the limit when $n \rightarrow \infty$ of the series $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}$.

(6+6+6)