Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Remedial Examination January/ February 2011

Subject code: 710901 **Subject Name: Theory of Elasticity**

Date: 31 /01 / 2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** "A small sphere immersed in the ideal fluid experiences isotropic stress at any point". Justify the statement with a mathematical proof. **06**

(b) "The strain tensor can not be specified arbitrarily and it must satisfy compatibility conditions". Substantiate the statement through example and explain the role of compatibility conditions as regards to permissible deformations. **06**

Q.2 **(a)** "The three values of principal stresses are always real and not imaginary". Comment on the degree of the validity of a given statement with a mathematical proof. **06**

(b) In a square plate in the region $-1 \leq x_1, x_2 \leq 1$ the following stresses hold: **06**

$\sigma_{11} = c\,x_1 + d\,x_2$ $\quad\quad$ $\sigma_{22} = a\,x_1 + b\,x_2$

$\sigma_{33} = \sigma_{31} = \sigma_{32} = 0$

where a, b, c and d are constants. Assuming that body forces and body moments are absent, what must be shear components σ_{12} in order to be ensure equilibrium? State the boundary conditions on the edges of the plate when $a = b = 0$.

OR

(b) In a plane stress problems, the stress component $\sigma_{33} = \sigma_{31} = \sigma_{32} = 0$, and the remaining ones are the functions of x_1 and x_2 only. Assuming that body forces and body moments are absent, find the relation between the non-zero stress components and a single stress function $\varphi(x_1, x_2)$, so that all equilibrium conditions are satisfied. **06**

Q.3 **(a)** The compatible strain field is given by **06**

$\varepsilon_{xx} = 5 + x^2 + y^2 + x^4 + y^4$

$\varepsilon_{yy} = 6 + 3x^2 + 3y^2 + x^4 + y^4$

$\gamma_{xy} = 10 + 4xy(x^2 + y^2 + 2)$

$\varepsilon_{zz} = \gamma_{yz} = \gamma_{zx} = 0$

Determine

(i) spherical and deviatoric part of the strain field

(ii) volumetric strain

(b) The displacement field for a body is given by **06**

$u = (x^2 + y)i + (3+z)j + (x^2 + 2y)k$

Determine the displacement matrix at a point (2, 3, 1) and comment on the displacement field.

OR

Q.3 **(a)** The displacement field for a body is given by **06**

$u = [(x^2 + y^2 + 2)i + (3x + 4y^2)j + (2x^3 + 4z)k]\ 10^{-4}$

Determine the displacement at a point originally at (1, 2, 3).

(b) The displacement field for a body is given by **06**

$u = [p(x^2 + 2z)i + p(4x + 2y^2 + z)j + (4pz^2)k]$

where k is a small constant. Determine the strain at a point (2, 2, 3) in the following directions:

(i) $n_x = 0, n_y = n_z = 1/\sqrt{2}$

(ii) $n_x = 1, n_y = n_z = 0$

Q.4 **(a)** A rubber cube is inserted in a cavity of the same form and size in a steel block and the top of cube is pressed by a steel block with pressure of 'p' Pascal's. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber. Find (i) the pressure of rubber against the walls and (ii) the extremum shear stress in rubber. The Poison's ratio $\nu \leq 0.5$ and in usual notations **06**

$\varepsilon_{xx} = [\sigma_x - \nu(\sigma_y + \sigma_z)]/E$

$\varepsilon_{yy} = [\sigma_y - \nu(\sigma_z + \sigma_x)]/E$

(b) A cubical element is subjected to the following state of stress. **06**

$\sigma_x = 100$ MPa, $\sigma_y = -20$ MPa, $\sigma_z = -40$ MPa

$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

Assuming the material to be homogeneous and isotropic determine the principal shear strain and octahedral shear strain, if $E = 2 \times 10^5$ MPa and $\nu = 0.25$. Assume in usual notations

$G = E/2(1+\nu)$

OR

Q.4 **(a)** The material constants for steel are $E = 207 \times 10^6$ kPa and $G = 80 \times 10^6$ kPa. Determine the stress matrix for the given strain matrix at a point **06**

$$\varepsilon_{ij} = \begin{bmatrix} 0.001 & 0 & -0.002 \\ 0 & -0.003 & 0.003 \\ -0.002 & 0.003 & 0 \end{bmatrix}$$

(b) A thin rubber sheet is enclosed between two fixed hard steel plates. The rubber plate is subjected to stresses in x and y directions as σ_x and σ_y respectively. Determine the strains ε_{xx}, ε_{yy} and the stress σ_z. The Poison's ratio should be considered as ν for the rubber. **06**

Q.5 **(a)** Explain the principle of superimposition and prove that the principle is valid for two different forces acting at two different points. **06**

(b) Derive the mathematical expression for Castigliano's first principle from the concepts of elastic strain energy. Illustrate the use of this principle for any one real time engineering application. **06**

OR

Q.5 **(a)** Consider a solid thin disk subjected to a temperature distribution which varies with radius 'r' and is independent of the angular displacement 'θ'. Derive the expression for radial and angular stresses for a given case. **06**

(b) Determine the thermal stresses induced in three directions (r, θ, z) on a long circular cylinder when the temperature is symmetrical about the axis and does not vary along its axis. **06**

****