Code: D-23 / DC-23 **Subject: MATHEMATICS - II**
Time: 3 Hours

December 2005

Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are vectors then $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)$ is equal to

(A) $\vec{a}\cdot\vec{c}-\vec{b}\,\vec{d}$

(B) $\vec{a}\times\vec{c}-\vec{b}\times\vec{d}$

(C) $\left(\vec{a}\cdot\vec{c}\right)\left(\vec{b}\cdot\vec{d}\right)-\left(\vec{a}\cdot\vec{d}\right)\left(\vec{b}\cdot\vec{c}\right)$

(D) none of above.

b. If A, B are square matrices of the same size then

(A) $(AB)^t = A^tB^t$ **(B)** $(AB)^t = B^tA^t$

(C) $(AB)^t = AB$ **(D)** $(AB)^t = BA$

c. If z_1 and z_2 are two complex numbers then $|z_1+z_2|$ is

(A) $=|z_1|+|z_2|$ **(B)** $\le|z_1|+|z_2|$

(C) $\le|z_1|-|z_2|$ **(D)** $\ge|z_1|+|z_2|$

d. The value of $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$ is equal to

(A) $3a^2x$ **(B)** $a^2(3x - a)$

(C) $a^2(3x + a)$ **(D)** $3ax^2$

e. If $I+A+A^2+\ldots+A^K=0$, then A^{-1} is equal to

(A) A^K **(B)** A^{K-1}
(C) A^{K+1} **(D)** $I+A$

f. If A is any real square matrix then $A+A^t$ is

(A) Hermitian. **(B)** Skew-hermition.
(C) Symmertic. **(D)** Skew-symmertic.

g. The Laplace transform $L(t^n)$ is

(A) $\frac{n!}{s^n}$. **(B)** $\frac{n!}{s^{n+1}}$.
(C) $\frac{1}{s}$. **(D)** $\frac{s^n}{n!}$

h. The solution of differential equation $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$ is

(A) $y=(c_1+c_2x)e^x$ **(B)** $y=(c_1+c_2x)e^{2x}$.
(C) $y=(c_1+c_2x)e^{3x}$. **(D)** $(c_1+c_2x)e^{-3x}$

i. The value of a_0 in the Fourier series $f(x)=\frac{a_0}{2}+a_1\cos x+a_2\cos 2x+\ldots a_n\cos nx+\ldots$ is given by

(A) $\frac{1}{\pi}\int_0^{2\pi}f(x)dx$ **(B)** $\frac{1}{2\pi}\int_0^{2\pi}f(x)dx$
(C) $\frac{1}{\pi}\int_0^{\pi}f(x)dx$ **(D)** 0

j. The inverse Laplace transform $L^{-1}\left(\frac{4}{s-2}\right)$ is

(A) e^t **(B)** $2e^{2^t}$
(C) $4e^{2t}$ **(D)** $4e^{4t}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Express $\frac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4}$ in the form x+iy. **(8)**

b. Write down all the values of $(1+i)^{1/4}$. **(8)**

Q.3 a. Using vector method prove that the altitudes of a triangle are concurrent. **(8)**

b. Find a unit vector perpendicular to the plane of vectors $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + 2\vec{k}$. **(8)**

Q.4 a. Prove that $\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{a}\times\vec{d}\right) + \left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{b}\times\vec{d}\right) + \left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right) = 0$ **(8)**

b. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $\left|\vec{a}\times\vec{b}\right| = \vec{a}\cdot\vec{b}$. **(8)**

Q.5 a. Let A be a square matrix. Prove that A can be written the sum of a symmetric and a skew-symmetric matrix. **(8)**

b. State Cayley Hamitton theorem and use it to find the inverse of $A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$, if the inverse exists. **(8)**

Q.6 a. Prove that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$. **(8)**

b. Give condition under which we can find λ so that the following system of linear equations has a non-trivial solution.

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$(p_1 + \lambda q_1)x + (p_2 + \lambda q_2)y + (p_3 + \lambda q_3)z = 0$$ **(8)**

Q.7 a. Find the Fourier series of the function defined by

$$f(x)=\begin{cases} x+\pi & : 0\le x\le \pi \\ -x-\pi & : -\pi\le x<0 \end{cases}$$ **(8)**

b. Find the Fourier series representing the function

$f(x)=x \qquad 0<x<2\pi$ **(8)**

Q.8 a. If F(t) is piecewise continuous and satisfies $|F(t)|\le Me^{at}$ for all $t\ge 0$ and for some constants a and M then

$$L\left\{\int_0^t F(x)\,dx\right\}=\frac{1}{s}L\{F(t)\},\ (s>0, s>a)$$ **(8)**

b. Define Inverse Laplace Transform of a function F(t). Prove that

$$L^{-1}\left\{\frac{1}{s^3+1}\right\}=\frac{t^2}{2!}-\frac{t^5}{5!}+\frac{t^8}{8!}-\frac{t^{11}}{11!}+\ldots\ldots\ldots$$ **(8)**

Q.9 a. Solve $(D^4+2D^2+1)\,y=0$. **(8)**

b. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin 2x$. **(8)**