IIT-JEE 2004 Mains Questions & Solutions – Maths – Version 2
(The questions are based on memory)

Break-up of marks:

Algebra	Trigonometry	Co-ordinate Geometry	Calculus	Vector/3D
24(40%)	0	8(13%)	22(37%)	6(10%)

1. A parallelepiped S with base ABCD and top A'B'C'D' is compressed to a parallelepiped T with the same base ABCD and top A"B"C"D". Volume of T is 90% that of S. Prove that the locus of A" is a plane. **[2]**

Solution

Volume of parallelepiped = Area of base ABCD X perpendicular distance between ABCD and A'B'C'D'
= A X h

Base area ABCD of the parallelepiped = A = constant

If volume is now 0.9 times the initial volume,
height between ABCD and A"B"C"D" = h' = 0.9 h

Therefore the plane A"B"C"D" will always be at a fixed height 0.9 h from ABCD.

Hence locus of A" is a plane parallel to the plane ABCD and at a fixed distance from it.

2. Using Rolle's theorem, prove that there is a root of
$p(x) = 51x^{101} - 2323x^{100} - 45x + 1035$ in $\left(45^{1/100}, 46\right)$ **[2]**

Solution

Consider, $q(x) = \int p(x)dx = \frac{x^{102}}{2} - 23x^{101} - \frac{45}{2}x^2 + 1035x + C$

q(x) being polynomial function is differentiable and continuous in $\left(45^{1/100}, 46\right)$ and

$$q\left(45^{1/100}\right) = q(46) = C$$

By Rolle's theorem

$\therefore\ q'(x) = p(x) = 51x^{101} - 2323x^{100} - 45x + 1035 = 0$ has at least one root in $\left(45^{1/100},\ 46\right)$.

3. Given, $y_{(x)} = \int\limits_{\pi^2/16}^{x^2} \frac{\cos x.\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$, find $\frac{dy}{dx}$ at x = π. **[2]**

Solution

$$y(x) = \int\limits_{\pi^2/16}^{x^2} \frac{\cos x.\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$$y'(x) = \cos x\left[\frac{d}{dx}\int\limits_{\pi^2/16}^{x^2} \frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta\right] - \sin x\int\limits_{\pi^2/16}^{x^2} \frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$$= \frac{\cos x.\cos|x|}{1+(\sin|x|)^2}.2x - \sin x\int\limits_{\pi^2/16}^{x^2} \frac{\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$$

$$\Rightarrow \quad y'(\pi) = 2\pi$$

4. A plane passing through (1, 1, 1) parallel to the lines having direction ratios (1, -1, 0) and (1, 0, -1) respectively makes intersection on the X, Y , Z axes at A, B, C respectively. Find the volume of tetrahedron formed with A, B, C and origin. **[2]**

Solution

Vector normal to the plane $= \left(\hat{i} + 0.\hat{j} - \hat{k}\right)\times\left(\hat{i} - \hat{j} + 0\hat{k}\right)$

$= -\left(\hat{i} + \hat{j} + \hat{k}\right)$

Direction ratios of normal to the plane is (1, 1, 1)

$\therefore$ Equation of plane: x + y + z = 3

$\Rightarrow \quad \overrightarrow{OA} = 3\hat{i}$

$\Rightarrow \quad \overrightarrow{OB} = 3\hat{j}$

$\Rightarrow \quad \overrightarrow{OC} = 3\hat{k}$

volume of tetrahedron =

$$\frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{9}{2}$$ cubic unit

5. $f(x)$ is defined as $f:(-1,\ 1)\to R$ and is differentiable on (–1, 1). It is given that $f\,'(0)=\lim_{n\to\infty} n\left(f\left(\frac{1}{n}\right)\right)$ also $f(0)=0$. Find the value of $\lim_{n\to\infty}\left(\frac{2}{\pi}(n+1)\cos^{-1}\frac{1}{n}-n\right)$ given that $\left|\cos^{-1}\left(\frac{1}{n}\right)\right|\le\frac{\pi}{2}$.

[2]

Solution

$$\lim_{n\to\infty}\left(\frac{2}{\pi}(n+1).\cos^{-1}\left(\frac{1}{n}\right)-n\right)$$

$$=\lim_{n\to\infty}\left(\frac{2}{\pi}(n+1)\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{n}\right)-n\right)$$

$$=\lim_{n\to\infty}\left((n+1)-\frac{2}{\pi}(n+1)\sin^{-1}\frac{1}{n}-n\right)$$

$$=\lim_{n\to\infty}\left(1-n.\frac{2}{\pi}\sin^{-1}\frac{1}{n}-\frac{2}{\pi}\sin^{-1}\frac{1}{n}\right)$$

$$=1-\frac{2}{\pi}.f'(0)-\frac{2}{\pi}\lim_{n\to\infty}\sin^{-1}\frac{1}{n}$$

[$\because \lim_{n\to\infty} n.f\left(\frac{1}{n}\right)=f'(0)$

Taking $f(n)=\sin^{-1}n$]

$$=1-\frac{2}{\pi}.f'(0)$$

$$=1-\frac{2}{\pi}$$

here $f(x)=\sin^{-1}x$

$$f'(x)=\frac{1}{\sqrt{1-x^2}}$$

$$f'(0)=1$$

6. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, prove that $\vec{a}.\vec{b}+\vec{c}.\vec{d}\ne\vec{a}.\vec{c}+\vec{b}.\vec{d}$, where $\vec{a},\vec{b},\vec{c},\vec{d}$ are distinct vectors. **[2]**

Solution

Here $(\vec{a}-\vec{d})\times(\vec{c}-\vec{b})=(\vec{a}\times\vec{c})+(\vec{d}\times\vec{b})-(\vec{a}\times\vec{b})-(\vec{d}\times\vec{c})=0$

$\Rightarrow\quad (\vec{a}-\vec{d})\,||\,(\vec{c}-\vec{b})$

$\therefore\quad (\vec{a}-\vec{d}).(\vec{c}-\vec{b})\ne 0$

$\Rightarrow\quad \vec{a}.\vec{b}+\vec{c}.\vec{d}\ne\vec{a}.\vec{c}+\vec{b}.\vec{d}$

7. Using permutation or otherwise, prove that $\dfrac{\lfloor n^2}{(\lfloor n)^n}$ is an integer, where n is a positive integer. **[2]**

Solution

Let r +1, r + 2 …. r + n are n consecutive integers
Product of these = (r + 1) (r + 2) … (r + n)

$$= \frac{1\ldots r.(r+1)\ldots.(r+n)}{1\ldots r}$$

$$= \frac{(n+r)!}{r!} = {}^{n+r}P_n$$

Which is an integer

$$n^2! = (1\times 2.....n)\times((n+1)....2n)\times((2n+1)....3n)......((n^2-n+1)....n^2)$$

There are n groups of n consecutive integers and each will be divisible by n!
So $n^2!$ is divisible by $(n!)^n$.

8. Find the center and radius of the circle $\left|\dfrac{z-\alpha}{z-\beta}\right| = k$; $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$, $k \neq 1$, $z = x + iy$. **[2]**

Solution
Here

$$|z-\alpha|^2 = k^2\,|z-\beta|^2$$

$$\Rightarrow \quad (z-\alpha)(\bar{z}-\bar{\alpha}) = k^2(z-\beta)(\bar{z}-\bar{\beta})$$

$$\Rightarrow \quad (k^2-1)z\bar{z} + z(\bar{\alpha}-k^2\bar{\beta}) + \bar{z}(\alpha-k^2\beta) + (k^2.|\beta|^2 - |\alpha|^2) = 0$$

Centre $\equiv \dfrac{k^2\beta-\alpha}{k^2-1}$

Radius $= \dfrac{1}{(k^2-1)}\sqrt{|\alpha-k^2\beta|^2 - (k^2.|\beta|^2-|\alpha|^2)(k^2-1)}$

9. A and B are two independent events. C is the event that exactly one of them takes place, then prove that $P(A\cup B).P(\bar{A}\cap\bar{B}) \leq P(C)$. **[2]**

Solution

Here $P(C) = P(A)P(\bar{B}) + P(B)P(\bar{A})$

and $P(\bar{A}\cap\bar{B}) = P(\bar{A})P(\bar{B})$ [events are independent]

$$P(A\cup B) = P(A) + P(B) - P(A)P(B)$$

$$\Rightarrow \quad P(A\cup B).P(\bar{A}\cap\bar{B}) \leq (P(A)+P(B)).(P(\bar{A}).P(\bar{B}))$$

$\Rightarrow \quad P(A \cup B).P(\overline{A} \cap \overline{B}) \le P(A).P(\overline{A})P(\overline{B}) + P(B).P(\overline{B}).P(\overline{A})$

$\Rightarrow \quad P(A \cup B).P(\overline{A} \cap \overline{B}) \le P(A)P(\overline{B}) + P(B).P(\overline{A})$

[since $(P(\overline{A}))$ and $(P(\overline{B}))$ are less than or equal to one]

$\Rightarrow \quad P(A \cup B).P(\overline{A} \cap \overline{B}) \le P(C)$.

10. M is a 3 X 3 matrix.
Det (M) = 1
$MM^T = I$
Prove that Det (M – I) = 0. **[2]**

Solution

Let

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & t \end{bmatrix}$$

Since $MM^T = I$

$\therefore \quad M^T$ is inverse of M

$\Rightarrow \quad M^T$ is adjoint M [Since det (M) = 1]

$$\Rightarrow \quad \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & t \end{bmatrix} = \begin{bmatrix} et - fh & ch - bt & bf - ce \\ gf - dt & at - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now,

$$|M - I| = \begin{vmatrix} a-1 & b & c \\ d & e-1 & f \\ g & h & t-1 \end{vmatrix}$$

On expanding above determinant and using (1) $|M - I| = 0$

11. Prove that $\sin x + 2x \geq \frac{3x(x+1)}{\pi}, x \in \left[0, \frac{\pi}{2}\right]$. Justify any inequality used in solving the question. **[4]**

Solution

Consider the function,

$$f(x) = \sin x + 2x - \frac{3x(x+1)}{\pi}$$

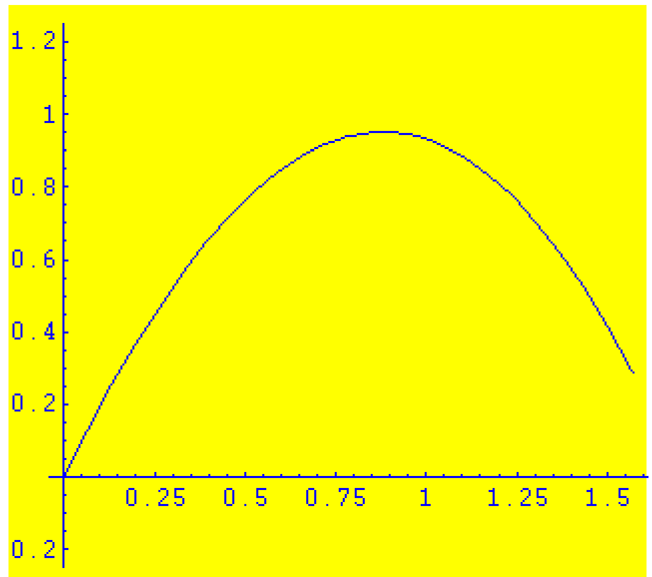
$$f'(x) = \cos x + 2 - \frac{6x}{\pi} - \frac{3}{\pi}$$

$$f'(0) = +ve$$

$$f'\left(\frac{\pi}{2}\right) = -ve$$

The function increases initially and decreases towards the end.
There must be a point of maxima somewhere in between 0 and $\frac{\pi}{2}$.

Now, we are in a position to plot the graph of the function.



The only point that needs to be checked is $\frac{\pi}{2}$.

$$f\left(\frac{\pi}{2}\right) = 1 + \pi - 1.5\left(\frac{\pi}{2} + 1\right)$$

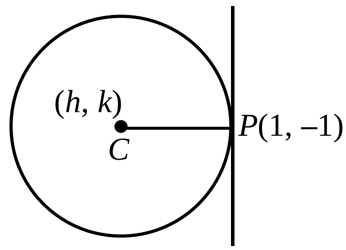
$$\Rightarrow f\left(\frac{\pi}{2}\right) = -0.5 + \frac{\pi}{4} = +ve$$

The function is +ve at this point as well.
Hence, it is clear that the function does not cut x axis anywhere in $\left[0, \frac{\pi}{2}\right]$.

Hence, the function is +ve everywhere.

12. A line $2x+3y+1=0$ touches a circle C at (1, -1). Another circle cuts circle 'C' orthogonally and the end points of its diameter are (0, -1) and (-2, 3). Find the equation of the circle 'C'. **[4]**

Solution



Slope CP = $\dfrac{k+1}{h-1} = \dfrac{3}{2}$

$$2k+2=3h-3$$
$$3h-2k=5 \quad \ldots\ldots\ldots \ (1)$$

Equation of second circle is

$$(x-0)(x+2)+(y+1)(y-3)=0$$

$$x^2+y^2+2x-2y-3=0 \quad \ldots\ldots\ldots \ (2)$$

Centre (–1, 1) and radius = $\sqrt{5}$

Above circle is orthogonal to the circle having centre (*h*, *k*).

$$\therefore \quad (h-1)^2+(k+1)^2+5=(h+1)^2+(k-1)^2$$
$$4h-4k=5 \quad \ldots\ldots\ldots\ldots (3)$$

Solving (1) and (3)

$$h=\frac{5}{2},\ k=\frac{5}{4}$$

$\therefore$ Equation of circle is

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{5}{4}\right)^2=\frac{117}{16}.$$

13. A curve passes through (2, 0) and tangent at a point P (x, y) on it has slope $\dfrac{(x+y)^2+(y-3)}{(x+1)}$. Find the equation of the curve and also find the area bounded by the curve in the fourth quadrant with the x axis. **[4]**

Solution

$$\frac{dy}{dx}=\frac{(x+1)^2+(y-3)}{(x+1)}$$

$$(x+1)\frac{dy}{dx}=(x+1)^2+(y-3)$$

Let $X=x+1, Y=y-3$

$$X\frac{dY}{dX}=X^2+Y$$

$$\frac{dY}{dX} - \frac{Y}{X} = X$$

I.F = $e^{\int -\frac{1}{X}.dX} = e^{-\log X} = \frac{1}{X}$

Solution is

$$Y.\frac{1}{X}. = \int \frac{1}{X}.X.dX$$

$$\frac{Y}{X} = X + C$$

$$\frac{y-3}{x+1} = x + 1 + C \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

As curve passes through (2, 0) C = –4

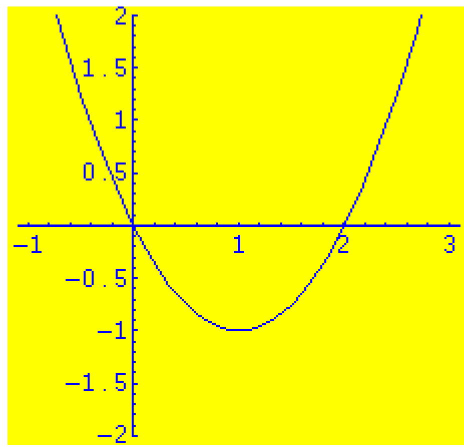
$\therefore$ (1) becomes

$$(x-3)(x+1) = (y-3)$$

$$\Rightarrow y = x^2 - 2x$$

Area $= \int_0^2 (x^2 - 2x)dx = \left[\frac{x^3}{3} - x^2\right]_0^2$

$= \left|\frac{8}{3} - 4\right| = \frac{4}{3}$ sq. units



14. Prove that $(1+a)^7(1+b)^7(1+c)^7 \geq 7^7 a^4 b^4 c^4$ where a, b, c are positive real numbers. **[4]**

Solution

A.M. $\geq$ G.M. (since the numbers are positive)

$$\frac{a+b+c+ab+bc+ca+abc}{7} \geq \left(a^4b^4c^4\right)^{1/7}$$

$$a+b+c+ab+bc+ca+abc \geq 7\left(a^4b^4c^4\right)^{1/7}$$

$$\Rightarrow \quad 1+a+b+c+ab+bc+ca+abc > 7\left(a^4b^4c^4\right)^{1/7}$$

$$\Rightarrow \quad (1+a)(1+b)(1+c) > 7\left(a^4b^4c^4\right)^{1/7}$$

$$\Rightarrow \quad (1+a)^7(1+b)^7(1+c)^7 > 7^7\left(a^4b^4c^4\right)$$

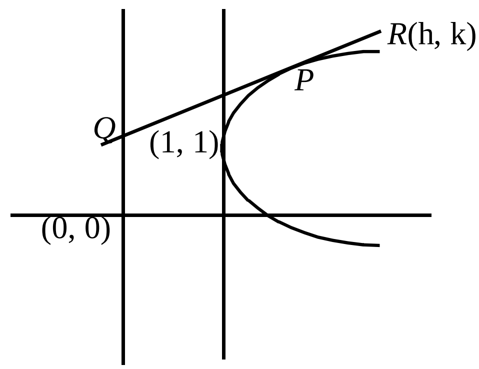
15. Given parabola $y^2-2y-4x+5=0$. If tangent at a point 'P' on the curve meets the directrix at Q, and a point R divides the line segment QP externally in the ratio $\frac{1}{2}:1$, find the locus of R. **[4]**

Solution

Sol. Parabola given $(y-1)^2=4(x-1)$
Equation to directrix $x = 0$; focus (2, 1)
Let point $P(t^2+1, 2t+1)$ be on the parabola
Tangent at any point P

$$ty-x=t^2+t-1$$

Co-ordinate of point Q $\left(0, \dfrac{t^2+t-1}{t}\right)$



Let the point $R(h, k)$ whose locus is to be found for which R divides PQ externally in the ratio 1 : 2

$$h=\frac{2(t^2+1)-1.0}{2-1}=2t^2+2 \Rightarrow t^2=\frac{h-2}{2} \quad \text{..........} \quad (1)$$

$$k=\frac{2(2t+1)-1.\left(\dfrac{t^2+t-1}{t}\right)}{2-1}=\frac{3t+\dfrac{1}{t}+1}{1}=\frac{3t^2+t+1}{t} \quad \text{..........} \quad (2)$$

Using (1) and (2)

$$\Rightarrow \quad k=\frac{3(t^2+1)+(t-2)}{t}$$

$$\Rightarrow \quad k=\frac{\dfrac{3h}{2}-2}{t}+1$$

$$\Rightarrow \quad (k-1)^2=\frac{(3h-4)^2}{4t^2} \quad \Rightarrow \quad (k-1)^2=\frac{(3h-4)^2}{2(h-2)}$$

The locus of point R is

$$2(y-1)^2(x-2)=(3x-4)^2$$

16. There are 18 balls in a box, 12 red and 6 white. 6 draws are made of one ball at a time without replacement of which at least 4 are found to be white. What is the probability that in the next 2 draws, exactly one ball is white? (Leave the answer in terms of C (n, r). **[4]**

Solution

This question is a direct application of Baye's theorem.
Let E be the event when there are minimum 4 white balls in 1'st 6 draws (without replacement)

$$E = E_4 \cup E_5 \cup E_6$$

$E_4 \equiv$ exactly 4 white balls are drawn

$E_5 \equiv$ exactly 5 white balls are drawn

$E_6 \equiv$ exactly 6 white balls are drawn

Let F be the event such that out of next two drawn exactly one is white.

$$P(F) = \frac{P(F \cap E_4) + P(F \cap E_5) + P(F \cap E_6)}{P(E_4) + P(E_5) + P(E_6)}$$

$$= \frac{P(F/E_4)P(E_4) + P(F/E_5)P(E_5) + P(F/E_6)P(E_6)}{P(E_4) + P(E_5) + P(E_6)}$$

$$= \frac{\dfrac{{}^2C_1\,{}^{10}C_1}{{}^{12}C_2}\dfrac{{}^6C_4\,{}^{12}C_2}{{}^{18}C_6} + \dfrac{{}^1C_1\,{}^{11}C_1}{{}^{12}C_2}\dfrac{{}^6C_5\,{}^{12}C_1}{{}^{18}C_6} + 0}{\dfrac{{}^6C_4\,{}^{12}C_2}{{}^{18}C_6} + \dfrac{{}^6C_5\,{}^{12}C_1}{{}^{18}C_6} + \dfrac{{}^6C_6\,{}^{12}C_0}{{}^{18}C_6}}$$

$$= \frac{{}^2C_1\,{}^{10}C_1\,{}^6C_4\,{}^{12}C_2 + {}^1C_1\,{}^{11}C_1\,{}^6C_5\,{}^{12}C_1}{{}^{12}C_2\left({}^6C_4\,{}^{12}C_2 + {}^6C_5\,{}^{12}C_1 + {}^6C_6\,{}^{12}C_0\right)}$$

17. Evaluate, $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$. **[4]**

Solution

$$\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} dx$$

$$= \int_{-\pi/3}^{\pi/3} \frac{\pi}{2 - \cos\left(|x| + \frac{\pi}{3}\right)} + \int_{-\pi/3}^{\pi/3} \frac{4x^3 dx}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}$$

$= I_1 + I_2$

$I_2 = 0$ [odd function], I_1= Even function

$I_1 = 2\pi\int_0^{\pi/3}\frac{\sec^2\left(\frac{x}{2}+\frac{\pi}{6}\right)}{1+3\tan^2\left(\frac{x}{2}+\frac{\pi}{6}\right)}dx$

$= \frac{4\pi}{3}\int_{1/\sqrt{3}}^{\sqrt{3}}\frac{dt}{\frac{1}{3}+t^2} = \frac{4\pi}{3}\times\sqrt{3}\left[\tan^{-1}t\sqrt{3}\right]_{1/\sqrt{3}}^{\sqrt{3}} = \frac{4\pi}{\sqrt{3}}\left[\tan^{-1}3-\frac{\pi}{4}\right]$

The following answers are also correct:

$-\frac{4\pi\,\text{ArcTan}[\frac{1}{2}]}{\sqrt{3}} + \frac{4\pi\,\text{ArcTan}[\frac{4}{3}]}{\sqrt{3}}$ &

$\frac{4\pi}{\sqrt{3}}\left[ArcTan\frac{1}{2}\right]$

18. $f(x) = \begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right) & -\frac{1}{2}<x<0 \\ \frac{1}{2} & x=0 \\ \frac{e^{ax/2}-1}{x} & 0<x<\frac{1}{2} \end{cases}$

If f(x) is differentiable at x = 0, find the value of 'a' and prove that $64b^2 = 4-c^2$.**[4]**

Solution

RHD = $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{\frac{e^{ah/2}-1}{h}-\frac{1}{2}}{h}$

$\lim_{h\to 0}\frac{1+\frac{ah}{2}+\frac{a^2h^2}{2^2}.\frac{1}{2!}......-1-\frac{h}{2}}{h^2}$

For the limit to exist a = 1;

Hence RHD = $\frac{1}{8}$

LHD = $\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$$\lim_{h\to 0}\frac{b\sin^{-1}\left(\frac{c-h}{2}\right)-\frac{1}{2}}{-h}$$

$$\lim_{h\to 0}\frac{b\left\{\frac{c-h}{2}+\frac{1^2}{3!}\left(\frac{c-h}{2}\right)^3+\frac{1^2.3^2}{5!}\left(\frac{c-h}{2}\right)^5.....\right\}-\frac{1}{2}}{-h}$$

As function is differentiable so this limit is equal to $\frac{1}{8}$

For this constant part must be zero and coefficient of h in the numerator must be equal to $-\frac{1}{8}$

Coefficient of h in numerator is equal to

$$-\frac{b}{2}\left[1+\frac{1^2}{3!}3\left(\frac{c}{2}\right)^2+\frac{1^2.3^2}{5!}\times 5\left(\frac{c}{2}\right)^4+....\right]=-\frac{1}{8} \qquad (1)$$

Clearly left hand side is derivative of

$-b\sin^{-1}\frac{x}{2}$ at x = c

$\Rightarrow$ Left hand side of equation (1) is

$$-b\frac{1}{\sqrt{1-\frac{c^2}{4}}}.\frac{1}{2}=-\frac{1}{8}$$

$\Rightarrow$ Squaring both side

$$64b^2=4-c^2$$

19.

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, \quad B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, \quad U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}.$$

If AX = U has infinitely many solution. Prove that BX = V has no unique solution, also prove that if afd ≠ 0, then BX = V has no solution. X is a vector.

[4]

Solution

AX = U

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 0 & : & x \\ 1 & b & d & : & y \\ 1 & b & c & : & z \end{bmatrix} = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1-ab & 0-ad & : & x \\ 1 & b & d & : & y \\ 0 & 0 & d-c & : & z \end{bmatrix} = \begin{bmatrix} f-ag \\ g \\ h-g \end{bmatrix}$$

as AX = U has infinitely many solution

$\Rightarrow \quad d = c$ and $h = g$

Now BX = V

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 1 & : & x \\ 0 & d & c & : & y \\ f & g & h & : & z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & 1 & 1 & : & x \\ 0 & d & c & : & y \\ 0 & 0 & h-\frac{f}{a}-\frac{c}{d}\left(g-\frac{f}{a}\right) & : & z \end{bmatrix} = \begin{bmatrix} a^2 \\ 0 \\ -af \end{bmatrix} \qquad (1)$$

If c = d = 0 and h = g

clearly BX = V has no unique solution

if c = d ≠ 0 $\quad h-\frac{f}{a}-\frac{c}{d}\left(g-\frac{f}{a}\right) = 0$

$\Rightarrow \quad [A : X] = 0$

$\Rightarrow \quad BX = V$ has no unique solution

Now as afd ≠ 0

$\Rightarrow \quad a \neq 0$, f ≠ 0 d ≠ 0

$\Rightarrow \quad$ [A : X] = 0 but as $\quad$ –af ≠ 0

$\therefore \quad$ from equation (1)

BX = V has no solution.

20. P_1 and P_2 are planes passing through origin. L_1 and L_2 also passes through origin. L_1 lies on P_1 not on P_2 and L_2 lies on P_2 but not on P_1. Show that there exists points A, B, C and whose permutation $A'.B'.C'$ can be chosen such that

(i) A is on L_1, B on P_1 but not on L_1 and C not on P_1.

(ii) A' in on L_2, B' on P_2 but not on L_2 and C' not on P_2.

[4]

Solution

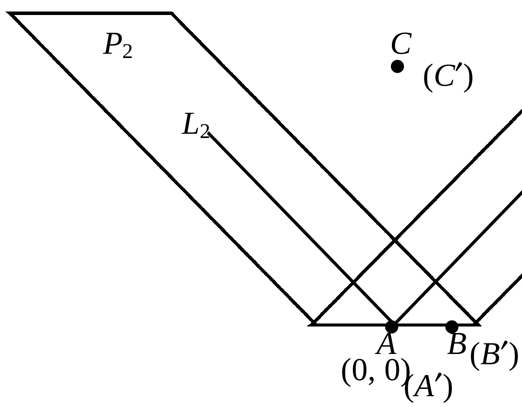
We take

$A = A' =$ origin

$B = B'$ = any point other than origin on the line of intersection of P_1 and P_2.

$C = C'$ = any point neither on P_1 nor on P_2

In this case both conditions (i) and (ii) are fulfilled.



Similarly if we take

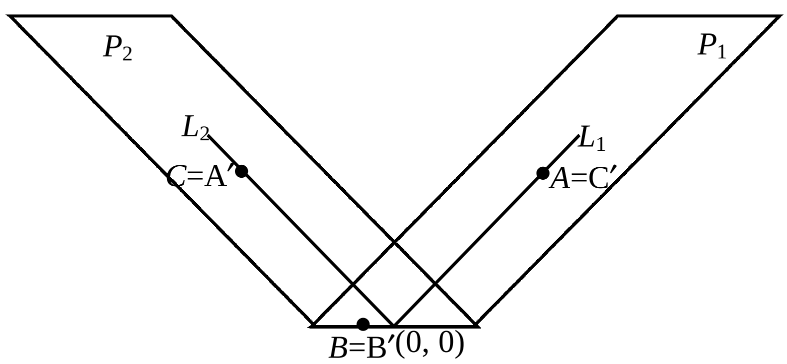
A = non–origin point on L_1

B = non–origin point on the line of intersection of P_1 and P_2

C = non–origin point on L_2

If we take A = C′, B = B′ and C = A′

Both the conditions (i) and (ii) are fulfilled



INDIANET, Where technology meets education!

Visual Physics, Chemistry & Maths