Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

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S.E. (Chem./Petrole./Polymer/Biotech./Printing)

(First Semester) EXAMINATION, 2010

ENGINEERING MATHEMATICS—III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of logarithmic tables, slide rule, electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(1) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = e^{-x}\sec^3 x$

P.T.O.

(2) $\left(D^2 - 4D + 4\right) y = e^{2x} + x^3 + \cos 2x$

(3) $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + x = 0$

(4) $\dfrac{d^2y}{dx^2} + y = \sec x \tan x$ (method of variation of parameters)

(5) $\left(D^2 - 1\right) y = x \sin x + \left(1 + x^2\right) e^x$.

(*b*) Solve :

$$2\frac{dx}{dt} - x + 3y = \sin t$$

$$2\frac{dy}{dt} + 3x - y = \cos t$$

and obtain x and y if $x = 1/4$ and $y = -1/20$ at $t = 0$. [5]

Or

2. (*a*) Solve the following (any *three*) : [12]

(1) $(1 + x)^2 \dfrac{d^2y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2\sin\left[\log(1 + x)\right]$

(2) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{e^x}$ (method of variation of parameters)

(3) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = e^{-2x} \sin 2x + 4x^2 e^x$

(4) $\left(D^2 - 3D + 2\right) y = \cos\left(\dfrac{1}{e^x}\right)$

(5) $\dfrac{d^2y}{dx^2} - y = \cosh x \cos x$.

(*b*) Solve :

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}. \qquad [5]$$

3. (*a*) The D.E. satisfied by a beam uniformly loaded with one end fixed and second subjected to a tensile force P is given by :

$$\mathrm{EI}\frac{d^2y}{dx^2} - \mathrm{P}y = -\frac{\mathrm{W}}{2}x^2.$$

Show that the elastic curve for the beam under conditions $y = 0, \ \frac{dy}{dx} = 0$ when $x = 0$ is given by :

$$y = \frac{\mathrm{W}}{2\mathrm{P}}\left[x^2 - \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right]$$

where $\mathrm{EI} = \frac{\mathrm{P}}{n^2}$. [8]

(*b*) Solve :

$$\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2} \quad \text{if}$$

(*i*) $v \neq \infty$ as $t \to \infty$

(*ii*) $\left(\frac{\partial v}{\partial x}\right)_{x=0} = 0 \quad \forall \ t$

(*iii*) $v(l, t) = 0 \quad \forall \ t$

(*iv*) $v(x, 0) = v_0$ for $0 < x < l$. [8]

Or

4. (*a*) In a certain chemical reaction, the temperature u and v satisfy the equations :

$$\frac{du}{dx} + v = \sin x$$

$$\frac{dv}{dx} + u = \cos x$$

given that when $x = 0$, then $u = 1$ and $v = 0$. Find the values of u and v. [8]

(*b*) An infinitely long plane uniform plate is bounded by two parallel edges in the y-direction and an end at right angles to them. The breadth of plate is π. This end is maintained at temperature u_0 at all points and other edge at zero temperature. Find the steady state temperature function $u(x, y)$. [8]

5. (*a*) Use Fourier transform to solve :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \infty, \; t > 0$$

where $u(x, t)$ satisfies the conditions :

(*i*) $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0 \quad t > 0$

(*ii*) $u(x, 0) = \begin{cases} x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $|u(x, t)| < M$. [7]

(*b*) Solve the integral equation :

$$\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1 & 0 \le \lambda < 1 \\ 2 & 1 \le \lambda < 2. \\ 0 & \lambda \ge 2 \end{cases}$$ [5]

(*c*) Find the Fourier sine and cosine transforms of the following function :

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2. \\ 0 & x > 2 \end{cases}$$ [5]

Or

6. (*a*) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ and hence evaluate :

$$\int_0^\infty \tan^{-1} \frac{x}{a} \sin x \, dx.$$ [6]

(*b*) Using the Fourier integral representation show that :

$$\int_0^\infty \frac{\sin \pi\lambda \, \sin \lambda x}{1 - \lambda^2} \, d\lambda = \begin{cases} \frac{\pi}{2} \sin x & 0 \le x \le \pi \\ 0 & x > \pi \end{cases}.$$ [6]

(*c*) Using inverse Fourier sine transform, find $f(x)$ if :

$$F_s(\lambda) = \frac{\lambda}{1 + \lambda^2}.$$ [5]

SECTION II

7. (*a*) Find Laplace transform (any *three*) : [12]

(*i*) $e^{4t} \int_0^t \frac{1 - \cos 2t}{t} \, dt$

(*ii*) $\frac{\cos\sqrt{t}}{\sqrt{t}}$

(*iii*) $f(t) = \cos \frac{1}{3}(2\pi - 3t), \quad t > \frac{2\pi}{3}$

$\qquad = 0, \quad 0 < t < \frac{2\pi}{3}$

(*iv*) $f(t) = \sin \omega t, \quad 0 < t < \frac{\pi}{\omega}$

$\qquad = 0, \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega},$

Given $f(t) = f\left(t + \frac{2\pi}{\omega}\right).$

(*b*) Find Laplace transform of $erf\left(\sqrt{t}\right)$ and hence evaluate :

$$\int_0^\infty e^{-t} \, erf\left(\sqrt{t}\right) dt.$$ [4]

Or

8. (*a*) Find inverse Laplace transform (any *three*) : [12]

(*i*) $\frac{5s + 3}{(s + 1)\left(s^2 + 2s + 5\right)}$

(*ii*) $\log \sqrt{\dfrac{s^2 + 4}{s^2 + 9}}$

(*iii*) $\dfrac{e^{-ps}}{\sqrt{2s + 3}}$

(*iv*) $\dfrac{1}{s} \sin \left(\dfrac{1}{s}\right).$

(*b*) Use convolution theorem to find :

$$L^{-1}\left\{\frac{s}{\left(s^2 + a^2\right)^2}\right\}. \qquad [4]$$

9. (*a*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at the point (2, –1, 1) along the line equally inclined with co-ordinate axes. [6]

(*b*) Show that the vector field :

$$\bar{F} = \left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$$

is irrotational. Find the scalar point function ϕ such that $\bar{F} = -\nabla\phi$. [6]

(*c*) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ for :

$$\bar{F} = (2y + 3)\,\bar{i} + xz\,\bar{j} + (yz - x)\,\bar{k}$$

along the curve $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [6]

Or

10. (*a*) Use Stokes' theorem to evaluate :

$$\iint_s \nabla \times \overline{\mathrm{F}} \,.\, \hat{h}\; ds$$

over the surface of cylinder $x^2 + y^2 = 4$ bounded by $z = 9$, and open at $z = 0$, where :

$$\overline{\mathrm{F}} = (2x - y + z)\,\overline{i} + \left(x + y - z^2\right)\overline{j} + (3x - 2y + 4z)\,\overline{k}\,. \qquad [6]$$

(*b*) Evaluate :

$$\iint_s \left(x^3\,\overline{i} + y^3\,\overline{j} + z^3\,\overline{k}\right).\, d\overline{s}$$

over the surface of sphere $x^2 + y^2 + z^2 = 16$ by using Gauss's divergence theorem. [6]

(*c*) Establish the vector identities : [6]

(*i*) $\nabla f(r) = \dfrac{f'(r)}{r}\,\overline{r},\ \left(\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}\right)$

(*ii*) $\nabla \times \left(\dfrac{\overline{a} \times \overline{r}}{r}\right) = \dfrac{\overline{a}}{r} + \dfrac{(\overline{a}\,.\,\overline{r})\,\overline{r}}{r^3}$

(*iii*) $\nabla \,.\, \left(\dfrac{\overline{a} \times \overline{r}}{r^n}\right) = 0.$

11. (*a*) Solve the differential equation by using Laplace transform method :

$$\frac{d^2y}{dt^2} + 4y = f(t),$$

where

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$$

with $y(0) = 0$, $y¢(0) = 1$. [6]

(*b*) If the velocity potential of a fluid motion is given by $f = \log(xyz)$, find the equations of streamlines. [5]

(*c*) The transfer function of a second order system (with $x < 1$) is given by :

$$G(s) = \frac{Y(s)}{X(s)} = \frac{6}{s^2 + 1.8s + 1}.$$

Find over shoot, decay ratio and period of oscillation. [5]

Or

12. (*a*) A liquid is in equilibrium under the action of field force $\bar{F}$ per unit mass is :

$$\bar{F} = 1\left\{(y+z)\,\bar{i} + (z+x)\,\bar{j} + (x+y)\,\bar{k}\right\}.$$

Find the pressure at any point of the field. [5]

(*b*) Solve by using Laplace transform method :

$$\frac{dy}{dt} + 2y(t) + \int_0^t y(t)\, dt = \sin t,$$

with $y(0) = 1$. [6]

(*c*) Two non-interacting tanks are connected in series. The time constants are $T_2 = 1$ and $T_1 = 0.5$ and $R_2 = 1$. Find the response using transfer function. [5]