Total No. of Questions—12] [Total No. of Printed Pages—8+2

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S.E. (Chemical/Print/Poly/Petro/Petro. Chem./Biotech) (I. Sem.)

EXAMINATION, 2010

ENGINEERING MATHEMATICS—III

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Attempt Q. Nos. **1** or **2**, **3** or **4**, **5** or **6** from Section **I**. Attempt Q. Nos. **7** or **8**, **9** or **10**, **11** or **12** from Section **II**.

(*ii*) Answers to the two sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of Electronic Calculator is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve (any *three*) : [12]

(*i*) $(D^4 - 1)y = \cos x . \cosh x$

(*ii*) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \dfrac{1}{1+e^x}$

P.T.O.

(*iii*) $(D^2 + 2D + 1)\, y = x.e^x . \sin x$

(*iv*) $\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$ [using variation of parameters]

(*v*) $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right)$.

(*b*) Solve the system :

$$\frac{du}{dx} + v = \sin x\,; \quad \frac{dv}{dx} + u = \cos x$$

Given that $x = 0$, $u = 1$, $v = 0$. [5]

Or

2. (*a*) Solve (any *three*) : [12]

(*i*) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + y = x^3 - 3x^2 + 1$

(*ii*) $(D^2 - 4D + 3)\, y = x^2\, e^{3x}$

(*iii*) $(D^2 + 1)\, y = \sin x . \sin 2x$

(*iv*) $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$ [Using variation of parameters]

(*v*) $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2.\sin[\log(1+x)]$.

(*b*) Solve : [5]

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}.$$

3. (*a*) The deflection of a strut with one end built in ($x = 0$) and other supported and subjected to end thrust p; satisfies the equation :

$$\frac{d^2y}{dx^2} + d^2y = \frac{a^2p^2}{p}(l - x).$$

Given that :

$$\frac{dy}{dx} = y = 0 \text{ when } x = 0 \text{ and } y = 0 \text{ when } x = l$$

prove that :

$$y = \frac{\mathrm{R}}{\mathrm{P}}\left[\frac{\sin ax}{a} - l\cos ax + l - x\right];$$

where $al = \tan al$. [8]

(*b*) Solve :

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \text{ if :}$$ [8]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \dfrac{u_0 x}{l}$ for $0 \le x \le l$.

Or

4\. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $u = a \sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end by using wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \qquad [8]$$

(b) In a chemical transformation of certain substance the equation appear as :

$$\frac{dx}{dt} + lx = 0, \quad \frac{dz}{dt} = my \quad \text{and}$$

$x + y + z = n$; where l, m, n are constants. Obtain a differential equation for z. Also prove that if

$$z = \frac{dz}{dt} = 0, \quad t = 0$$

then :

$$z = n + \frac{n}{l-m}\left(m\,e^{-lt} - l\,e^{-mt}\right). \qquad [8]$$

5\. (a) Using Fourier integral representation; show that :

$$\int_0^{\infty} \frac{1-\cos\pi\lambda}{\lambda} \sin\lambda x \, d\lambda = \begin{cases} \frac{\pi}{2} & ; \quad 0 < x < \pi \\ 0 & ; \quad x > \pi \end{cases} \qquad [6]$$

(b) Find the Fourier cosine transforms of the following function :

$$f(x) = \begin{cases} x & ; \quad 0 \le x \le 1 \\ 2 - x & ; \quad 1 \le x \le 2 \\ 0 & ; \quad x > 2 \end{cases}.$$ [5]

(c) Use Fourier transform to solve the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, \; t > 0$$

subject to the following conditions :

(i) $u(0, t) = 0 ; \; t > 0$

(ii) $u(x,0) = \begin{cases} 1 & ; \quad 0 < x < 1 \\ 0 & ; \quad x > 1 \end{cases}$

(iii) $u(x, t)$ is bounded. [6]

Or

6. (a) Find the Fourier sine transform of $\frac{1}{x}$. [5]

(b) Solve the following integral equation :

$$\int_0^\infty f(x) . \cos \lambda x \, dx = e^{-x} \; ; \; \lambda > 0.$$ [5]

(c) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 - x^2 & ; \quad |x| \le 1 \\ 0 & ; \quad |x| > 1 \end{cases}$$

and hence evaluate :

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} \, dx.$$ [7]

SECTION II

7. (a) Find Laplace transform of (any *two*) : [6]

(i) $e^{-t} \sin t \, U(t-\pi)$

(ii) $\displaystyle\int_0^t \frac{\sin t}{f} \, dt$

(iii) $\sin t^2$.

(b) Find inverse Laplace transform of (any *two*) : [6]

(i) $\cot^{-1}\left(\dfrac{s-2}{3}\right)$

(ii) $\dfrac{1}{(s+2)(s^2+2s+2)}$

(iii) $\dfrac{s+2}{s^2(s-1)^2}$

use convolution theorem.

(c) Find Laplace transform of periodic function,

$$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases} \quad f(t+2\pi) = f(t).$$ [4]

Or

8. (a) Evaluate the integral :

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} \, dt.$$ [4]

(*b*) Find Laplace transform of (any *two*) : [6]

(*i*) $t^2 \, u(t-2) - \cosh t \, \delta(t-4)$

(*ii*) $e^{-3t} \int_0^t t \sin 2t \; dt$

(*iii*) $\dfrac{1-\cos t}{t}$.

(*c*) Find inverse Laplace transform of (any *two*) : [6]

(*i*) $\dfrac{2s}{(s^2-4)^2}$

(*ii*) $\dfrac{s-1}{s^2-6s+25}$

(*iii*) $\dfrac{e^{-2s}}{\sqrt{s+5}}$.

9. (*a*) Find the directional derivative of $\phi = e^{2x-y-z}$ at (1, 1, 1) in the direction tangent to the curve :

$$x = e^{-t}, y = 2\sin t + 1, z = t - \cos t \text{ at } t = 0.$$ [5]

(*b*) If

$$\overline{\mathrm{F}}_1 = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}, \quad \overline{\mathrm{F}}_2 = (\overline{a}\,.\,\overline{r})\,\overline{a},$$

show that $\overline{\mathrm{F}}_1 \times \overline{\mathrm{F}}_2$ is solenoidal. [5]

(c) Verify Stokes theorem for $\overline{F} = -y^3 \hat{i} + x^3 \hat{j}$ and the closed curve c is the boundary of an ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ [7]

Or

10. (a) Use divergence theorem to evaluate :

$$\iint_S \overline{F} \cdot d\overline{S}$$

where $\overline{F} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$

and S is the part of surface of sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [6]

(b) Prove the following (any *two*) : [6]

(i) $\nabla^4 (\log r) = \dfrac{2}{r^4}$

(ii) $\nabla \cdot \left(\dfrac{\overline{a} \times \overline{r}}{r^n} \right) = 0$

(iii) $\nabla \times \left(\dfrac{\overline{a} \times \overline{r}}{r^5} \right) = \dfrac{-3\overline{a}}{r^5} + \dfrac{5(\overline{a} \cdot \overline{r})\overline{r}}{r^7}.$

(c) Show that :

$$\overline{F} = (2xz^3 + 6y)\,\hat{i} + (6x - 2yz)\,\hat{j} + (3x^2z^2 - y^2)\,\hat{k}$$

is irrotational. Hence find scalar potential ϕ s.t. $\overline{F} = \nabla\phi$. [5]

11. (a) Using Laplace transform, solve :

$$\frac{dy}{dt} + 3y(t) + 2\int_0^t y(t)\,dt = t$$

Given : $y(0) = 0.$ [6]

(b) Show that : The velocity potential

$$\phi = \frac{1}{2} a (x^2 + y^2 - 2z^2)$$

satisfies the Laplace equation. Also determine the stream-lines. [5]

(c) The transfer function of a second order system is given as

$$G(s) = \frac{10}{s^2 + 1.6s + 4}$$

Determine its properties such as overshoot, $y(t)_{max}$, period of oscillations. [6]

Or

12. (a) Find the surfaces of equipressure in the case of steady motion of a liquid which has velocity potential $\phi = \log x + \log y + \log z$ and is under the action of force :

$$\overline{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$$ [5]

(b) Using Laplace transform, solve :

$$\frac{dx}{dt} + \frac{dy}{dt} = t$$

$$\frac{d^2x}{dt^2} - y = e^{-t}.$$

subject to $x(0) = 3$, $x'(0) = -2$, $y(0) = 0$. [6]

(c) A tank having a time constant 1 min and resistance $\frac{1}{9}$ ft/cfm is operating at steady-state with an inlet flow of 10 ft^3/min. At time $t = 0$, the flow is suddenly increased to 100 ft^3/min for 0.1 min by adding an additional 9 ft^3 of water to the tank uniformly over a period of 0.1 min. Plot response in tank level and compare with impulse response. [6]