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S.E. (Chemical/Print/Poly/Petro/Petro. Chem./Biotech) (I. Sem.)

## EXAMINATION, 2010

## ENGINEERING MATHEMATICS—III

# (2003 COURSE)

Time: Three Hours

Maximum Marks: 100

- (i) Attempt Q. Nos. 1 or 2, 3 or 4, 5 or 6 from N.B. :-Section I. Attempt Q. Nos. 7 or 8, or 10, 11 or 12 from Section II.
  - Answers to the two sections should be written in separate (ii) answer-books.
  - Figures to the right indicate full marks. (iii)
  - Use of Electronic Calculator is allowed. (iv)
  - Assume suitable data, if necessary. (v)

# SECTION I

(a) Solve (any three): 1.

[12]

 $(-1)y = \cos x. \cosh x$ 

$$(it) \qquad \frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1 + e^x}$$

(iii) 
$$(D^2 + 2D + 1) y = x \cdot e^x \cdot \sin x$$

(iv) 
$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$
 [using variation of parameters]

(v) 
$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right).$$

Solve the system : (b):

$$\frac{du}{dx} + v = \sin x \; ; \quad \frac{dv}{dx} + u = \cos x$$

Given that x = 0, u = 1, v = 0. [5]

2. (a) Solve (any three):

(i) 
$$\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = x^3 - 3x^2 + 1$$

(ii) 
$$(D^2 - 4D + 3) y = x^2 e^{0x}$$

(iii) 
$$(D^2 + 1) y = \sin x \cdot \sin 2x$$

(iii) 
$$(D^2 + 1) \ y = \sin x \cdot \sin 2x$$
  
(iv)  $\frac{d^2y}{dx^2} + y = \csc x$  [Using variation of parameters]  
(v)  $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \cdot \sin [\log (1+x)].$   
(b) Solve : 
$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}.$$

(v) 
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2.\sin[\log(1+x)].$$

[5]

$$\frac{dx}{x(2y^4-z^4)} = \frac{dy}{y(z^4-2x^4)} = \frac{dz}{z(x^4-y^4)}.$$

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(a) The deflection of a strut with one end built in (x = 0) and other supported and subjected to end thrust p; satisfies the equation :

$$\frac{d^2y}{dx^2} + d^2y = \frac{a^2p^2}{p}(l-x).$$

Given that:

$$\frac{dy}{dx} = y = 0$$
 when  $x = 0$  and  $y = 0$  when  $x = l$ 

prove that :

$$y = \frac{R}{P} \left[ \frac{\sin ax}{a} - l \cos ax + D - x \right];$$

where  $al = \tan al$ . [8]

(b). Solve :

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$
, if : [8]

- (i)  $u(0, t) = C_{0}$
- $(ii)\quad u\left(l,\,\star\right)\ =\ 0$

(iii) 
$$u(x, t)$$
 is bounded and 
$$(x_0) \quad u(x, 0) = \frac{u_0 x}{l} \quad \text{for } 0 \le x \le l.$$

4. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form  $u = a \sin \frac{\pi x}{l}$  from which it is released at time t = 0. Find the displacement u(x, t) from one end by using wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$
 [8]

(b) In a chemical transformation of certain substance the equation appear as:

$$\frac{dx}{dt} + lx = 0$$
,  $\frac{dz}{dt} = my$  and

x + y + z = n; where l, m, n are constants. Obtain a differential equation for z. Also prove that if

$$z = \frac{dz}{dt} = 0, \quad t = 0$$

then:

$$n + \frac{n}{l-m} (m e^{-lt} - l e^{-mt}).$$
 [8]

5. (a) Using Fourier integral representation; show that :

$$\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin \lambda x \, d\lambda = \begin{cases} \frac{\pi}{2} & ; \quad 0 < x < \pi \\ 0 & ; \quad x > \pi \end{cases}$$
 [6]

Find the Fourier cosine transforms of the following function: (b)

$$f(x) = \begin{cases} x & ; & 0 \le x \le 1 \\ 2 - x & ; & 1 \le x \le 2 \\ 0 & ; & x > 2 \end{cases}$$
 [5]

Use Fourier transform to solve the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, \ t > 0$$

subject to the following conditions:

$$(i) \quad u\,(0,\ t) \ = \ 0\;;\; t \ > \ 0$$

(ii) 
$$u(x,0) = \begin{cases} 1 & ; & 0 < x < 1 \\ 0 & ; & x > 1 \end{cases}$$

(iii) u(x, t) is bounded. [6]

Or

- Find the Fourier sine transform of [5] (a) 6.
  - Solve the following integral equation : (b)

$$\int_{0}^{\infty} f(x) \cos \lambda x \, dx = e^{-x} ; \lambda > 0.$$
 [5]

• 
$$f(x) = \begin{cases} 1 - x^2 & ; |x| \le 1 \\ 0 & ; |x| > 1 \end{cases}$$

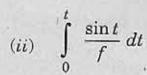
Find the Fourier transform of : 
$$f(x) = \begin{cases} 1 - x^2 & ; & |x| \le 1 \\ 0 & ; & |x| > 1 \end{cases}$$
 and hence evaluate : 
$$\int_0^\infty \left( \frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} \, dx \,.$$
 [7]

#### SECTION II

7. (a) Find Laplace transform of (any two):

[6]

(i)  $e^{-t} \sin t U(t-\pi)$ 



(iii)  $\sin t^2$ .



(b) Find inverse Laplace transform of (any two):

[6]

(i)  $\cot^{-1}\left(\frac{s-2}{3}\right)$ 

(ii) 
$$\frac{1}{(s+2)(s^2+2s+2)}$$

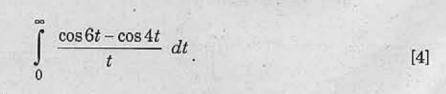
 $(iii) \quad \frac{s+2}{s^2 (s-1)^2}$ 

use convolution theorem.

(c) Find Laplace transform of periodic function,

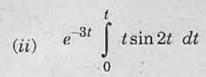
$$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi & t & \pi < t < 2 \pi \end{cases} \quad f(t + 2\pi) = f(t) . \tag{4}$$

8. (a) Evaluate the integral :

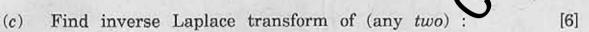


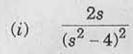
(b) Find Laplace transform of (any two):

(i)  $t^2 u(t-2) - \cosh t \delta (t-4)$ 



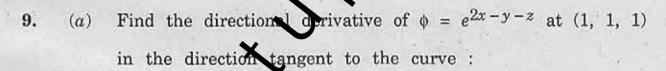
 $(iii) \quad \frac{1-\cos t}{t}.$ 





$$(ii) \quad \frac{s-1}{s^2-6s+25}$$

(iii)  $\frac{e^{-2s}}{\sqrt{s+5}}$ 



$$x = e^{-t}$$
,  $y = 2\sin t + 1$ ,  $z = t - \cos t$  at  $t = 0$ . [5]

(b) If

$$\overline{\mathbf{F}}_1 = yz \stackrel{\wedge}{i} + zx \stackrel{\wedge}{j} + xy \stackrel{\wedge}{k}, \quad \overline{\mathbf{F}}_2 = (\overline{a} \cdot \overline{r}) \stackrel{\sim}{a},$$

show that  $\overline{F}_1 imes \overline{F}_2$  is solenoidal.

[5]

(c) Verify Stokes theorem for  $\overline{F} = -y^3 \hat{i} + x^3 \hat{j}$  and the closed curve c is the boundary of an ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$
Or

10. (a) Use divergence theorem to evaluate:

$$\iint\limits_{\mathbf{S}} \, \overline{\mathbf{F}} \, . \, d\overline{\mathbf{S}}$$

where

$$\overline{F} = yz \hat{i} + zx \hat{j} + xy \hat{k}$$

and S is the part of surface of sphere  $x^2 + y^2 + z^2 = 1$  which lies in the first octant. [6]

[6]

(b) Prove the following (any two):

$$(i) \qquad \nabla^4 \ (\log r) = \frac{2}{r^4}$$

(ii) 
$$\nabla \cdot \left( \frac{\overline{a} \times \overline{r}}{r^n} \right) = 0$$

(iii) 
$$\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^5}\right) = \frac{-3\overline{a}}{r^5} + \frac{5(\overline{a} \cdot \overline{r})\overline{r}}{r^7}.$$

(c) Show that:

$$\overline{F} = (2xz^3 + 6y) \hat{i} + (6x - 2yz) \hat{j} + (3x^2z^2 - y^2) \hat{k}$$

is irrotational. Hence find scalar potential  $\varphi$  s.t.  $\overline{F}=\nabla \varphi$  . [5]

11. (a) Using Laplace transform, solve:

$$\frac{dy}{dt} + 3y(t) + 2\int_{0}^{t} y(t) dt = t$$

Given :

$$y(0) = 0.$$

(b) Show that: The velocity potential

$$\phi = \frac{1}{2} \ a \ (x^2 + y^2 - 2z^2)$$

satisfies the Laplace equation. Also determine the stream-lines. [5]

(c) The transfer function of a second order system is given as

$$G(s) = \frac{10}{s^2 + 1.6s + 4}$$

Determine its properties such as overshoot,  $y(t)_{max}$ , period of oscillations.

12. (a) Find the surfaces of equipressure in the case of steady motion of a liquid which has velocity potential  $\phi = \log x + \log y + \log z$  and is under the action of force :

$$\overline{F} = yz \hat{i} + zx \hat{j} + xy \hat{k}.$$
 [5]

(b) Using Laplace transform, solve:

$$\frac{dx}{dt} + \frac{dy}{dt} = t$$

$$\frac{d^2x}{dt^2} - y = e^{-t}.$$

subject to x(0) = 3, x'(0) = -2, y(0) = 0.

level and compare with impulse response.

(c) A tank having a time constant 1 min and resistance  $\frac{1}{9}$  ft/cfm is operating at steady-state with an inlet law of 10 ft<sup>3</sup>/min. At time t=0, the flow is suddenly increased to 100 ft<sup>3</sup>/min for 0.1 min by adding an additional 9 ft<sup>3</sup> of water to the tank uniformly over a period of 0.1 min. Plot response in tank

[6]

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