M-50/ 8/M.S c STATIS

Entrance Examination, 2006

M.Sc. (Mathematics/Applied Mathematics)

Hall Ticket No.							

Time: 2 hours

Max. Marks: 100
Part A: 25
Part B: 75

Instructions

1. Calculators are not allowed.
2. Part A carries 25 marks. Each correct answer carries **1 mark** and each wrong answer carries $-\frac{1}{4}$ **mark**. So do not gamble. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part B carries 75 marks. There are 15 questions in Part B and each question carries 5 marks. The answers should be written in the space provided below each question.
4. Do not detach any pages from this answer book. It contains 16 pages. Pages 15 and 16 are for rough work.

Answer Part A by **circling** the correct letter in the array below:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

Part A

1. Let A and B be two sets having m and n elements respectively. If the number of elements in $A \cap B$ is 10, then the number of elements in $(A \setminus B) \cup (B \setminus A)$ is

   a) $m+n-20$ b) $m+n-10$ c) 10 d) $m+n$ e) 0.

2. If $\mathcal{A} = \{E \subset \mathbf{N} \mid E$ is disjoint from the set of all multiples of $3\}$, where $\mathbf{N}$ is the set of all positive integers, then which of the following is always true:

   a) $\{6\} \in \mathcal{A}$ b) $6 \in E$, whenever E does not belong to $\mathcal{A}$.

   c) $6 \in \mathcal{A}$ d) If $6 \in E$, then E does not belong to $\mathcal{A}$.

   e) none of the above.

3. Let (a_n) be a sequence of real numbers. Let $b_n = a_n + a_{n+1}$, for $n = 1, 2, \cdots$. Which of the following is always true:

   a) If (b_n) converges, then (a_n) converges.

   b) If (a_n) converges, then (b_n) diverges.

   c) If (a_n) converges, then (b_n) converges.

   d) (a_n) is a subsequence of (b_n).

   e) none of the above.

4. Consider the series $\sum \frac{1}{\sqrt{n}}$ and $\sum \frac{1}{n^{\frac{3}{2}}}$. Then

   a) both the series converge to the same value.

   b) both the series converge to different values.

   c) both the series are divergent.

   d) first series is divergent and second series is convergent.

   e) first series is convergent and second series is divergent.

5. $\lim_{x \to 2\pi} \frac{Sin\ x}{x - 2\pi}$ is equal to

   a) 0 b) 1 c) ∞ d) -1 e) none of these.

6. Let $f(x) = |x|^3$.

   a) f is continuous at $x = 0$ but not differentiable at $x = 0$.

   b) f is differentiable at $x = 0$, but $f'(x)$ is not continuous at $x = 0$.

   c) f is differentiable at $x = 0$ and $f'(x)$ is continuous at $x = 0$.

   d) $f''(x)$ exists at $x = 0$, but $f''(x)$ is not continuous at $x = 0$.

   e) none of the above.

7. Which of the following inequalities is always true for $x > 1$:

   a) $\frac{x-1}{x} > \log x$ b) $\frac{x-1}{x} < \log x < x - 1$

   c) $\log x > x - 1$ d) $\frac{x-1}{x} > x - 1$

   e) none of the above.

8. If $f(x) = [x^2] - [x]^2$, where $[x]$ denotes the greatest integer $\leq x$, then $\int_1^2 f(x)dx$ is

   a) $4 - \sqrt{3} + \sqrt{2}$ b) $4 - \sqrt{3} - \sqrt{2}$

   c) $4 + \sqrt{3} + \sqrt{2}$ d) 0

   e) none of the above.

9. The slope of the tangent to the unit circle $x^2 + y^2 = 1$ at $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ is

   a) $-\sqrt{3}$ b) $\sqrt{3}$ c) $\frac{1}{\sqrt{3}}$ d) $-\frac{1}{\sqrt{3}}$ e) none of these.

10. The angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x + y - 2z - 3 = 0$ is

    a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$ e) none of these.

11. Let $\mathbf{Z}_{29}^* = \{1, 2, 3, \cdots, 28\}$ be the group under multiplication modulo 29. The inverse of 28 in $\mathbf{Z}_{29}^*$ is

    a) 28 b) 27 c) 26 d) 1 e) none of these.

12. The value of $5^{5!}$ (modulo 11) is

    a) 2 b) 3 c) 4 d) 5 e) none of these.

13. Consider the following statements:

    S1) Every group of prime order must be cyclic.

    S2) Every group of prime order must be abelian.

    S3) Every group of prime order has only one subgroup other than itself.

    Which of the following is always true?

    a) S1 and S2 are true but S3 is false.

    b) S2 and S3 are true but S1 is false.

    c) S1 and S3 are true but S2 is false.

    d) S1, S2 and S3 are true.

    e) none of the above.

14. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. For the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ which of the following is true?

    a) The nullity is 1 and the rank is 2.

    b) The nullity is 2 and the rank is 1.

    c) The nullity is 0 and the rank is 3.

    d) The nullity is 3 and the rank is 0.

    e) none of the above.

15. The eigen values of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$ are

    a) 1, 2, 3 b) 3, 5, 6 c) 4, 5, 6 d) 0, 1 e) none of these.

16. Let A be a 3×3 matrix with real entries. If A commutes with all 3×3 matrices with real entries, then the number of distinct real eigen values of A is

    a) 0 b) 1 c) 2 d) 3 e) none of these.

17. Let a and b be the zeroes of the quadratic polynomial $x^2 + \sqrt{\pi}x + \frac{22}{28}$. Then

a) a and b are distinct real numbers.

b) a and b are real and equal.

c) a and b are distinct and complex conjugate.

d) $a = b$ and a is not a real number.

e) none of the above.

18. Let $A = \begin{pmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \\ 6 & 12 & 18 \end{pmatrix}$ and $b = \begin{pmatrix} 12 \\ 24 \\ a \end{pmatrix}$. The system of equations $AX = b$ has a solution if a is equal to

a) 12 b) 24 c) 28 d) 82 e) none of these.

19. A non-empty set in the plane ($\mathbf{R}^2$) is said to be *convex* if the line segment joining any two points in the set is completely contained in the set. Consider the following sets.

I) the intersection of two circular discs of radius 2 having centers $(0,0)$ and $(3,0)$.

II) the set of all (x, y) with $x \geq 0$ and $y \geq 0$.

Then

a) both I and II are convex. b) both I and II are not convex.

c) I is convex but II is not. d) II is convex but I is not.

e) none of the above.

20. Let

$$f(x) = \begin{cases} e^x + a\sin x & \text{if } x < 0 \\ b(x-1)^2 + x - 2 & \text{if } x \geq 0 \end{cases}.$$

The function f is differentiable at $x = 0$ if

a) $a = 6$ and $b = 3$. b) $a = -6$ and $b = 3$.

c) $a = -6$ and $b = -3$. d) $a = 6$ and $b = -3$.

e) none of the above.

21. Consider the element $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix}$ of the symmetric group S_5 on five elements. Then

a) the order of α is 2. b) the order of α is 3.

c) the order of α is 6. d) the order of α is 5.

e) none of the above.

22. If the statement "All elements of A are in B" is false, then

a) no element of A is in B.

b) all elements of B are in A.

c) some element of A is not in B.

d) some element of B is not in A.

e) none of the above.

23. The value of the integral $\int_{-1}^{0} \sqrt{\frac{1+x}{1-x}}\, dx$ is

a) 0 b) ∞ c) $\frac{\pi}{2} - 1$ d) $\frac{\pi}{2} + 1$ e) none of these.

24. A fair coin is being tossed. The probability that after 4 trails, more heads than tails have appeared is

a) $\frac{1}{8}$ b) $\frac{3}{16}$ c) $\frac{4}{16}$ d) $\frac{5}{16}$ e) none of these.

25. 5 boys and 5 girls are made to sit in a row. Then the probability that all the 5 girls sit together is

a) twice the probability that no two girls sit next to each other.

b) half the probability that no two girls sit next to each other.

c) 3 times the probability that no two girls sit next to each other.

d) equal to the probability that no two girls sit next to each other.

e) none of the above.

Part-B

1. Let $v_1 = (1, 0, -1, 2)$, $v_2 = (0, 0, 3, 0)$, $v_3 = (1, 1, 0, -1)$ in $\mathbb{R}^4$. Give an example of a vector which does not belong to the linear span of v_1, v_2 and v_3. Show why your example works.

2. Find the dimension of the subspace $\{(x_1, x_2, x_3, x_4, x_5)/3x_1 - x_2 + x_3 = 0, \;\; x_2 - x_3 = 0, \;\; x_1 = 0\}$ of $\mathbb{R}^5$.

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3. Let $\mathbb{R}^*$ be the group of all nonzero real numbers under multiplication and ${\mathbb{R}^*}^2$ the subset of $\mathbb{R}^*$ consisting of all squares. Show that ${\mathbb{R}^*}^2$ is a subgroup of $\mathbb{R}^*$ and find the order of the quotient group $\mathbb{R}^*/{\mathbb{R}^*}^2$.

4. If m is an odd integer then show that $m^2 - 1$ is divisible by 8.

5. Give an example of a 2×2 matrix A which is not equal to the identity matrix I and $A^3 = I$.

6. Find the eigenvalues of the matrix $\begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$.

7. Find the maximum and minimum of the function $f(x) = x^3 - 27x$ in the interval $[-4, 7]$.

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. If f is not one-one then show that $f'(c) = 0$ for some point $c \in \mathbb{R}$. Give an example of differentiable function $g : \mathbb{R} \to \mathbb{R}$ such that $g'(c) = 0$ for some point $c \in \mathbb{R}$ and g is one-one.

9. Let $f : [-1, 2] \longrightarrow \mathbb{R}$ defined as $f(x) = \min(x, x^2)$. Find $\int_{-1}^{2} f(x)dx$.

10. Evaluate $\int (x^2 y dx + xy^2 dy)$ from $(0, 0)$ to $(2, 4)$ along the straight line joining these two points and also along the curve $y = x^2$.

11. Find the equations of the lines of intersection of the plane $x + 7y - 5z = 0$ and the cone $3yz + 14xz - 30xy = 0$.

12. Prove that $3y^2 - 8xy - 3y^2 - 29x + 3y - 18 = 0$ represents two straight lines. Find the point of intersection and the angle between them.

13. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} x^n \sqrt{\log(n)}$.

14. 3 fair die of different colours are thrown together. Evaluate

(a) The probability that more sixes show up than ones.

(b) An equal number of ones and threes show up.

(c) The expected number of sixes.

15. There are 10 slips numbered 1, 2, ... , 10 in a bag. Anita puts her hand in the bag and takes out two slips. Evaluate the probabilities of the following events

(a) One of the numbers drawn is 6.

(b) 6 is the larger of the two numbers drawn.

(c) The sum of the two numbers drawn is divisible by 3.

(d) The product of the two numbers drawn is a perfect square.