

Fourth Semester Examination – 2007

QUANTITATIVE TECHNIQUES - I

Full Marks – 70

Time – 3 Hours

*Answer Question No. 1 which is compulsory
and any five from the rest.*

*The figures in the right-hand margin
indicate marks.*



1. Answer the following questions : 2x10

(a) Define the following terms :

Basic solution, Surplus variable, Pseudo-optimal solution.

P.T.O.

- (b) Give a mathematical formulation of the traveling salesman problem.
- (c) Write Little's formula.
- (d) Using North-West corner method, find the starting basic solution of the transportation problem.

				Supply	
	10	2	20	11	15
	12	7	9	20	25
	4	14	16	18	10
Demand	5	15	15	15	

- (e) What do you mean by queue-discipline ?
- (f) Two red and two black cards are lying face down on a table. If you guess their colours, what is the probability that your guess is right ?
- (g) A person has 0.8 probability of passing a test. If the person takes six tests, what

is the probability of his (her) passing at least four test ?

- (h) You purchase a lottery ticket worth 20 rupees. If the probability of your winning the first prize of one million rupees is one in one million, what is your expected profit ? [Assume that there is just one prize in this lottery]

(i) A fair coin is tossed 100 times. If X is the number of heads obtained, find the expected value and variance of X .

- (j) A book has 2 misprints per page on average. If you open the book at random, what is the probability that there are more than 2 misprints on that page ?

2. (a) Solve graphically :

$$\text{Maximize } z = 5x + 4y$$

$$\text{subject to : } 6x + 4y \leq 24$$

$$x + 2y \leq 6$$

$$-x + y \leq 1$$

$$y \leq 2$$

$$x, y \geq 0$$

(b) Solve :

$$\text{Minimize } z = 4x + y$$

$$\text{subject to : } 3x + y = 3$$

$$4x + 3y \geq 6$$

$$x + 2y \leq 4$$

$$x, y \geq 0$$

3. (a) Find the starting solution of the transportation problem given below by Vogel's approximation method :

	Supply				
	10	2	20	11	15
	12	7	9	20	25
	14	14	16	18	10
Demand	5	15	15	15	

(b) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution.

Find - 5

(i) The average number of births per year

(ii) The probability that no births will occur in any one day.

4. Use branch and bound method to solve the following : 10

$$\text{Maximize } z = 5x + 7y$$

$$\text{subject to : } 2x + y \leq 13$$

$$5x + 9y \leq 41$$

$$x, y \geq 0 \text{ are integers.}$$

5. (a) Write short notes on : 5

Finite population models.

(b) The coefficients of a linear equation $ax + b = 0$ are determined by throwing a die twice. Find the probability that the equation has an integral root. 5

6. (a) A bag contains 100 tokens, numbered 1 through 100. If you pick up one token at random, what is the probability that the number on the token is divisible by 2, 3 or 5 but not by all these three numbers?

(b) The mean and the variance of the mathematics marks of an examination are 46 and 15 respectively. Assuming that the marks are normally distributed: 5

(i) find the probability that a randomly picked person has scored more than 70

(ii) find the probability that a randomly picked person has scored less than 30.

7. (a) Define the moment generating function of a random variable X . Find the moment generating function of X if - 5

$$P_r(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

(b) Let x_1, x_2, \dots, x_n be independent random variables with same mean μ and same variance σ^2 , find the mean and variance of - 5

$$Y = \frac{x_1 + x_2 + \dots + x_n - n\mu}{\sqrt{n}\sigma}$$

8. (a) The joint p.d.f. of two random variables X and Y , is given by

$$f(x,y) = \begin{cases} 6e^{-2x-3y}, & x > 0, y > 0 \\ 0 & \text{elsewhere;} \end{cases}$$

Find the following probabilities. 5 C

(i) $P, (1 \leq X \leq 2, 2 \leq Y \leq 3)$

(ii) $P, (X \geq 2, Y \geq 2)$

(b) Determine the marginal densities. Are X and Y independent? 5

