INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

22nd May 2008

Subject ST6 – Finance and Investment B

Time allowed: Three hours (14.15* pm – 17.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer sheet.*
2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*
3. *The answers are not expected to be any country or jurisdiction specific However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
4. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor.*
5. *Mark allocations are shown in brackets.*
6. *Attempt all questions, beginning your answer to each question on a separate sheet.*
7. *Candidates should show calculations where this is appropriate.*
8. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

Professional Conduct:

It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI.

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answersheets and this question paper to the supervisor seperately.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

Q. 1) A collar is established by buying a share of stock for Rs 1000, buying a 3-month put option with exercise price Rs. 950, and writing a 3-month call option with exercise price Rs. 1050. Based on the volatility of the stock, you calculate that for a strike price of Rs. 950 and maturity of 3 months $\Phi(d_1) = 0.55$, whereas for the exercise price of Rs. 1050 and maturity of 3 months $\Phi(d_1) = 0.30$, where $\Phi(d) = \int_{-\infty}^{d} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx$ and d_1 has the same meaning as under Black Scholes.

a) What will be gain or loss on the collar if the stock price decreases by Rs.1. (3)

b) What happens to the delta of the portfolio if the stock price becomes very large? Very small? (3)

[6]

Q. 2) Suppose the 3-month futures price on NSE Nifty is 5580, the Nifty index currently is 5500, the 3-month risk-free rate is 9% per annum with continuous compounding and the dividend that will be paid at the end of three months on a Rs. 5500 investment in the Nifty index portfolio is Rs. 40.

a. By how much is the contract mispriced from fair value? (1)

b. Formulate a zero-net-investment arbitrage portfolio and show that you can lock in risk-less profit equal to futures mispricing. (2)

c. Now assume (as is true for small investors) that if you short sell the stocks in the market index, the proceeds of short sale are kept with the broker, and you do not receive any interest income on the funds. Besides, you would have to pay a flat brokerage of 50 on the stocks under Nifty basket and 10 on Futures transaction. What is the no arbitrage band for the stock-futures price relationship? That is, given the stock index of 5500, how high and how low can the futures price be without giving rise to arbitrage opportunities? (4)

d. List the assumptions which will not hold in reality which have been implicitly assumed by you while answering part c above. (3)

[10]

Q. 3) Let F be the futures price of the asset with:

$$dF = \mu F dt + \sigma F dZ$$

Where Z is a Brownian motion process
Let f be the price of the derivative contingent on F

a. Write down the expression for the stochastic derivative df of f. (1)

b. Consider a portfolio consisting of:
   - -1 unit of a derivative

- $\frac{\partial f}{\partial F}$ units of futures contracts

Show that:

$$\frac{\partial f}{\partial t}+\frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf$$

(3)
[4]

Q. 4) a. Consider a two-year bond with 10% annual coupon due in one year's time and two year's time. Suppose that the continuously compounded gross redemption yield on the bond is 8% and that the continuously compounded risk-free yield is flat 6% per annum. Default can happen just before each coupon payment and has probability Q at both times. In the event of default 30% of the principal is recovered. Compute the value of Q. (5)

b. Explain the concept of Credit Default Swap. Explain the two ways a credit default swap can be settled. (3)
[8]

Q. 5) a. Explain how the Monte Carlo simulation method can be used to value an Asian option. Explain why the Monte Carlo simulation cannot easily be used for American style derivatives. (5)

b. Explain control variate technique. How can it be used to improve the price of a call option on a non-dividend paying stock with stochastic volatility which is computed using the Monte Carlo simulation? (5)
[10]

Q. 6) a. Suppose that the short rate is currently 9% and its standard deviation is 2% per annum. What happens to the standard deviation when the short rate increases to 16% in (a) Vasicek's model; (b) the Cox, ingersoll, and Ross model; (c) Hull-White (1990) model; and (d) Rendleman and Bartter's model? (4)

b. Your friend is a trader of caps and floors. While having a drink with you, he said "deltas and gammas on these derivatives make me mad. It was far easier when I was handling put and calls on stocks. " Discuss. (4)

c. The Hull and White model is specified by the equation:

$$dr(t) = \alpha[\mu(t)-r(t)] + \sigma dW(t)$$

Identify the items that need to be estimated in order to calibrate this model to the current term structure of interest rates and state how this can be done. (4)
[12]

Q. 7) The fund manager of HDFC Asset Management Company Limited has a portfolio worth Rs. 800 million with a beta of 0.80. The manager is concerned about the performance of the market over the next one month and plans to use 2-month futures contract on NSE Nifty to reduce the risk. The current level of the index is 5400, one contract is on 50 times the index.

a. What position should the fund manager take to reduce the beta of the portfolio to 0.5? (2)

b. After two months the fund manager finds that the beta of the portfolio over the two months was 0.8 and not 0.5. Explain the possible reasons for the same. (3)

[5]

Q. 8) a. It is March 31 2008. The cheapest to deliver bond in a May 2008 Treasury bond futures contract is a 12% coupon bond and the delivery is expected to be made on May 31, 2008. Coupon payments on the bond are made on April 3 and October 3 each year. The term structure is flat, and the rate of interest with continuous compounding is 10% per annum. The conversion factor for the bond is 1.4269. The current quoted bond price is Rs. 120 (face value of the bond is Rs. 100). Calculate the futures price for the contract. (5)

b. Suppose that the Treasury bond futures price is Rs. 104.50. Which of the following five bonds is cheapest to deliver?

Bond	**Price**	**Conversion Factor**
A	130.25	1.3241
B	147.50	1.4753
C	119.25	1.1326
D	149.50	1.4962
E	120.25	1.1563

(3)

[8]

Q. 9) List the financial instruments which can be used to hedge longevity risk. Explain the uses of any three of these.

[10]

Q. 10) Explain why portfolio insurance schemes can have a destabilizing effect on the market? **[6]**

Q. 11) Identify the types of European exotic option that have the following payoff functions:

a. $Max(K – S_{avg} , 0)$
b. $S_T - S_{min}$
c. $Max(K – S_T , 0)$ if $\min (S_t \text{ for } t<=T) >H$, 0 otherwise
d. S_T if $S_T > K$, 0 otherwise
e. $Abs(S_T – K)$
f. $Max(V_T – U_T , 0)$

[6]

Q. 12) The formula for the exchange option assuming that both the assets follow lognormal models and will pay no dividends during the life of the option is (option to get one share of company2 in exchange for company 1) :

$$c = S_2\Phi(d_1) - S_1\Phi(d_2) \text{ where}$$

$$d_1 = \frac{\log\left(S_2/S_1\right) + 1/2\,\sigma^2 T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T} \text{ and } \sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

Also, S_i is corresponding current price of the ith stock, σ_i is the volatility of the corresponding stock, T is the time to expiry and ρ is the instantaneous correlation between them.

a) Explain how the correlation ρ affects the value of this option stating when the option price would be highest. (3)

b) The price of the above exchange option with six months to expiry is observed to be 22. Compute the price of an option to get one share of company1 in exchange for company 2 when the following estimates are given:

$$\rho = 0.5,\quad \sigma_2 = 0.3,\ \sigma_1 = 0.21,\ S_1 = 420,\ S_2 = 400$$ (3)

c) By considering an appropriate put call parity relationship, comment on your answer obtained in b. (2)

[8]

Q. 13) Consider two money managers, Alice and Bob. Each starts out with a million dollars under management. Customers come along in a sequence, each with one million dollars. The first customer gives a million dollars to Alice or Bob at random. The second customer gives Alice or Bob another million, but the customer choose the money manager with probabilities that are proportional to the amount of money they already have under management. In general, let A_n and B_n denote the numbers of millions that Alice and Bob have under management after the nth customer. Now, when the n + 1'st customer arrives he gives his million to Alice with probability A_n / (A_n + B_n) and otherwise he gives it to Bob.

(a) Let M_n denote the fraction of money that is invested with Alice. Show that $\{M_n\}$ is a martingale. (2)

(b) Assume that there is zero investment returns for both Alice and Bob. Show that $M_n = A_n / (n + 2)$. (1)

(c) Assuming that condition in b holds, show that $P(A_n = k) = 1 / (n+1)$. (Hint: Use induction) (5)

[8]

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