Test Code : RE I / RE II (Short Answer type) 2007
(Junior Research Fellowship in Economics)

The candidates for Junior Research Fellowship in Economics are required to take two short answer type tests - RE I (Mathematics) in the forenoon session and RE II (Economics) in the afternoon session.

Syllabus for RE I

1. Permutations and combinations.
2. Theory of quadratic equations.
3. Elementary set theory; Functions and relations; Matrices.
4. Convergence of sequences and series.
5. Functions of one and several variables: limits, continuity, differentiation, applications, integration of elementary functions and definite integrals.
6. Constrained and unconstrained optimization, convexity of sets and concavity and convexity of functions.
7. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions and distributions of functions of random variable.

Syllabus for RE II

1. Theory of consumer behaviour; theory of production; market structure; general equilibrium and welfare economics; international trade and finance; public economics.
2. Macroeconomic theories of income determination, Rational Expectations, Phillips Curve, Neo-classical Growth Model, Inequality.
3. Game Theory: Normal and extensive forms, Nash and sub-game perfect equilibrium.
4. Statistical inference, regression analysis (including heteroscedasticity, autocorrelation and multicollinearity), least squares and maximum likelihood estimation, specification bias, endogeneity and exogeneity, instrumental variables and elementary time-series analysis.

Sample questions for RE I

1. Solve

$$\max_{x,y}(3xy - y^3)$$

subject to

$$2x + 5y \geq 20\,,$$

and

$$x - 2y = 5\,.$$

where $x, y \geq 0$. Interpret the solution graphically.

2a. Consider functions $f(x), g(x),$ and $h(x)$ defined on the real line such that $h(x) \leq f(x) \leq g(x)$. Suppose $\lim_{x\to\infty} h(x) = \lim_{x\to} g\ (x) = 0$. What is $\lim_{x\to\infty} f(x)$? Provide an argument for your answer.

2b. Suppose that sequences $< x^n >$ and $< y^n >$ do not converge. Provide examples such that

$< x^n + y^n >$ converges.

$< x^n + y^n >$ is unbounded.

$< x^n + y^n >$ does not converge, but is bounded.

3. Consider a square of side length 4 units. Place 17 points randomly inside this square. Show that no matter where you place these 17 points, you can always find at least two points whose distance is less than or equal to $\sqrt{2}$ units.

4. Consider the following constrained maximization problem which we denote as (P).

$$\text{Maximize } \Pi_{i=1}^{n} x_i$$

subject to

$$\sum_{i=1}^{n} x_i = n,$$

where, $x_i \geq 0, \quad i = 1,2\ldots\ldots n$.

- (a) If $(x_1^*,.....,x_n^*)$ solves (P), then show that $\Pi_{i=1}^{n} x_i^* = 1$.

- (b) Let $(a_1, a_2, \ldots\ldots, a_n)$ be n positive real numbers. Use the conclusion of the above part to prove that

$$\sum_{i=1}^{n} a_i \geq n(\Pi_{i=1}^{n} a_i)^{\frac{1}{n}}$$

5a. Two balls are drawn at random from an urn containing 5 white and 3 red balls. The first ball is not replaced before the second ball is drawn. Find the probability that the first ball is red, conditional on the second ball being red.

5b. In the game of Risk, the attacker wins a territory if the larger of the two numbers he gets when he rolls two dice is greater than or equal to the number obtained by the defender rolling one die. What is the probability that the attacker will win a territory?

6. An irregular six faced die is thrown and the probability that in 10 throws it will give five even numbers is twice the probability that it will give four even numbers. How many times, in 10,000 sets of 10 throws each, would you expect it to give no even number?

7. Let $X \sim N(\mu, \sigma^2)$. If $\sigma^2 = \mu^2$ $(\mu > 0)$, express

$$P(X < -\mu \mid X < \mu)$$

in terms of the cumulative distribution of $N(0,1)$.

8a. State the intermediate value theorem.

8b. Let $f:[a,b] \to [a,b]$ be a continuous function on $[a,b]$. Prove that there exists some $\beta \in [a,b]$ such that $f(\beta) = \beta$.

9. Suppose A is a convex set in $\Re^n$ and $f: A \to \Re$. Prove that f is a concave function *if and only if* the set $\{(x, \alpha)\ \varepsilon\ A \times \Re : f(x) \geq \alpha\}$ is a convex set in $\Re^{n+1}$.

10 Find the conditions on a, b and c (all real numbers) such that the roots of the quadratic equation $ax^2 + bx + c = 0$ are of unequal magnitude and of opposite sign.

Sample questions for RE II

1(a).A game in normal form consists of a set of players $N = \{1, \ldots.., n\}$, strategy sets S_i, and payoff functions $\pi_i :\ S_1$ x……x $S_n \rightarrow R$, for each player, $i \in N$.

- (i) Provide a definition of Nash equilibrium of this game.

- (ii) Give an example to show that Nash equilibria may not exist if the sets S_i are finite.

1(b). There are three voters 1,2,3 who have to vote for either candidate *A* or candidate *B*. The candidate who gets at least two votes wins the election. If *A* wins, all voters get a payoff of 1; if *B* is elected, all of them get 0.

- Which of the following strategy profiles constitute a Nash equilibrium?

(i) Voter 1 votes for B while 2 and 3 vote for A.
(ii) Voter 2 votes for A while the other two vote for B.
(iii) All three voters vote for A.
(iv) All three voters vote for B.

- Do any of the Nash equilibria involve players playing weakly dominated strategies?

(A strategy $s_i^* \in S_i$ is said to be *weakly dominated* in G if there exists $s_i^{'} \in S_i$ such that $\pi_i(s_i^{'}, s_{-i}) \geq \pi_i\ (s_i^*, s_{-i})$ for all $s_{-i} \in S_{-i}$, with strict inequality for some $s_{-i} \in S_{-i.}$)

2 Suppose voters are uniformly distributed over the interval [0,1]. A voter's position x on this interval represents her political position. For example, if $x = 0$ for some voter, then this voter has an extreme left ideology, and so on. There are two political parties whose strategies are to announce their political positions, i.e., select some $P_i \in [0,1]$. A voter will vote for a party nearest to her own position. In case parties are equidistant, a voter votes for either party with equal probability. The objective of each party is to maximize its vote share.

- (a) Where will the two parties locate?
- (b) Generalize part (a) to the case of a continuous distribution function, F, of voters over [0,1].

- (c) Show that if there are three political parties, there is no equilibrium in pure strategies.

3. Consider a monopoly manufacturer which sells its products to a monopolist retailer which in turn re-sells these products to final users. The market demand for the product is linear and is given by

$$p = a - bq$$

$a, b > 0$. The unit cost of producing the good by the manufacturer is $c, 0 < c < a$; retailer has no additional retailing cost.

- (a) Suppose the manufacturer provides a linear contract, $\{r\}$, under which the retailer can buy any amount of the good at price r per unit. Find the optimal values of r, p, and profits of the manufacturer and the retailer.

- (b)Suppose the manufacturer gives a two part tariff contract $\{F, r\}$ where F is the fixed fee and r is the price charged per unit. Again solve for F, r, p and profits of the manufacturer and the retailer.

- (c) If both the manufacturer and retailer integrate to form a single firm, what will happen to industry profits and consumer welfare compared to parts (a) and (b) above?

4(a) A consumer consumes two goods, x and y. Suppose that the government imposes a per unit tax, t, on good x and gives a per unit subsidy, s, on y. Assume that this happens in such a way that the government's expenditure on the subsidy is exactly balanced by the revenue from the tax. Show that with this tax-subsidy scheme, the consumer is not better off.

4(b). A consumer with money income y consumes two goods, X_1 and X_2, with price p_1 and p_2. Which of the following functions(s) legitimately represent his demand for X_1?

- $X_1 = p_2 - p_1 + y$
- $X_1 = \frac{p_2 y}{p_1}$

- $X_1 = \frac{(p_2 y)^{\frac{1}{2}}}{p_1}$
- All of the above.

Give reasons for your answers

5. Mr. A has just graduated from school and entered the job market. After the usual job search, he has the following job offers.

A job at RESTAURANT is always available – it pays Rs. 20 per hour and it has the flexibility that a worker can work for any duration as he wishes.

Company X offers him a salaries (not hourly) position as a Supervisor. If he takes the job, he has to work *exactly* 8 hours per day at this job, and will receive a *fixed salary* of Rs. 320 per day.

Company Y offers him the position of a technician. If he takes this job, he has to work *at least* 4 hours per day at his job and will receive a *fixed salary* of Rs. 160 for these 4 hours. In addition, there is the overtime rate – if he works for more than 4 hours per day, Company Y pays him Rs. W for every additional hour worked. He cannot work for less than 4 hours in Company Y, but he is free to choose his overtime work hours there. [Note that Company X has no overtime opportunity.]

[Important] No matter how long he works for Company X or Company Y, he can always work additional hours at RESTAURANT if he wishes to do so. But, if he works for Company X, he *cannot* work additional hours at Company Y. Similarly, if he works for company Y, he *cannot* work extra hours at Company X.]

Mr. A likes both consumption and leisure and both are *normal* goods to him. Assume that he spends all his income on consumption.

- (a) Suppose that Company *Y* offers the overtime rate of $W = 24$.

(i) Given the job options, draw and describe Mr. A's **daily** budget line. (Plot leisure (in hours) on the x-axis, and consumption (in Rs.) on the y axis. Label all relevant points clearly.)

(ii) Facing the job options, Mr. A makes optimal leisure-consumption decision where he has chosen to work for strictly more than 4 hours at RESTAURANT. How long is he working for *Y*? For Company *X*? Explain briefly.

- (b) Now suppose that you do not know the overtime rate (W) Company Y has offered. But you know that Mr. A has accepted Company X's Offer and has chosen to earn and consume Rs. 400 per day. Find out the value of *W* such that you know for sure that Company *Y* has not offered any *W* higher that that. Give a brief explanation for your answer.

6. Consider a closed economy in which household's labor supply, L^s, to firms is determined by the amount which maximizes their utility

$$U = C^{\alpha}(1 - L^s)^{\beta},$$

where $\alpha, \beta > 0, C$ denotes household real consumption expenditure which is taken to equal its wage income (the total labour time is normalized to unity).

- (a) Find the first order condition for utility maximization and also the household's labour supply (L^s) for a given real wage rate, ω. Does L^s depend on ω? Explain your answer.

- (b) Assume that this economy is a Keynesian economy in which (real) investment expenditure (*I*) is autonomous and output (*Y*) is determined by aggregate demand, i.e. $Y = C + I$. The aggregate production function is given by

$$Y = AL^{\theta}$$

where *A* is a positive parameter, and $\theta \in [0,1]$. Find the equilibrium values of *Y* for given value of *I*? What role does θ play here? Provide an intuitive explanation for this.

7a. Let the time series $\{Z_t\}$ be independently and identically distributed (i.i.d) as normal with mean 0 and variance 1. Define the time series $\{X_t\}$ as

$$X_t = Z_t \text{ if } t \text{ is even;}$$

$$X_t = \frac{Z_t^2 - 1}{\sqrt{2}} \text{ if } t \text{ is odd.}$$

- (i) What is $E(X_t)$?
- (ii) What is $V(X_t)$?
- (iii) What is $Cov(X_{t,} X_{t-k})$, $\forall$ *k* not equal to 0?

7b. Suppose that for a moving average process of order 1, given by

$$x_t = \mu + a_t + .6a_{t-1},$$

where a_t is a white noise process with mean 0 and variance 2, the sample mean – in a sample size of 100 – obtained was $\bar{x}_{100} = 0.27$. Construct a 95% confidence interval for μ.

8. Suppose the production function is

$$Y(t) = K(t)^{\alpha} H(t)^{1-\alpha}$$

with $0 < \alpha < 1$, and that *K* and *H* evolve according to $\dot{K}(t) = s_k Y(t)$, and $\dot{H}(t) = s_H Y(t)$.

- (a) Show that regardless of the initial levels of K and H (as long as both are positive), the ratio $\frac{K}{H}$ converges to some balanced growth path level, $(\frac{K}{H})^*$.
- (b) $\frac{K}{H}$ has converged to $(\frac{K}{H})^*$, what are the growth rates of *K*, *H* and *Y*?
- (c) How, if at all, does the growth rate of *Y* on the balanced growth path depend on s_k, s_H? Explain your answer in terms of level and growth effects.

9. A researcher estimates the following equation by OLS,

$$C = \alpha + \beta Y + u$$

where *C* denotes aggregate consumption, *Y* denotes GNP, and *u* is the exogenous shock to the economy.

- (a) Under what conditions is the OLS estimator of β consistent? Are these conditions likely to be valid?
- (b) If the conditions are such that the OLS estimator of β is inconsistent, compare and contrast two methods that could rectify the problem.

- (c) How will your estimators differ across these methods? Explain.

10a. From a data set on consumption patterns of rural households in Maharashtra in 1993, a researcher obtained the following results from an ordinary least squares (OLS) linear regression,

$$\hat{y}_i = 1.2 + .51x_i$$

where y_i is the log of expenditures on cereals by household i, and x_i is the log of income of household i. The standard errors associated with the intercept and slope coefficients were 0.1 and 0.14 respectively. The data set consisted of 9042 observations and the R^2 from the regression was 0.13.

- Test the hypothesis that the elasticity of cereal expenditures with respect to income is one against the alternative hypothesis that it is less than one. (Note: 1%, 5%, and 10% critical values of t distribution with large (> 120) degrees of freedom are 2.576, 1.96, and 1.645, respectively).

10b. A researcher analyzed the determinants of hospital costs by case mix. The data are for 177 hospitals. The dependent variable is hospital costs. The explanatory variables are *proportions* of total patients treated in each category. There are nine categories: M = Medical, P = pediatrics, S = general surgery, E = ENT, T = traumatic and orthopedic surgery, OS = other surgery, G = gynecology, Ob = obstetrics, Other = miscellaneous others. The results are

VARIABLE	REGRESSION COEFFICIENT	STANDARD ERROR
M	44.97	18.89
P	-44.54	28.51
S	-36.81	14.88
E	-54.26	16.52
T	-29.82	17.18
OS	28.51	20.27
G	-10.79	21.47
OB	-34.63	16.34
Constant	69.51	24.6

- (i) Why did the researcher not include the 'other' category in the regression?
- (ii) How would you interpret the coefficient on pediatrics?