INSTITUTE OF MATHEMATICS & APPLICATIONS, BHUBANESWAR

ENTRANCE TEST-2011

B.Sc.(Honours): Mathematics & Computing

Maximum Marks :100 **Time Alloted: 2 Hr.**

(Multiple choice questions)

All questions are compulsory. Each question has 4 choices $(A), (B), (C), (D)$, out of which *ONLY ONE* is correct. Choose the correct answer. Each question carries +4 marks for correct answer and −1 mark for wrong answer.

1. A question "Who have studied Mathematics ?" was asked to three students X,Y and Z. The question was answered correctly as: it is true that if X studied Mathematics, then Y also studied Mathematics, but it is false statement that if Z studied Mathematics, then Y also studied Mathematics. Then Mathematics was studied by

   (A) Both X and Y (B) only X

   (C) only Y (D) only C

2. The set of values of x for which the inequality:$|x-1|+|x+1|<4$ is true is

   (A) $(-2,2)$ (B) $(-\infty,2)\cup(2,\infty)$

   (C) $(-\infty,-1]\cup[1,\infty)$ (D) none of these.

3. The equation $|z+i|-|z-i|=k$ represents a hyperabola, if

   (A) $-2<k<2$ (B) $k>2$

   (C) $0<k<2$ (D) none of these.

4. If p,q and r are positive integers , ω is a cube root of unity, and $f(x)=x^{3p}+x^{3q+1}+x^{3r+2}$, then the value of $f(\omega)$ is

   (A) ω (B) $-\omega^2$

   (C) 0 (D) 1

5. If $(a+ib)^{1/3}=x+iy$, then $\dfrac{a}{x}+\dfrac{b}{y}$ is equal to

   (A) $2(x^2-y^2)$ (B) $4(x^2-y^2)$

   (C) $8(x^2-y^2)$ (D) none of these.

6. Let $f(n)=\left[\dfrac{1}{2}+\dfrac{n}{100}\right]$, where $[x]$ denotes the integral part of x. Then the value of $\sum_{n=1}^{100} f(n)$ is

   (A) 1 (B) 50

   (C) 51 (D) 101.

7. If $\begin{vmatrix} 1+y & 1-y & 1-y \\ 1-y & 1+y & 1-y \\ 1-y & 1-y & 1+y \end{vmatrix} = 0$, then the values of y are

(A) $0, 3$ (B) $2, -1$

(C) $-1, 3$ (D) $0, 2$

8. If $\cos x - \sin x \geq 1$ and $0 \leq x \leq 2\pi$, then the solutions set for x is

(A) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$ (B) $\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

(C) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$ (D) none of these.

9. The area enclosed by the curve $|x| + |y| = \sqrt{3}$ in the first quadrant in (sq. units) is

(A) $3/2$ (B) 6

(C) 9 (D) none of these

10. The value of the integral $\int_{1/2}^{2} \frac{1}{x} \operatorname{cosec}^{101}\left(x - \frac{1}{x}\right) dx$ is

(A) 0 (B) $3/2$

(C) $100/101$ (D) $101/102$.

11. If the Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ on the interval $[-1, 1]$ at the point $c = 1/2$, then the value of $10a + b$ is

(A) 0 (B) $3/2$

(C) $4/3$ (D) 3

12. A boat is to be manned by eight men of whom 2 of them can only row on bow side and 3 can only row on stroke side. The number of ways in which the crew can be arranged is

(A) 4360 (B) 5760

(C) 5960 (D) 6970

13. The maximum and minimum value of a 3×3 determinant whose elements belongs to $\{0, 1, 2, 3\}$ is

(A) $0, 0$ (B) $2, -2$

(C) $9, -9$ (D) $54, -54$

14. The probability that a teacher will give an unannounced test during any class meeting is $1/5$. If a student is absent twice, then the probability that the student will miss at least one test is

(A) $4/5$ (B) $2/5$

(C) $7/75$ (D) $9/25$

15. $\lim_{x\to -1^+} \dfrac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is equal to

(A) $1/\sqrt{2}$ (B) $1/\sqrt{2\pi}$

(C) $1/\sqrt{\pi}$ (D) none of these.

16. The value of $f(0)$ for which the function $f(x) = \dfrac{2-\sqrt{x+4}}{\sin 2x}$ is continuous is

(A) $-1/8$ (B) $1/8$

(C) $1/3$ (D) $1/6$

17. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$ is

(A) continuous at $x = 1$, not differentiable at $x = 1$

(B) differentiable only at $x = 3$

(C) continuous at $x = 1, x = 3$, and differentiable at $x = 1$

(D) continuous at $x = 1$, but not continuous at $x = 3$.

18. For a whole number n, if $f(x) = x^{n-1}\sin(1/x), x \neq 0$, and $f(0) = 0$, then in order that f is differentiable for all x, the smallest value of n can be

(A) 0 (B) 1

(C) 2 (D) 3

19. A point is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$. As it passes through the point$(1/2, \sqrt{3}/2)$, its $y-$coordinate is decreasing at the rate of 3 units per second. The rate at which the $x-$coordinate changes is(in units per second)

(A) 2 (B) $3\sqrt{3}$

(C) $2\sqrt{3}$ (D) $\sqrt{3}$

20. A point moves such that the sum of the squares of its distances from the sides of a square of side unity is 9. the locus of such a point is a circle

(A) inscribed in the square (B) circumscribing the square

(C) inside the square (D) containing the square

21. AB is a chord of the parabola $y^2 = 8x$ with end A at the vertex of the given parabola. BC is drawn perpendicular to AB meeting the axis of the parabola at C. The projection of BC on this axis is

(A) 2 (B) 4

(C) 8 (D) 16

22. If the coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$, and the coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal, then

(A) $ab=1$ (B) $ab=11$

(C) $ab=7$ (D) $ab=5$

23. Let $f:\mathbb{R}\to[2,\infty)$ be defined by $f(x)=e^{e^x}+e^{-e^x}$. Consider the following statements:

(i) f is an onto function (ii) the range of f is $[2,\infty)$.

(A) both (i) and (ii) are wrong

(B) both (i) and (ii) are correct, and (ii) is the correct reason for (i)

(C) (ii) is correct, but (i) is wrong

(D) (i) is correct, but (ii) is wrong

24. If $f(x)=\max\{\tan x,\cot x\}$, then

(A) f is continuous at $x=0,\pi/4$, and $5\pi/4$ (B) f is continuous at $x=\pi/2$ and $3\pi/2$

(C) $\int_0^{\pi/2} f(x)\,dx = 2\ln\sqrt{2}$ (D) f is periodic with period π.

25. If $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has

(A) at least one root in $(0,1)$ (B) one root in $(1,2)$, other root in $(-1,0)$

(C) both imaginary roots. (D) none of these