

Entrance Test -2007

B.Sc.(Mathematics and Computing)

Institute of Mathematics and Applications

Bhubaneswar

Full Marks :150

Time-11.30 A.M. - 1.30 P.M.

The questions are of equal value

Answer as many questions as you can

1. Examine whether the proposition $p \implies (p \vee q)$ is a tautology .
2. If $3 + 4i$ is a root of the quadratic equation $x^2 + bx + c = 0$, then find the values of b and c .
3. if $A = \{x \in \mathbb{R} : x^2 - 3x + 2 > 0\}$, then find $A \cap B$.
4. In the complex plane, find the set of points defined by the equation $|z - i| = |z + i|$.
5. Prove that $101^{50} > 99^{50} + 100^{50}$.
6. Find the value(s) of λ for which the following system of equations has a unique solution.

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

7. If $A = \begin{pmatrix} 3x & 0 \\ x & x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$, then find the value of x .
8. Evaluate the following :
- (i) $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x}\right)^x$ (ii) $\lim_{x \rightarrow \infty} \frac{\tan x - \sin x}{x^3}$
9. The function $f(x) = \frac{\sqrt{1+x}-1}{x}$ is not defined at $x = 0$. What should be its value at $x = 0$, so that f becomes continuous?
10. Find $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$.
11. Sketch the region bounded by the curve $|x| + |y| = 1$ and find its area.
12. Find the number of common tangents to the circle $x^2 + y^2 - x = 0$ and $x^2 + y^2 + x = 0$.
13. Find the image of the point $(4, -13)$ with respect to the line $5x + y + 6 = 0$.
14. In how many ways can a committee of 5 members be selected from 6 men and 5 ladies consisting of 3 men and 2 ladies? Justify your answer.
15. A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same color.