

1. Question No. 1 is **compulsory**.
 2. Attempt any **four** questions out of remaining **six** questions.
 3. Assume any suitable data, wherever **required** but justify the **same**.
 4. **Figures** to the **right** indicate **full marks**.

a) Define Laplace transforms, and If $L\{f(t)\} = f(s)$ and $g(t)$ is a function defined as 5

$$g(t) = \begin{cases} 0, & 0 < t < a \\ f(t-a), & t > a \end{cases} \text{ then prove that } L\{g(t)\} = e^{-as} f(s)$$

b) Determine the value of b such that the rank of A is 3 where $A =$ 5

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$$

c) If $w = f(z)$ is analytic then show that $|f'(z)|^2 =$ 5

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

(d) Express $f(x) = \frac{1}{2}(\pi - x)$ in a Fourier series with period 2π to be valid in the interval $(0, 2\pi)$. 5

(a) Find (1) $L\left\{\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3\right\}$ (2) $L\{e^{2t} \sin^4 t\}$ 5

(b) Find the Fourier series of $f(x) = x^2, 0 < x < 4$ and hence deduce that — 5

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

(c) Determine P such that the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{Px}{y}$ is analytic. 5

(d) Using row transformations find the inverse of the matrix $A =$ 5

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

a) Using Laplace transforms show that $\int_0^{\infty} e^{-\sqrt{2}t} \frac{\sinh t \sin t}{t} dt = \frac{\pi}{8}$. 5

b) Reduce the matrix $A =$ 5

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

to the normal form and hence find its rank.

c) Find the Fourier series expansion of the function— 5

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$$

d) If $f(z) = u + iv$ be analytic function of $z = x + iy$, and $u - v = (x - y)(x^2 + 4xy + y^2)$ 5
 then find $f(z)$. [TURN OVER

4. (a) Find non-singular matrix P and Q so that PAQ is a normal form where

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(b) Find (1) $L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}$ (2) $L^{-1} \left\{ \frac{s + 29}{(s + 4)(s^2 + 9)} \right\}$.

- (c) If $u = \lambda (1 + \cos \theta)$ then find V so that $u + iv$ is analytical.

- (d) Obtain half range sine series for $f(x)$, where $f(x) = mx, 0 \leq x \leq \pi/2$

$$= m(\pi - x), \frac{\pi}{2} \leq x \leq \pi.$$

5. (a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into s.t. line $4u + 3 = 0$.

(b) Use convolution theorem to find $L^{-1} \left\{ \frac{(s+2)^2}{(s^2+4s+8)^2} \right\}$

(c) If the matrix $A = \begin{bmatrix} 4 & 3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ then show that $\text{adj } A$ is symmetric.

- (d) Show that the set of functions $e^{\frac{x}{2}}, e^{-\frac{x}{2}}(1-x), e^{-\frac{x}{2}}(2-4x+x^2)$ are orthogonal over $(0, \infty)$.

6. (a) Find the Bilinear transformation which maps the points. $1, i, -1$ of z -plane on $i, 0, -i$ of w -plane and find the fixed pt's of this transformation.

- (b) If A is non-singular matrix of order n, prove that —
 (1) $A (\text{adj } A) = (\text{adj } A) A = |A| I_n$ (2) $| \text{adj } A | = |A|^{n-1}$

- (c) Obtain complex form of Fourier series for $f(x) = \cos h 3x + \sin h 3x$ in $(-3, 3)$

- (d) Solve $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^{-t} \sin t$ with $y(0) = 0$ and $y'(0) = 1$ by Laplace transform method.

7. (a) If $f(t)$ is a periodic function of period T, show that $L \{ f(t) \} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$

(b) If u is a regular function, then prove that — $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f'(z)|^2 = 4 |f''(z)|^2$

- (c) Test the consistency of following system of equation and solve them if possible

$$\begin{aligned} 6x + y + z &= -4 \\ 2x - 3y - z &= 0 \\ -x - 7y - 2z &= 7. \end{aligned}$$

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 where $f(x) = mx, 0 \leq x \leq \pi/2$

$$= m(\pi - x), \frac{\pi}{2} \leq x \leq \pi.$$

5. (a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into s.t. line $4u + 3 = 0$.

(b) Use convolution theorem to find $L^{-1} \left\{ \frac{(s+2)^2}{(s^2 + 4s + 8)^2} \right\}$

(c) If the matrix $A = \begin{bmatrix} 4 & 3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ then show that $\text{adj } A$ is symmetric.

- (d) Show that the set of functions $e^{\frac{x}{2}}, e^{-\frac{x}{2}}(1-x), e^{-\frac{x}{2}}(2-4x+x^2)$ are orthogonal over $(0, \infty)$.

6. (a) Find the Bilinear transformation which maps the points. $1, i, -1$ of z -plane on $i, 0, -i$ of w -plane and find the fixed pt's of this transformation.

(b) If A is non-singular matrix of order n, prove that —
 (1) $A (\text{adj } A) = (\text{adj } A) A = |A| I_n$ (2) $| \text{adj } A | = |A|^{n-1}$

- (c) Obtain complex form of Fourier series for $f(x) = \cos h 3x + \sin h 3x$ in $(-3, 3)$

- (d) Solve $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 5y = e^{-t} \sin t$ with $y(0) = 0$ and $y'(0) = 1$ by Laplace transform method.

7. (a) If $f(t)$ is a periodic function of period T, show that $L \{ f(t) \} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$

(b) If u is a regular function, then prove that — $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f'(z)|^2 = 4 |f''(z)|^2$

- (c) Test the consistency of following system of equation and solve them if possible

$$\begin{aligned} 6x + y + z &= -4 \\ 2x - 3y - z &= 0 \\ -x - 7y - 2z &= 7. \end{aligned}$$

- (d) Expand $f(x) = a \left(1 - \frac{x}{l} \right)$ in the range $(0, l)$ in a half range cosine series.