S.E (Biomedical Engg.) Sem-III (Rev.) Dec, 2010.

1487 p3 ksl apq Sec (tall-K) 9-10

Sub:- Applied Mathematics-III

Con. 5826–10. **GT–6375**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Figure to the **right** indicate **full** marks.

1. (a) Show that $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t}\, dt = \log\left(\frac{3}{4}\right)$ **5**

(b) Find complex form Fourier series for $f(x) = \sinh 3x + \cosh 3x$ in $[-\pi, \pi]$ **5**

(c) Find the image of $x + y = 1$ under $w = \frac{1}{z}$ interpret with sketches. **5**

(d) Obtain Laurent's series for $f(z) = \frac{1}{z^2 - 4z + 3}$ when $1 < |z| < 3$. **5**

2. (a) Find (i) $L\left[t\sqrt{1 + \sin 2t}\right]$ **4**

(ii) $L\left[e^{-3t}\left(\frac{1 - \cos 3t}{t}\right)\right]$ **4**

(b) Find the bilinear transformation which maps the points $z = 1, -i, -1$, into the points $w = i, 0, -i$. **6**

(c) Obtain the Fourier expansion of **6**

$$f(x) = \begin{cases} \cos x & -\pi < x < 0 \\ -\cos x & 0 < x < \pi \end{cases} \text{ and } f(x) = f(x + 2\pi)$$

3. (a) (i) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$ **4**

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)}\, dx$ **4**

(b) Find the Fourier expansion of $f(x) = 4 - x^2$ in the interval $(0, 2)$. **6**

(c) Solve using Laplace transform $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$; $y(o) = 1$ and $y^1(o) = 1$. **6**

4. (a) Obtain the half range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$ and hence show that $\sum \frac{1}{(2n - 1)^6} = \frac{\pi^6}{960}$ **7**

(b) Find an analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ **6**

[TURN OVER

(c) Find Laplace transform of 6

$$f(t) = t, \quad 0 < t < \pi$$
$$= \pi - t; \quad \pi < t < 2\pi$$
$$\text{and } f(t) = f(t + 2\pi)$$

5. (a) Find Inverse Laplace Transform of

(i) $\log\left[\dfrac{s^2 + a^2}{(s+b)^2}\right]$ 4

(ii) $\dfrac{(s+3)^2}{(s^2+6s+13)^2}$ 4

(b) State and prove Cauchy's Residue theorem. 6

(c) S.T the set of functions $\left\{\dfrac{\text{Sin } x}{\sqrt{\pi}}, \dfrac{\text{Sin } 2x}{\sqrt{\pi}}, \dfrac{\text{Sin } 3x}{\sqrt{\pi}}, \ldots\ldots\right\}$ from a orthonormal set in the interval $[-\pi, \pi]$ 6

6. (a) Evaluate $\displaystyle\int_0^{1+i} \left(x^2 + iy\right) dz$ along the curve $y = x^2$. 6

(b) Evaluate $\displaystyle\int_0^{\infty} e^{-2t}\left(1 - t + t^2\right) H(t-3)\, dt$ 6

(c) Express $f(x) = \dfrac{\pi}{2} e^{-x} \cos x$ for $x > 0$ as Fourier sine integral and show that 6

$$\int_0^{\infty} \frac{w^3 \text{ Sin} wx}{w^4 + 4} dx = \frac{\pi}{2} e^{-x} \cos x.$$

7. (a) If $f(k) = \displaystyle\int_C \frac{4z^2 + z + 4}{z - k} dz$ where C is the ellipse $4x^2 + 9y^2 = 36$. Find the values of (i) $f(1)$ (ii) $f(i)$, (iii) $f'(-1)$ (iv) $f''(i)$ 6

(b) Find the Fourier Expansion of $f(x) = x + x^2$ in $[-1, 1]$. 7

(c) If $f(z)$ is an analytic function prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^n = n^2 |f(z)|^{n-2} \left|f'(z)\right|^2$. 7
