BTS 115(A)

B.Tech. Degree III Semester (Supplementary) Examination in Information Technology/Computer Science and Engineering/Electronics and Communication Engineering/Civil Engineering/Mechanical Engineering, June2001

IT/CS/EC/CE/ME 301 MATHEMATICS - III
(1998 admissions)

Time: 3 Hours   *Max. Marks: 100*

(All questions carry equal marks)

I a) If $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$. Show that

$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$. Deduce that

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$

b) Find a fourier series expansion for
$f(t) = t \qquad 0 < t < 1$
$\quad\ = 1 - t \quad 1 < t < 2$

OR

II a) Obtain half range cosine series for $f(x) = x^2$ in the interval $0 \le x \le 2$.

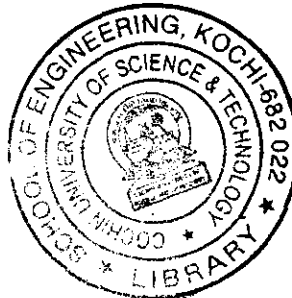b) Prove that $\beta(m+1, n) + \beta(m, n+1) = \beta(m, n)$

c) Prove that $\int_{0}^{\infty} x^3 e^{-x^3} dx = \frac{1}{9} \Gamma\left(\frac{1}{3}\right)$

III a) Verify that $z = f\left(x^2 + y^2\right)$ is a solution of the

PDE $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$

b) solve $(y+z)p + (z+x)q = x+y$

(P.T.O)

c)   Solve $z^2(p^2 + q^2 + 1) = a^2$

OR

IV   a)   Solve the heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$ by the method of separation of variables.

b)   Solve $yp = 2yx + \log q$

V   a)   Determine the binomial distribution with

$$\beta_1 = \frac{1}{120} \text{ and } \beta_2 = \frac{349}{120}$$

b)   If X is a normal variable with mean 20 and SD 5

find $p\{16 \le X \le 22\}$ and $p\{|X - 20| > 5\}$

c)   A poisson variate X is such that $p\{X = 1\} = 2p\{X = 2\}$

find $p\{X = 0\}$.

OR

VI   a)   An unbaised coin is tossed 100 times. Show that probability that the number of heads will be between 30 and 70 is greater than 0.93.

b)   Find the mean and variance of Geometric distribution.

VII   a) S^2 is the variance of a sample of size 10 taken from a population with SD 5. Find the probability that S^2 will lie between 8.4 and 42.3.

b)   The mean and SD of a sample 60 and found to be 145 and 40. Construct 95% confidence interval for population mean.

c)   A sample is taken from a population with unknown SD. The mean and SD of sample is found to be 195 and 50. If the statistician rejected the hypothesis at 5% level, find the least size of sample.

OR

VIII   a)   Find the maximum likelihood estimate of λ based on a sample taken from the population with p.d.f.

$$f(x) = \frac{\lambda^x \bar{e}^\lambda}{x!}, x = 0,1,2,3,........$$

b)   The heights of students studying in college classes it is believed to be distributed with SD 1.5. A sample of 400 students have their mean height 4.75 ft. Does this contradict the hypothesis that the mean height of students is 4.48 ft. (significance level 0.01).

c)   Random samples of sizes 500 and 400 are found to have means 11.5 and 10.9 respectively. Can the samples be regarded as random samples drawn from the same population whose SD is 5 (significance level 0.05).

IX   a)   The following figures give the prices in rupee of a certain commodity in a sample of shops selected at random from a city A. Assuming the prices are normally distributed, examine whether the S.D of prices is 0.30.

7.41, 7.77, 7.44, 7.40, 7.38, 7.93, 7.58, 8.28, 7.23, 7.52, 7.82, 7.71, 7.84, 7.63, 7.68

(significance level 0.05)

b)   Find the coefficient of correlation between x and y.

x:	1	2	4	7	8	9
y :	9	8	12	14	16	15

OR

X   a)   From a class of 20 children, 10 were selected at random and given orange juice each day for a certain period. The remaining 10 were given a ration of milk. The gains in weight in pound were

Group A: 4   2.5   3.5   4   1.5   1   3.5   3   2.5   3.5

Group B: 5   3.5   2.5   3   2.5   2   2   2.5   1.5   3

Test whether $\sigma_1^2 = \sigma_2^2$ (significance level 0.05)

b)   Two random variables have regression lines $3x + 2y = 26$ and $6x + y = 31$. Identify the nature of lines, means of x and y. Find coefficient of correlation and variance of y if variance of x is 8.

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