Mathematics

Class X

Board Paper - 2011

Time: **2½** hour Total Marks: 80

1. Answer to this paper must be written on the paper provided separately.
2. You will NOT be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt all questions from Section A and any four questions from Section B.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

Section - A (40 Marks)

Q.1.

(a) Find the value of 'k' if (x-2) is a factor of $x^3 + 2x^2 - kx + 10$. Hence determine whether (x+5) is also a factor. [3]

(b) If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible? Give a reason. If yes, find AB. [3]

(c) Mr. Kumar borrowed ₹ 15000 for two years. The rates of interest for two successive years are 8% and 10% respectively. If he repays ₹ 6200 at the end of first year, find the outstanding amount at the end of second year. [4]

Q.2.

(a) From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.

(i) a face card (King, Jack or Queen)

(ii) an even numbered red card [3]

(b) Solve the following equation:

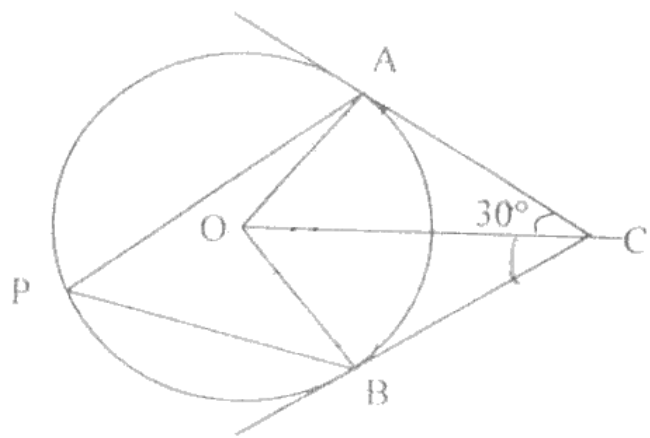
$x - \frac{18}{x} = 6$. Give your answer correct to two significant figures. [3]

(c) In the given figure O is the centre of the circle. Tangents A and B meet at C. If $\angle ACO = 30^{\circ}$, find

(i) $\angle BCO$

(ii) $\angle AOB$

(iii) $\angle APB$ [4]



Q.3.

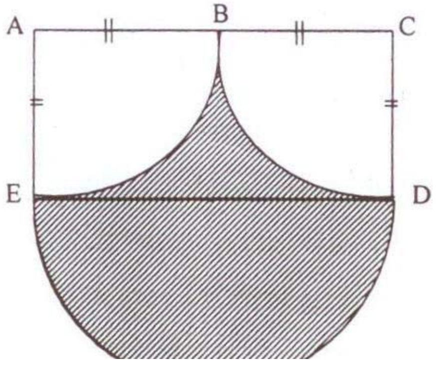
(a) Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find

(i) The interest paid by the bank

(ii) The rate of interest [3]

(b) Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm.

Take $\pi = \frac{22}{7}$. [3]



(c) ABC is a triangle and G(4,3) is the centroid of the triangle.If A=(1,3),B =(4,b) and C =(a,1),find 'a' and 'b'. Find length of side BC.
[4]

Q.4.

(a) Solve the following inequation and represent the solution set on the number line $2x - 5 \leq 5x + 4 < 11$, where $x \in I$ [3]

(b)Evaluate without using trigonometric tables.

$$2\left(\frac{\tan 35^o}{\cot 55^o}\right)^2 + \left(\frac{\cot 55^o}{\tan 35^o}\right) - 3\left(\frac{\sec 40^o}{\operatorname{cosec} 50^o}\right)$$ [3]

(c) A Mathematics aptitude test of 50 students was recorded as follows:

Matks	50 - 60	60 - 70	70 - 80	80 - 90	90 – 100
No. of Students	4	8	14	19	5

Draw a histogram from the above data using a graph paper and locate the mode.
[4]

Section - B (40 marks)

Q.5.

(a) A manufacturer sells a washing machine to a wholesaler for ₹15000. The wholesaler sells it to a trader at a profit of ₹1200 and the trader in turns sells it to a consumer at a profit of ₹1800. If the rate of VAT is 8% find:
(i) The amount of VAT received by the state government on the sale of this machine from the manufacturer and the wholesaler.
(ii) The amount that the consumer pays for the machine. [3]

(b) A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed.
[3]
(c) ABCD is a parallelogram where A(x,y), B(5,8), C(4,7) and D(2,-4). Find
(i) Coordinates of A
(ii) Equation of diagonal BD [4]

Q.6.

(a) Use a graph paper to answer the following questions (Take 1 cm = 1 unit on both axes)

(i) Plot A(4,4), B(4,-6) and C(8,0), the vertices of a triangle ABC.

(ii) Reflect ABC on the y-axis and name it A′B′C′.

(iii) Write the coordinates of the images A′, B′ and C′.

(iv) Give a geometrical name for the figure AA′ C′ B′ BC.

(v) Identify the line of symmetry of AA′ C′ BC′.
[4]

(b) Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below.

Date	Particulars	Withdrawals (in	Deposits (in	Balnce (in Rs.)

		Rs.)	Rs.)	
I^{st} April 2007	By Cash	-	8550.00	8550.00
12^{th}- April 2007	To Self	1200,00	-	7350.00
24^{th} April 2007	By Cash	-	4550.00	11900.00
8^{th} July 2007	By Cheque	-	1500.00	13400.00
10^{th} Sept. 2007	By Cheque	-	3500.00	16900.00
17^{th} Sept. 2007	To Cheque	2500.00	-	14400.00
11^{th} Oct. 2007	By Cash	-	800.00	15200.00
6^{th} Jan. 2008	To Self	2000.00	-	13200.00
9^{th} March 2008	By Cheque	-	950.00	14150.00

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1^{st} April. 2008. Give your answer correct to the nearest rupee.
[6]

Q.7.

(a) Using componendo and dividendo, find the value of x

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$ [3]

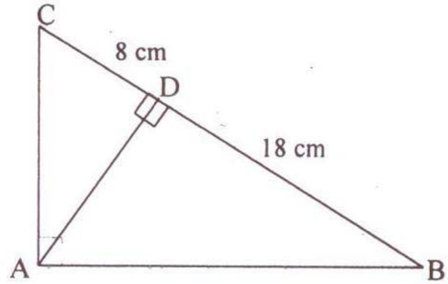
(b) If $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}4 & -2\\ -1 & 3\end{bmatrix}$ and I is the identity matrix of the same order and A^t is the transpose of matrix A, find $A^t.B + BI$.
[3]

(c) In the adjoining figure ABC is a right angled triangle with $\angle BAC = 90^o$.

(i) Prove $\Delta ADB \sim \Delta CDA$.

(ii) If BD = 18 cm CD = 8 cm Find AD.
[4]

(iii) Find the ratio of the area of ΔADB is to area of ΔCDA.



Q.8.

(a) (i) Using step – deviation method, calculate the mean marks of the following distribution.

(ii) State the modal class. [4]

Class interval	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80	80 - 85	85 – 90
Frequency	5	20	10	10	9	6	12	8

(b) Marks obtained by 200 students in an examination are given below:

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

(i) The median marks.

(ii) The number of students who failed if minimum marks required to pass is 40.

(iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination. [6]

Q.9.

(a) Mr. Parekh invested Rs. 52,000 on Rs. 100 shares at a discount of Rs. 20 paying 8% dividend. At the end of one year he sells the shares at a premium of Rs. 20. find

(i) The annual dividend.

(ii) The profit earned including his dividend.
[3]

(b) Draw a circle of radius 3.5 cm. Marks a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.
[3]

(c) Prove that (cosec A – Sin A)(sec A – cos A) $\sec^2 A$ = tan A.
[4]

Q.10.

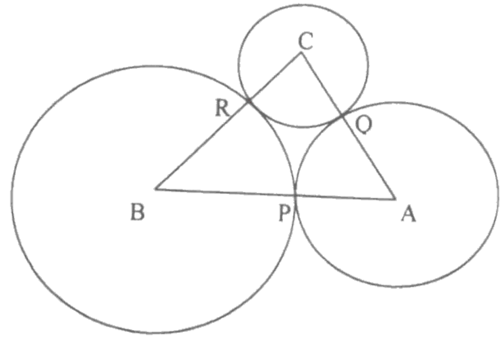
(a) 6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers.

(b) In what period of time will ₹12,000 yield ₹3972 as compound interest at 10% per annum, if compounded on an yearly basis?

(c) A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.

Q.11.

(a) ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.
[3]



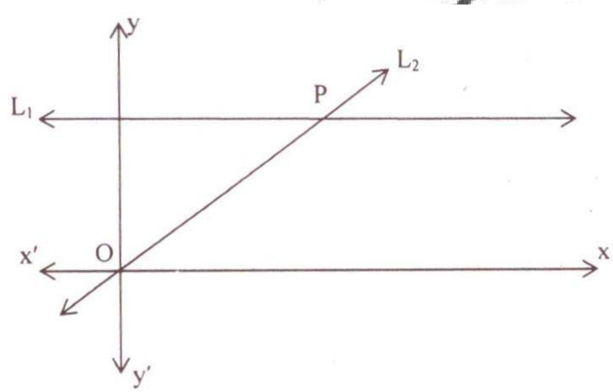
(b) Rs. 480 is divided equally among 'x' children. If the numbers of children were 20 more then each would have got Rs. 12 less. Find 'x'.
[3]

(c) Given equation of line L, is y = 4.

(i) Write the slope of line L_2, if L_2, is the bisector of angle O.

(ii) Write the co–ordinates of point P.

(iii) Find the equation of L_2.
[4]



Mathematics

Class X

Board Paper - 2011

Time: **2½** hour Total Marks: 80

1. Answer to this paper must be written on the paper provided separately.
2. You will NOT be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections.
   Attempt all questions from Section A and any four questions from Section B.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

Solution

Section - A (40 Marks)

Soln.1.

(a). Here, $p(x) = x^3 + 2x^2 - kx + 10$

For (x-2) to be the factor of $p(x) = x^3 + 2x^2 - kx + 10$

$p(2) = 0$

Thus, $(2)^3 + 2(2)^2 - k(2) + 10 = 0$

$\Rightarrow 8 + 8 - 2k + 10 = 0$

$\Rightarrow k = 13$

Thus p(x) becomes $x^3 + 2x^2 - 13x + 10$

Now, (x+5) would be the factor of p(x) iff $p(-5) = 0$

$p(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$

$= -125 + 50 + 65 + 10$

$= 0$

So, (x+5) is also a factor of $p(x) = x^3 + 2x^2 - 13x + 10$.

(b) Yes, product AB is possible since the number of columns of matrix A is equal to the number of rows of matrix B . (Matrix A is of the order 2 x 2 and B is of the order of 2 x 1)

The required product AB = $\begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 6+20 \\ 8-8 \end{bmatrix}$

$= \begin{bmatrix} 26 \\ 0 \end{bmatrix}$

(c). Here Principal, P = ₹15000

Rate of interest, R = 8% for first year and 10% for second year

Interest for 1st year = $\frac{PXRXT}{100} = \frac{15000x8x1}{100} =$ ₹1200

Amount at the end of first year = ₹15000 + 1200 = ₹16200

Kumar repays ₹ 6200

Principal for second year = ₹16200 - ₹ 6200 = ₹ 10000

Interest for second year = $\frac{PXRXT}{100} = \frac{10000x10x1}{100} =$ ₹ 1000

Amount at the end of second year = ₹ 10000 + 1000 = ₹ 11000

Soln.2.

(a) In a deck of cards, for each suit we have three cards with number 3, 6, 9 which are multiples of 3.

Thus for four different suits Spade, Heart, Diamond, Club , 3 x 4 = 12 such cards will be removed.

Total number of possible outcomes = 52 – 12 = 40

(i) Each suit has 3 face cards.
Four suits (Spade, Heart, Diamond, Club) will have 3 x 4 = 12 face cards.

So, required probability will be given by

P (getting a face card) = $\frac{12}{40} = \frac{3}{10}$

(ii) Each suit has 4 (cards with number 2, 4, 8, 10) even numbered cards.
Suits Heart and Diamond are of red colour.

Thus, two suits will have 2 x 4 = 8 even numbered cards.

So, required probability would be given by

P (getting an even numbered red card) = $\frac{8}{40} = \frac{1}{5}$

(b) $x - \frac{18}{x} = 6$

$$\frac{x^2 - 18}{x} = 6$$

$$\Rightarrow x^2 - 6x - 18 = 0$$

Here $a = 1, b = -6$ and $c = -18$

Thus the roots of the equation will be

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-18)}}{2(1)}$$

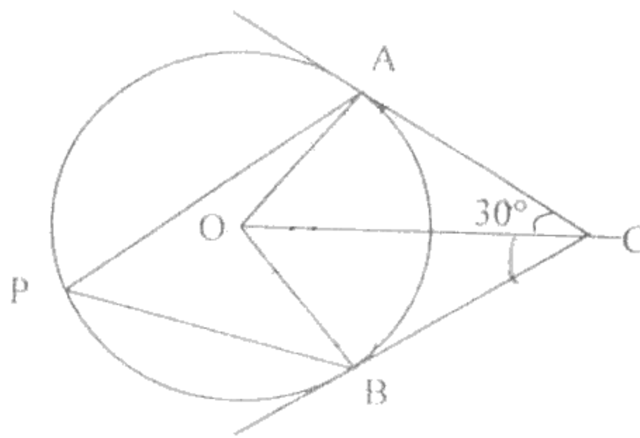
$$\Rightarrow x = \frac{6 \pm \sqrt{108}}{2}$$

$$\Rightarrow x = \frac{6 \pm 6\sqrt{3}}{2}$$

$$\Rightarrow x = 3 \pm 3\sqrt{3}$$

$$\Rightarrow x = 3 \pm 3 \times 1.73 \quad [\text{Using}, \sqrt{3} = 1.73]$$

$$\Rightarrow x = 8.19 \text{ and } -2.19$$



(c) In Δ AOC, $\angle ACO = 30^\circ$ (Given)

$\angle OAC = 90^\circ$ [radius is perpendicular to the tangent at the point of contact]

By angle sum property, $\angle ACO + \angle OAC + \angle AOC = 180^\circ$

$\angle AOC = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

Consider Δ AOC and Δ BOC

AO = BO (radii)

AC = BC (tangents to a circle from an external point are equal in length)

OC = OC (Common)

Δ AOC is congruent to Δ BOC.

(i) $\angle BCO = \angle ACO = 30^o$

(ii) $\angle AOC = \angle BOC = 60^o$

$\angle AOB = \angle AOC + \angle BOC = 120^o$

(iii) We know that, "If two angles stand on the same chord, then the *angle at the centre is twice* the *angle* at the *circumference.*

$\angle AOB$ and $\angle APB$ stand on the same chord AB.

$\angle AOB = 2 \angle APB$

So, $\angle APB = \frac{1}{2} \angle AOB = 60^o$

Soln.3.

(a) P = ₹ 2500, n = 2 years = (2 × 12) months= 24 months

Total Principal = ₹ 2,500 x 24 = ₹ 60,000

Amount = ₹ 66,250

Interest = Amount – Principal = ₹ 66,250 – ₹ 60,000 = ₹ Rs 6,250

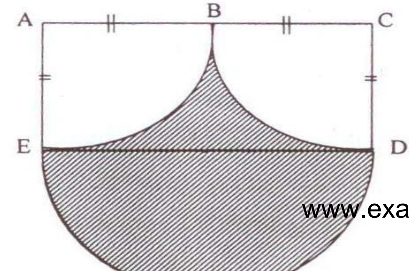
Thus, the interest paid by the bank is ₹ Rs 6,250.

Let r be the rate of interest.

$$N = \frac{n(n+1)}{2 \times 12} = \frac{24 \times 25}{2 \times 12} = 25 \text{ yrs}$$

This is equivalent to depositing ₹2,500 for 25 yrs.

$$\text{Interest} = \frac{P \times N \times R}{100}$$



$$6,250 = \frac{2,500 \times 25 \times R}{100}$$

$$R = 10$$

Thus, the rate of interest is 10%.

(b)

Diameter of the semi circle is 14 cm.

ED = AC = 14 cm

Therefore, AB = BC = AE = CD = 7 cm

Area of the shaded region = Area of semi circle EFD + Area of rectangle AEDC − Area of quadrant ABE − Area of quadrant BCD

Area of semi circle EFD = $\frac{\pi r^2}{2} = \frac{22}{7} \times \frac{1}{2} \times 7 \times 7 = 77\text{ cm}^2$

Area of rectangle AEDC = AC × AE = 14 cm × 7 cm = 98 cm^2

Area of quadrant ABE = Area of quadrant BCD = $\frac{90^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{8}\text{cm}^2$$

Area of the shaded region = 77 cm^2 + 98 cm^2 – 2 x $\frac{77}{8}$ cm^2 = 155.75 cm^2

(c) The coordinates of the vertices of ΔABC are A(1, 3), B(4, b) and C(a, 1).

It is known that $A(x_1, y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then the coordinates of centroid are G = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Thus, the coordinates of the centroid of ΔABC are

$$\left(\frac{1+4+a}{3}, \frac{3+b+1}{3}\right) = \left(\frac{5+a}{3}, \frac{4+b}{3}\right)$$

It is given that the coordinates of the centroid are G(4, 3).

Therefore, we have:

$$\frac{5+a}{3} = 4$$
$$5 + a = 12$$
$$a = 7$$
$$\frac{4+b}{3} = 3$$
$$4 + b = 9$$
$$b = 5$$

Thus, the coordinates of B and C are (4, 5) and (7, 1) respectively.

Using distance formula, we have:

$$\begin{aligned} BC &= \sqrt{(7-4)^2 + (1-5)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \text{ units} \end{aligned}$$

Soln.4.

(a) The given inequation is $2x - 5 \leq 5x + 4 < 11$, where $x \in I$

$2x - 5 \leq 5x + 4$

$2x - 5x \leq 4 + 5$

$-3x \leq 9$

$x \geq -3$

$5x + 4 < 11$

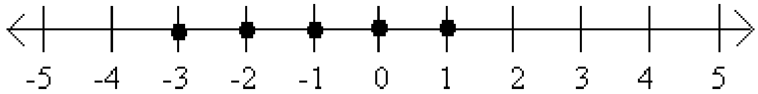
$5x < 11 - 4$

$5x < 7$

$x < 1.4$

Since $x \in I$, the solution set is {-3, -2, -1, 0, 1}.

The solution set can be represented on a number line as follows:



(b)

$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2+\left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2-3\left(\frac{\sec 40^\circ}{\cos ec50^\circ}\right)$$

$$=2\left(\frac{\tan(90^\circ-55^\circ)}{\cot 55^\circ}\right)^2+\left(\frac{\cot(90^\circ-35^\circ)}{\tan 35^\circ}\right)^2-3\left(\frac{\sec(90^\circ-50^\circ)}{\cos ec50^\circ}\right)$$

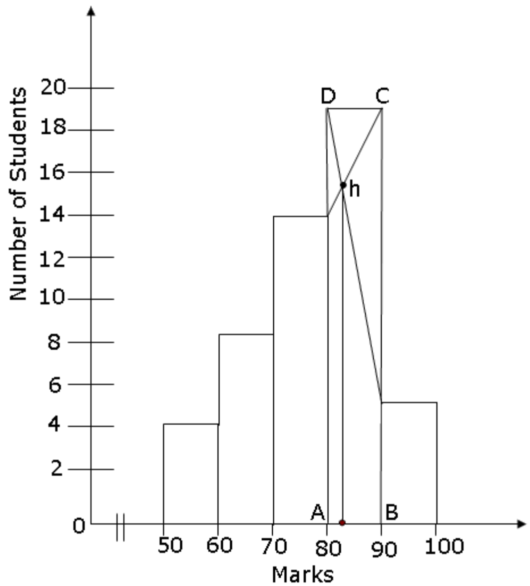
$$=2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2+\left(\frac{\tan 35^\circ}{\tan 35^\circ}\right)^2-3\left(\frac{\cos ec50^\circ}{\cos ec50^\circ}\right)\quad\left[\begin{array}{l}\because \tan(90^\circ-\theta)=\cot\theta\\ \cot(90^\circ-\theta)=\tan\theta\\ \sec(90^\circ-\theta)=\cos ec\theta\end{array}\right]$$

$$=2(1)^2+(1)^2-3(1)$$

$$=2+1-3=0$$

(c)

The histogram for the given data can be drawn by taking the marks on the x-axis and the number of students on the y-axis.



To locate the mode from the histogram, we proceed as follows:

i Find the modal class. Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. The modal class is 80 – 90.

ii Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these rectangles intersect at point H.

iii The x-value of the point H is the mode. Thus, mode of the given data is approximately 83.

Section – B (40 marks)

Soln.5.

(a) Given cost of washing machine = **₹15000**

(i) Amount of tax collected by manufacturer = 8% of ₹15000

$$= \frac{8}{100} \times 15000 = ₹1200$$

As profit of wholesaler is ₹1200, VAT to be payed by wholesaler

$$= 8\% \text{ of } ₹1200$$

$$= \frac{8}{100} \times 1200 = ₹96$$

As trader earns a profit of ₹1800 VAT to be payed by trader

$$= 8\% \text{ of } ₹1800$$

$$= \frac{8}{100} \times 1800 = ₹144$$

Amount of tax received by government = ₹(1200 + 96 +144) = ₹1440

(i) Value of machine paid by the consumer
= Price charged by manufacturer + Profit of wholesaler + Profit of trader

= ₹ (15000+1200+1800) = ₹18000

Tax paid by consumer = 8% of ₹18000

$$= \frac{8}{100} \times 18000 = ₹1440$$

Therefore, amount paid by customer = ₹(18000+1440) = ₹19440

(b) Volume of solid cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 8 = \frac{1}{3} \times \frac{22}{7} \times 25 \times 8$

Volume of a small sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{125}{1000}$

Number of spheres formed = $\frac{\text{Volume of cone}}{\text{Volume of sphere}} = \frac{\frac{1}{3} \times \frac{22}{7} \times 25 \times 8}{\frac{4}{3} \times \frac{22}{7} \times \frac{125}{1000}} = 400$

Thus 400 spheres are obtained by melting the solid cone.

(c) We know that the diagonals of a parallelogram bisect each other

So, coordinates of mid point of BD =
$\left(\frac{\text{x coordinate of B} + \text{x coordinate of D}}{2}, \frac{\text{y coordinate of B} + \text{y coordinate of D}}{2}\right)$

$= \left(\frac{5+2}{2}, \frac{8-4}{2}\right) = \left(\frac{7}{2}, 2\right)$ ..(1)

Now the midpoint of diagonal AC = $\left(\frac{x+4}{2}, \frac{y+7}{2}\right)$ ...(2)

From (1) and (2), we get

$$\left(\frac{x+4}{2}, \frac{y+7}{2}\right) = \left(\frac{7}{2}, 2\right)$$

Comparing we get, x+4 = 7 and y+7 = 4

Thus x = 3 and y = -3

So, the coordinates of point A are (3, -3)

(ii) Equation of a line is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Coordinates of point B and D are (5,8) and (2,-4) respectively.

Equation of a diagonal BD, $y - 8 = \frac{-4-8}{2-5}(x-5)$

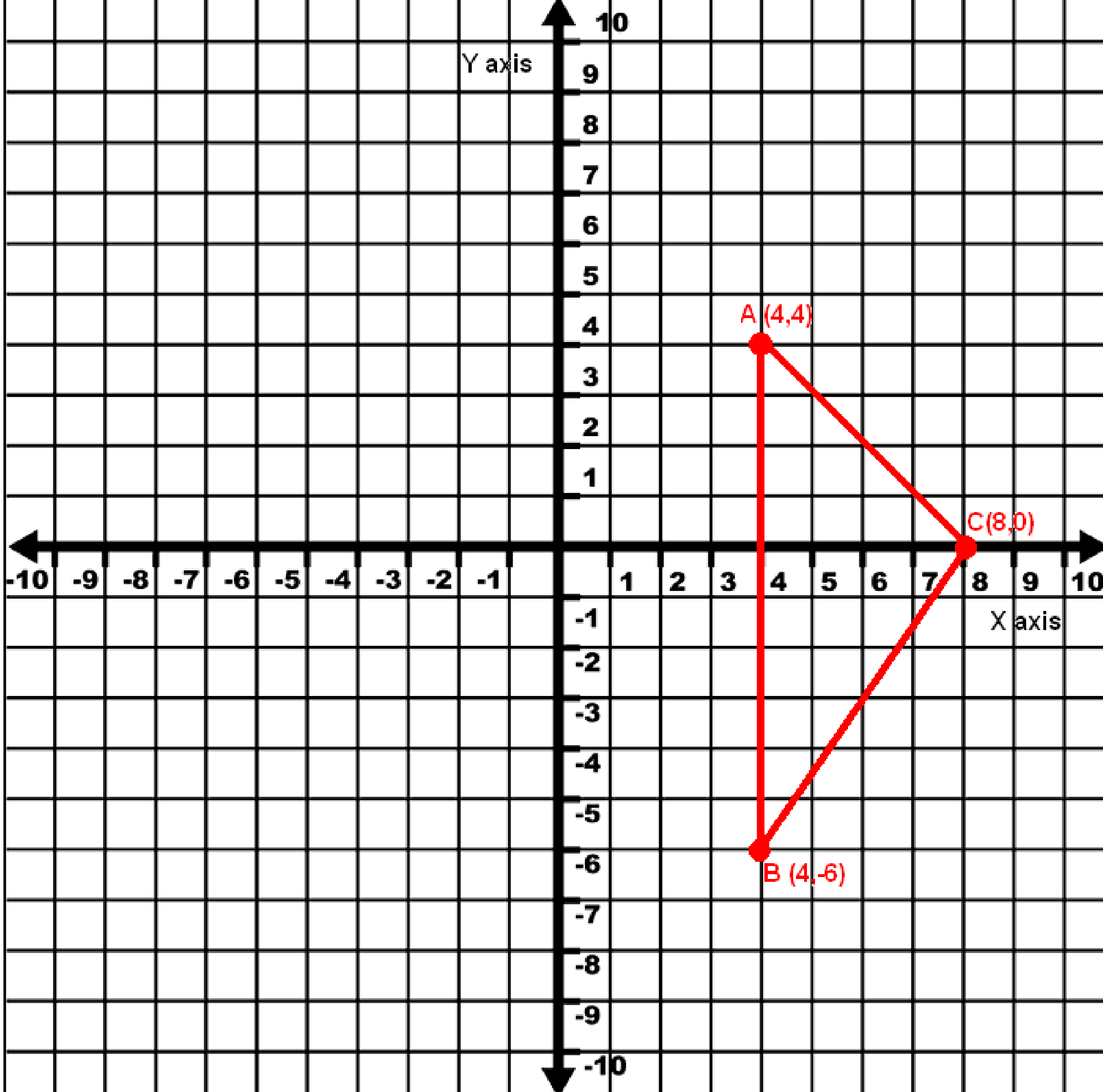
$\Rightarrow$ y - 8 = 4(x-5)

Or 4x – y = 12

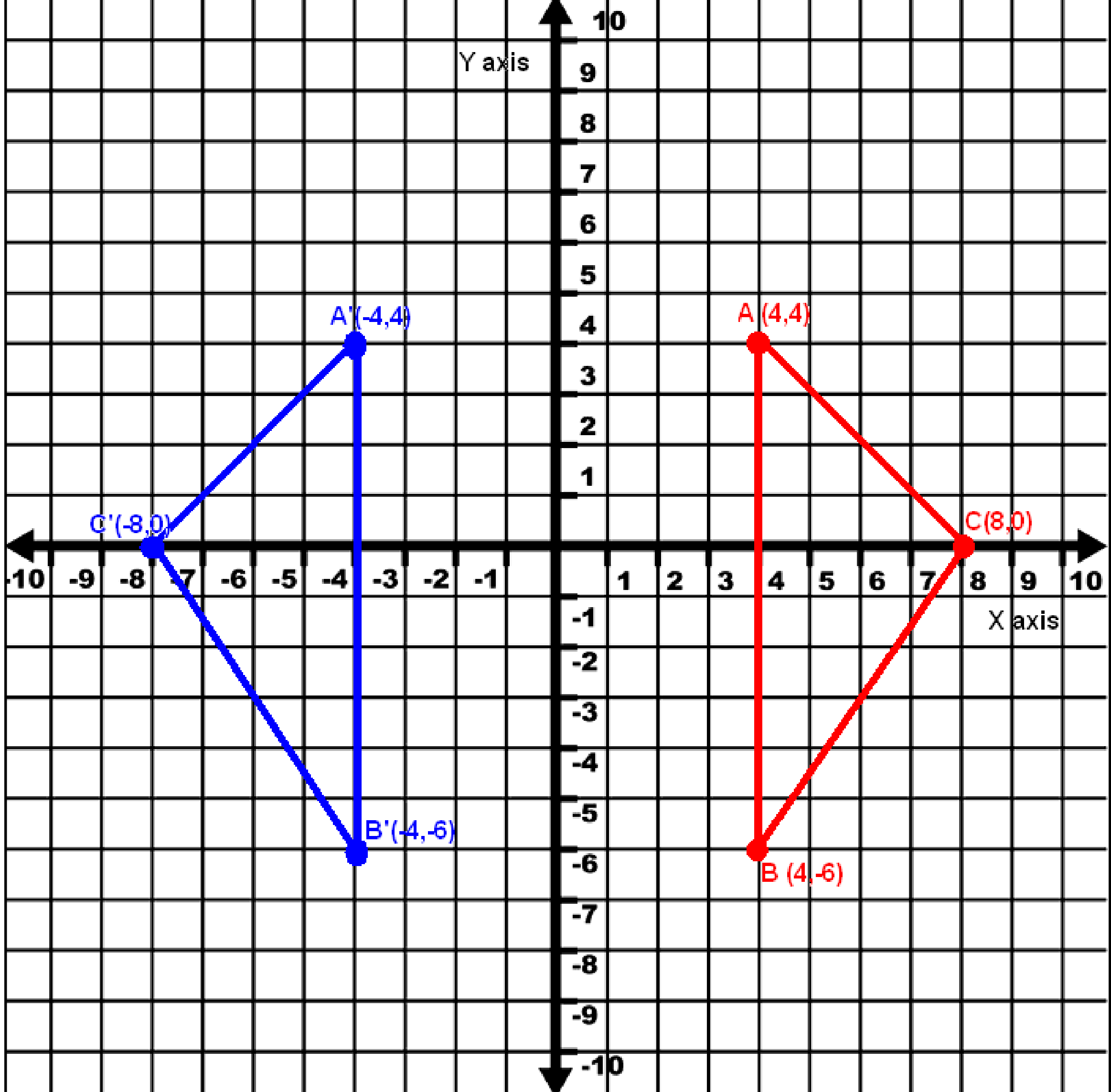
Ans.6.

(a)

(i) The points A(4,4), B(4,-6) and C(8,0) are plotted on the coordinate plane as follows:



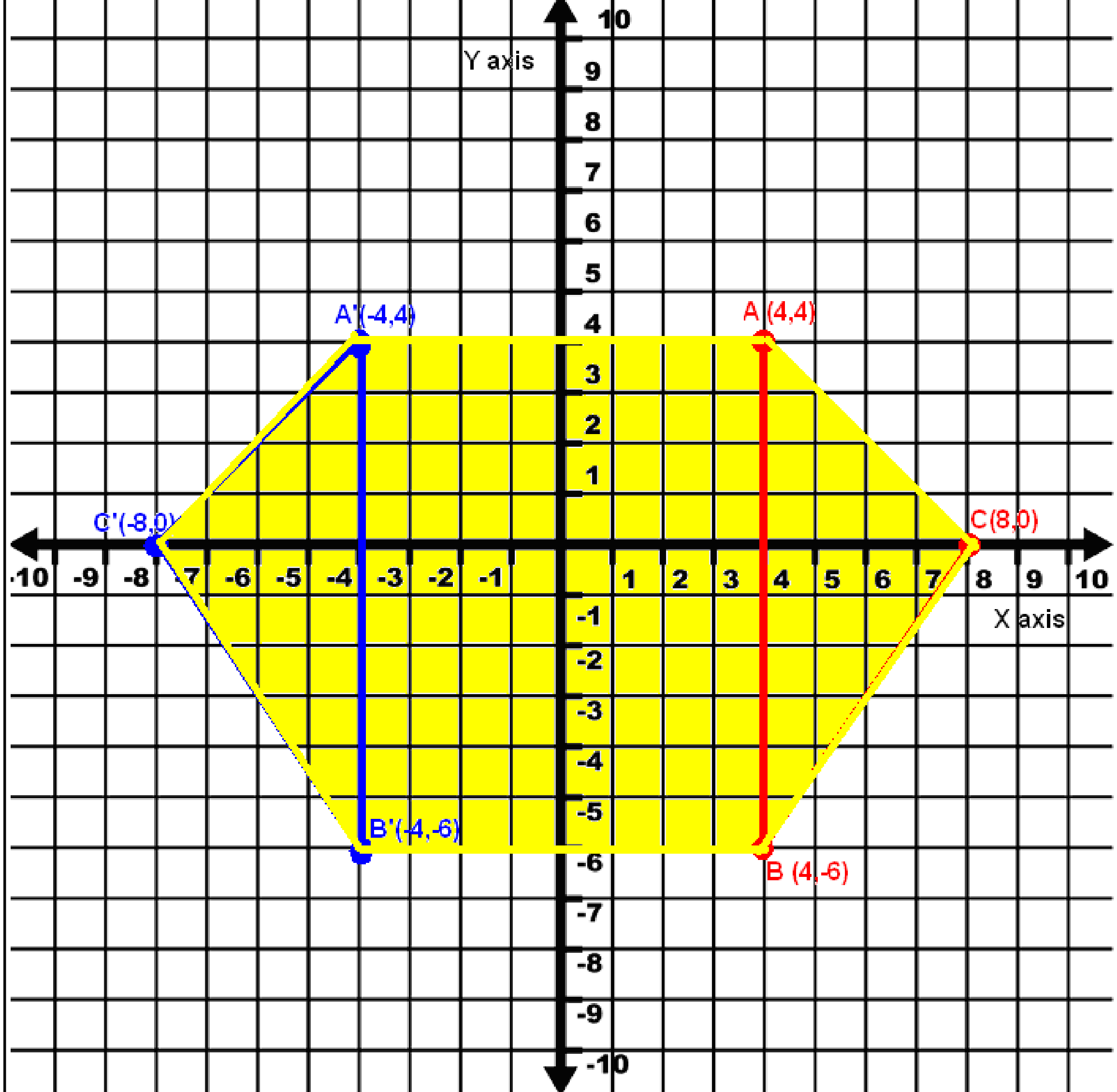
(ii) To reflect the points ABC across Y axis we will keep y coordinate as it is and negate the x coordinate.



The reflected image of the triangle is shown in blue.

(iii) The coordinates of point A′, B′, C′ are (-4, 4), (-4,-6) and (-8, 0) respectively.

(iv) The figure AA′C′B′BC obtained is a polygon with six sides. Thus such a figure would be called a hexagon.



(v) The line of symmetry of AA’C’B’BC would be the y axis.

(b) Here rate of interest = 5%

Principal for April, 07 = ₹7350

Principal for May, 07 = ₹ 11900

Principal for June, 07 = ₹ 11900

Principal for July, 07 = ₹ 13400

Principal for August, 07 = ₹ 13400

Principal for September, 07 = ₹14400

Principal for October, 07 = ₹14400

Principal for November, 07 = ₹15200

Principal for December, 07 = ₹15200

Principal for January, 07 = ₹13200

Principal for Feb, 07 = ₹13200

Principal for March, 07 = ₹ 14150

Total principal for April 2007 to April 2008 = ₹157700

$$\text{Interest paid} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = \frac{157700 \times 5 \times \left(\frac{1}{12}\right)}{100} = ₹657.08 = ₹657$$

Soln.7.

(a)

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$$

Using componendo and dividendo,

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides,

$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

$16(3x+4) = 25(3x-5)$

$48x + 64 = 75x - 125$

$75x - 48x = 64 + 125$

$27x = 189$

$x = 7$

(b) Given, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$$A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

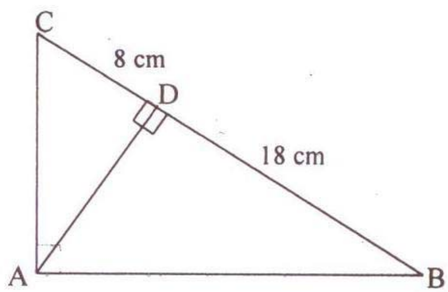
$$A^t.B + B.I = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} . \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix} + \begin{bmatrix} 4+0 & 0-2 \\ -1+0 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+4 & -1-2 \\ 17-1 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$



(c)

(i) In ΔADB and ΔCAB,

$\angle ADB = \angle CAB$ (both 90^o)

$\angle ABD = \angle CBA$ (common angle)

$\therefore$ □ABC ~ □DBA (AA similarity criterion)

In ΔADC and ΔBAC,

$\angle ADC = \angle BAC$ (both 90^o)

$\angle ACD = \angle ACB$ (common angle)

$\therefore$ □DAC ~ □ABC (AA similarity criterion)

If two triangles are similar to one triangle, then the two triangles are similar to each other.

$\therefore \square DAC \sim \square DBA$ or $\square CDA \sim \square ADB$

(ii) Since the corresponding sides of similar triangles are proportional.

$$\therefore \frac{CD}{AD} = \frac{DA}{DB}$$
$$AD^2 = DB \times CD$$
$$AD^2 = 18 \times 8$$
$$AD = 12 \text{ cm}$$

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

So $\frac{\text{Ar }(\Delta ADB)}{\text{Ar }(\Delta CDA)} = \frac{AD^2}{CD^2} = \frac{144}{64} = \frac{9}{4}$

Thus, the required ratio is 9: 4.

Soln.8.

(a)

Class interval	**Frequency (f)**	**x**	**d = x − A= x − 67.5**	**t = $\frac{d}{i}$ i=5**	**f × t**
50 – 55	5	52.5	-15	-3	-15
55 – 60	20	57.5	-10	-2	-40
60 – 65	10	62.5	-5	-1	-10
65 – 70	10	67.5	0	0	0
70 – 75	9	72.5	5	1	9
75 – 80	6	77.5	10	2	12
80 – 85	12	82.5	15	3	36
85 – 90	8	87.5	20	4	32

Total	80				24

Assumed mean (A) = 67.5

Class size, i = 5

Mean = A + i $\frac{\sum ft}{\sum f}$

$= 67.5 + 5 \times \frac{24}{80}$
$= 67.5 + 1.5$
$= 69$

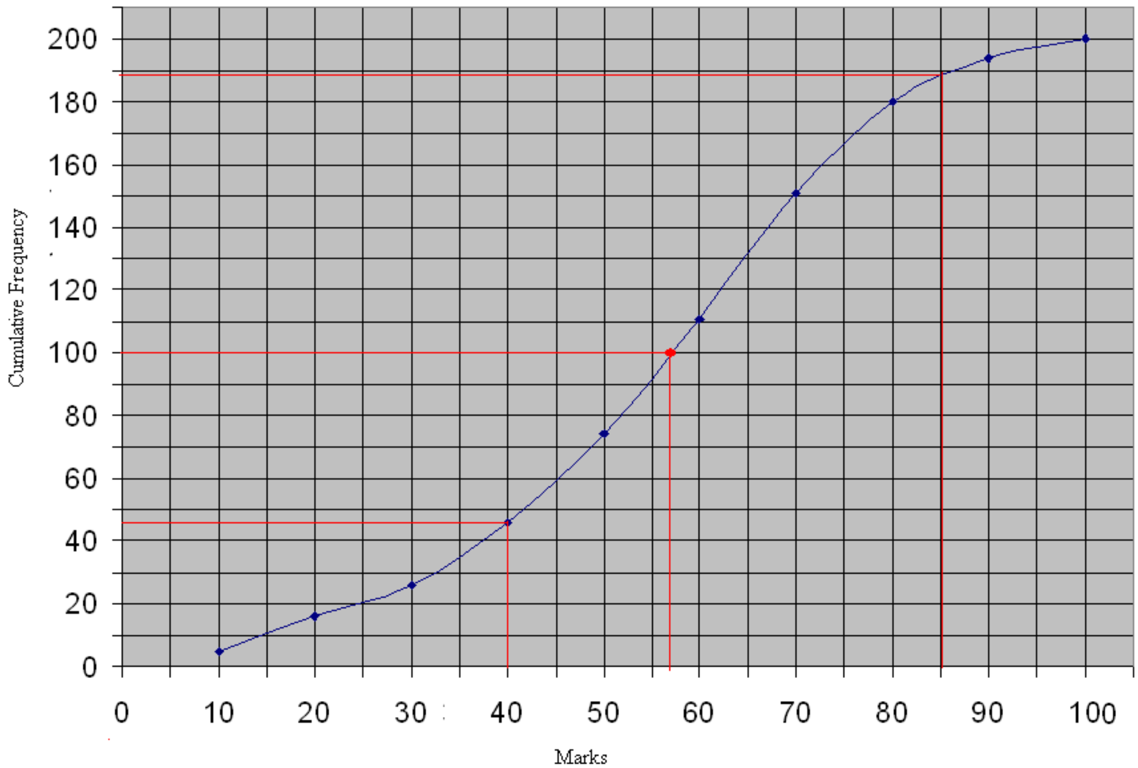
Thus, the mean of the given data is 69.

Modal class is the class corresponding to the greatest frequency. So, the modal class is 55 – 60.

(b)

Marks	**No. of students (f)**	**cf**
0 – 10	5	5
10 – 20	11	16
20 – 30	10	26
30 – 40	20	46
40 – 50	28	74
50 – 60	37	111
60 – 70	40	151
70 – 80	29	180
80 – 90	14	194
90 – 100	6	200

The ogive can be drawn as follows:



(i)

Median marks will be 57.5 as the x coordinate corresponding to n/2 i.e., 100 is 57.5.

(ii) The number of students who failed is 46, which is the y coordinate corresponding to 40 marks.

(iii) Number of students who secured more than 85 marks (grade one) = Total number of students – 184 = 200 – 184 = 16

Soln.9.

(a)

(i) Amount invested = ₹ 52,000

Face value of share = ₹ 100

Discount = ₹ 20

Market price = ₹ 100 – ₹ 20 = ₹ 80

Number of shares = ₹ 52,000/ ₹80 = 650

Dividend % = 8%

Total FV = FV of each share × Number of shares = ₹ 100 x 650 = ₹ 65,000

D = D% × Total FV = $\frac{8}{100} \times 65,000 = 5,200$

Thus, the annual dividend is ₹ 5,200.

(ii) Amount at which the shares were sold = ₹ 120 x 650 = ₹ 78,000

Profit earned including his dividend = ₹ (78,000 – 52,000) + ₹ 5,200 = ₹ 31,200

(b) For constructing the pair of tangents to the given circle following steps will be followed –

1. Taking any point O of the given plane as centre draw a circle of 3.5 cm. radius. Locate a point P, 6 cm away from O. Join OP.

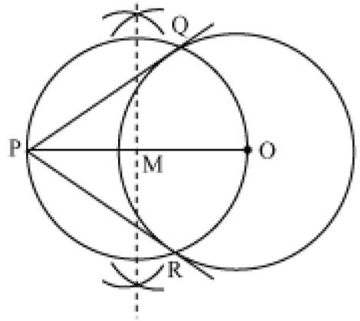
2. Bisect OP. Let M be the midpoint of PO.

3. Taking M as centre and MO as radius draw a circle.

4. Let this circle intersect our circle at point Q and R.

5. Join PQ and PR. PQ and PR are the required tangents.

We may find that length of tangents PQ and PR are 8 cm each.



(c) Consider LHS

$$\text{LHS} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)\sec^2 A$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{1}{\cos^2 A}\right)$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\left(\frac{1}{\cos^2 A}\right)$$
$$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \cdot \frac{1}{\cos^2 A}$$
$$= \frac{\sin A}{\cos A} = \tan A = \text{RHS}$$

Soln.10

(a) 6 is the mean proportion between two numbers x and y,

i.e. $6 = \sqrt{xy}$

So, $36 = xy$ ....(1)

It is given that 48 is the third proportional of x and y

So, $y^2 = 48x$ ... (2)

From (1) and (2), we get

$y^2 = 48\left(\frac{36}{y}\right)$

$\Rightarrow y^3 = 1728$

Hence, $y = 12$

(b) Here Principal = ₹12,000, Rate = 10%, Compound Interest = ₹3972

$$\text{Compound Interest} = P\left[\left(1+\frac{R}{100}\right)^N - 1\right]$$

$$\Rightarrow 3972 = 12000\left[\left(1+\frac{10}{100}\right)^N - 1\right]$$

$$\Rightarrow 3972 = 12000\left[\left(\frac{11}{10}\right)^N - 1\right]$$

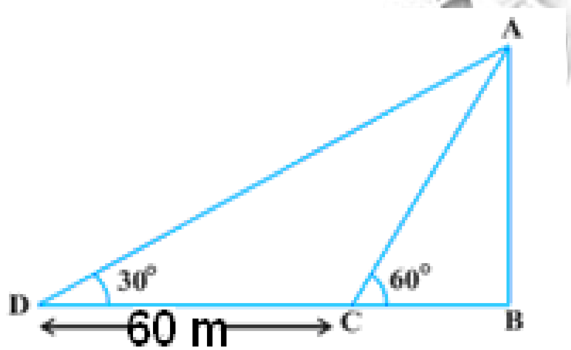
$$\Rightarrow \frac{3972}{12000} + 1 = \left(\frac{11}{10}\right)^N$$

$$\Rightarrow \frac{1331}{1000} = \left(\frac{11}{10}\right)^N$$

$$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^N$$

$$\Rightarrow N = 3 \text{ years}$$

(c) Let the height of the building be AB = h and BC = x



In ΔABC,

$\tan 60^\circ = \frac{h}{x}$

$\Rightarrow x\sqrt{3} = h \quad ...(1)$

In ΔADB,

$\tan 30^\circ = \frac{h}{x+60}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+60}$

$\Rightarrow x+60 = h\sqrt{3}$ ...(2)

From (1) and (2)

$\Rightarrow x+60 = x\sqrt{3}.\sqrt{3}$

$\Rightarrow 2x = 60$

$\Rightarrow x = 30$

Thus, $h = 30\sqrt{3} = 51.96m$

Soln.11.

(a) Let the radii of the circles with A, B and C as centres be r_1, r_2 and r_3 respectively.

According to the given information,

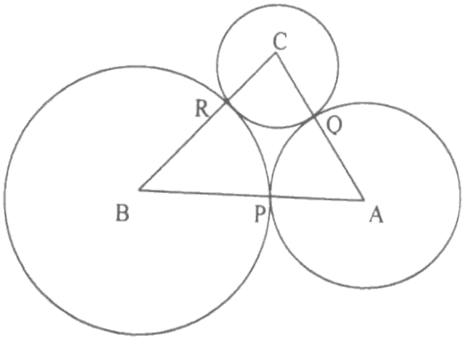
AB = 10 cm = $r_1 + r_2$ ... (1)

BC = 8 cm = $r_2 + r_3$ ... (2)

CA = 6 cm = $r_1 + r_3$ ... (3)

Adding equations (1), (2) and (3),

$2(r_1 + r_2 + r_3) = 24$



$r_1 + r_2 + r_3 = 12$ ... (4)

Subtracting (1) from (4),

$r_3 = 12 - 10 = 2$

Subtracting (2) from (4),

$r_1 = 12 - 8 = 4$

Subtracting (3) from (4),

$r_2 = 12 - 6 = 6$

Thus, the radii of the three circles are 2 cm, 4 cm and 6 cm.

(b) Number of children = x.

Share of each child = ₹ $\frac{480}{x}$

If number of children are x + 20, then share of each child = ₹ $\frac{480}{x+20}$

According to the given information:

$$\frac{480}{x} - \frac{480}{x+20} = 12$$

$$480\left(\frac{1}{x} - \frac{1}{x+20}\right) = 12$$

$$480\left(\frac{x+20-x}{x(x+20)}\right) = 12$$

$$480\left(\frac{20}{x(x+20)}\right) = 12$$

$$\frac{480 \times 20}{12} = x(x+20)$$

$$x^2 + 20x - 800 = 0$$

$$x^2 - 20x + 40x - 800 = 0$$

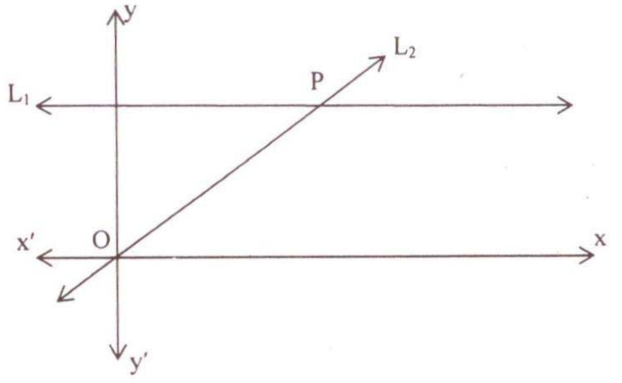
$$x(x-20) + 40(x-20) = 0$$

$$(x-20)(x+40) = 0$$

$$x = 20 \text{ or } x = -40$$

But, the number of children cannot be negative.

Therefore, $x = 20$



(c)

The equation of the line L_1 is $y = 4$.

It is given that L_2 is the bisector of angle O and $\angle O = 90^\circ$.

Thus, the line L_2 makes an angle of 45° with the x-axis.

Thus, slope of line $L_2 = \tan 45^\circ = 1$

The line L_2 passes through (0, 0) and its slope is 1. So, its equation is given by

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$y = x$

Now, the point P is the point of intersection of the lines L_1 and L_2.

Solving the equations $y = 4$ and $x = y$, we get

$x = y = 4$

Thus, the coordinates of the point P are (4, 4).