B.E.

Seventh Semester Examination, May-2010

OPERATION RESEARCH

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) Define operation research and discuss its development.

Ans. Operational research, also known as operations research, is an interdisciplinary branch of applied mathematics and formal science that uses advanced analytical methods such as mathematical modeling, statistical analysis, and mathematical optimization to arrive at optimal or near-optimal solutions to complex decision-making problems. It is often concerned with determining the maximum (of profit, performance, or yield) or minimum (of loss, risk, or cost) of some real-world objective. Originating in military efforts before World War II, its techniques have grown to concern problems in a variety of industries.

Operational research encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency. Some of the tools used by operational researchers are statistics, optimization, probability theory, queuing theory, game theory, graph theory, decision analysis, mathematical modeling and simulation. Because of the computational nature of these fields, OR also has strong ties to computer science. Operational researchers faced with a new problem must determine which of these techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power. Work in operational research and management science may be characterized as one of three categories :

Fundamental or foundational work takes place in three mathematical disciplines: probability, optimization, and dynamical systems theory.

Modeling work is concerned with the construction of models, analyzing them mathematically, implementing them on computers, solving them using software tools, and assessing their effectiveness with data. This level is mainly instrumental, and driven mainly by statistics and econometrics.

Application work in operational research, like other engineering and economics' disciplines, attempts to use models to make a practical impact on real-world problems.

The major subdisciplines in modern operational research, as identified by the journal Operations Research, are :

Computing and information technologies

Decision analysis.

Environment, energy, and natural resources

Financial engineering,

Manufacturing, service sciences, and supply chain management

Policy modeling and public sector work

Revenue management r

Simulation

Stochastic models

Transportation.

Q. 1. (b) Discuss applications and limitations of OR in industry.

Ans. Applications of management science :

Applications of management science are abundant in industry as airlines, manufacturing companies, service organizations, military branches, and in government. The range of problems and issues to which management science has contributed insights and solutions is vast. It includes scheduling airlines, including both planes and crew, deciding the appropriate place to site new facilities such as a warehouse, factory or fire station, managing the flow of water from reservoirs, identifying possible future development paths for parts of the telecommunications industry, establishing the information needs and appropriate systems to supply them within the health service, and identifying and understanding the strategies adopted by companies for their information systems Management science is also concerned with so-called "soft-operational analysis", which concerns methods for strategic planning, strategic decision support, and Problem Structuring Methods (PSM). In dealing with these sorts of challenges mathematical modeling and simulation are not appropriate or will not suffice. Therefore, during the past 30 years, a number of non-quantified modelling methods have been developed. These include:

stakeholder based approaches including metagame analysis and drama theory morphological analysis and various forms of influence diagrams approaches using cognitive mapping the Strategic Choice Approach robustness analysis.

Q. 2. Solve the following LPP by graphical method

Maximize $Z = 5X_1 + 10X_2 + 8X_3$

Subject to :

$$3X_1 + 5X_2 + 2X_3 \le 60;$$

$$4X_1 + 4X_2 + 4X_3 \le 72$$

$$2X_1 + 4X_2 + 5X_3 \le 100;$$

$$X_1, X_2, X_3 \ge 0$$

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Subject to :

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$$X_1, X_2, X_3 \ge 0$$

The problem is exhibited graphically in fig. The feasible region is shaded.

Shaded portion will give the maximum area under the curve.

Q. 3. (a) How can you formulate an assignment problem as a standard Linear Programming Problem? Illustrate.

Ans. The problem of solving a system of linear inequalities dates back at least as far as Fourier, after whom the method of Fourier-Motzkin elimination is named. Linear programming arose as a mathematical model developed during World War II to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy. It was kept secret until 1947. Postwar, many industries found its use in their daily planning.

The founders of the subject are Leonid Kantorovich, a Russian mathematician who developed linear programming problems in 1939, George B. Dantzig, who published the simplex method in 1947, and John von Neumann, who developed the theory of the duality in the same year. The linear programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior point method for solving linear programming problems.

Dantzig's original example of finding the best assignment of 70 people to 70 jobs exemplifies the usefulness of linear programming. The computing power required to test all the permutations to select the best assignment is vast; the number of possible configurations exceeds the number of particles in the universe. However, it takes only a moment to find the optimum solution by posing the problem as a linear program and applying the Simplex algorithm. The theory behind linear programming drastically reduces the number of possible optimal solutions that must be checked.

Q. 3. (b) Using the following cost matrix, determine

(i) Optical job assignment

(ii) The cost of assignments

Machinist Job

	1	2	3	4	5
A	10	3	3	2	8
B	9	7	8	2	7
C	7	5	6	2	4
D	3	5	8	2	4
E	9	10	9	6	10

Ans. Iteration 1–Reduced Cost Table :

Machinist	**Job**				
	1	2	3	4	5
A	8	1	1	0	6
B	7	5	6	0	6
C	5	3	4	0	2
D	1	3	6	0	2
E	3	4	3	0	4

Iteration-2- Reduced cost table.

Machinist	Job				
	1	2	3	4	5
A	7	0	0	0	4
B	6	4	5	0	3

C	4	2	3	0	0
D	C	2	5	0	0
E	2	3	2	0	2

Since the numoer of lines covering all zeros is less than the number of columns/rows, we modify above9 table. The least of the uncovered cell values is 2. Accordingly, the new table would be as table

Machinist	Job				
	1	2	3	4	5
A	7	0	×	2	4
B	4	2	3	0	3
C	2	0	1	0	0
D	0	2	5	2	2
E	0	1	0	0	2

The optimal assignment can be made as the least number of lines covering zeros equals 5. Considering rows and columns, the assignments can be made in the following order B-4, D-1, C-5, A-2 and finally E-3, as shown by squares in the table. The total cost associated with the optimal machinist-job assignment pattern A-2, B-4, C-5, D-1 and E-3 is $3+2+4+3+9=21$.

Q. 4. Obtain the dual of the L.P.P. and solve the dual by simplex method.

$$\text{Max } Z = 108Y_1 + 36Y_2 + 100Y_3$$

Subject to

$$36Y_1 + 3Y_2 + 20Y_3 \le 20;$$

$$6Y_1 + 12Y_2 + 10Y_3 \le 40;$$

$$Y_1, Y_2, Y_3 \ge 0.$$

Ans. $\text{Max. } Z = 180Y_1 + 36Y_2 + 100Y_3$

Subject to

$$36Y_1 + 3Y_2 + 20Y_3 \leq 20$$

$$6Y_1 + 12Y_2 + 10Y_3 \leq 40$$

$$Y_1, Y_2, Y_3 \geq 0$$

The dual of this problem is given here

Minimize $G = 20x_1 + 40x_2$

Subject to,

$$36x_1 + 3x_2 + 2x_3 \geq 180$$

$$6x_1 + 12x_2 + 10x_3 \geq 36$$

$$x_1, x_2, x_3 \geq 0$$

Introducing the surplus and artificial variables, we get

Minimize $G = 20x_1 + 40x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$

Subject to,

$$36x_1 + 3x_2 + 2x_3 - s_1 + A_1 = 180$$

$$6x_1 + 12x_2 + 10x_3 - s_2 + A_2 = 36$$

$$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$$

Basis	x_1	x_2	x_3	s_1	s_2	A_1	A_2	b_i	b_i / a_{ji}	
A_1	M	36	3	2	−1	0	1	0	180	5 ←
A_2	M	6	12	10	0	−1	0	1	36	6

C_j	20	40	0	0	0	M	M
Sol.	0	0	0	0	0	180	36
Δj	20–42M	40–15M	0	M	M	0	0
	↑						

Q. 5. The following table shows the normal duration of tasks of a project along with crash duration, normal cost and crash cost. The indirect cost for the project is Rs. 300 for each day.

(i) Draw the network of the project.

(ii) What is the normal duration and cost of the project.

(iii) Find the optimal duration and minimum project cost.

(iv) If all the activities are crashed, what will be the project duration and the corresponding cost?

	Job	1–2	1–3	2–3	2–4	2–5	3–6	4–5	5–6
Normal	Time (days)	9	15	7	7	12	12	6	9
	Cost (Rs.)	1300	1000	7000	1200	1700	600	1000	900
Crash	Time (days)	4	13	4	3	6	11	2	6
	Cost (Rs.)	2400	1380	1540	1920	2240	700	1600	1200

Ans. The network rk of the project :

From the diagram, we have

Path	Normal Time	Crash Time
1–2–5–6	30 (critical)	16
1–3–6	27	24 (critical)
1–2–3–6	28	19
1–2–4–5–6	31	15

Crashing cost : The per day crashing cost for each activity can be obtained as follows :

$$\text{Crashing cost} = \frac{\text{Crash cost–Normal cost}}{\text{Normal time– Crash time}}$$

Accordingly for activity 1–2, we have

$$\text{Crashing cost} = \frac{2400-1300}{9-4}$$

$$= 220 \text{ per day}$$

Q. 6. (a) Explain the basic queuing process. State the meaning of queue discipline and give its role in queuing problem.

Ans. Queueing theory is the mathematical study of waiting lines, or queues. The theory enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue (essentially a storage process), and being served at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service, and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served.

Queueing theory has applications in diverse fields, including telecommunications, traffic engineering, computing and the design of factories, shops, offices and hospitals.

The word queue comes; via French, from the Latin cauda, meaning tail. The spelling "queueing" over "queuing" is typically encountered in the academic research field. In fact, one of the flagship journals of the profession is named "Queueing Systems". "Queueing" - the correct spelling–is the only word in the English language with five consecutive vowels. Queueing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide service. It is applicable in a wide variety of situations that may be encountered in business, commerce, industry, health care public service and engineering. Applications are frequently encountered in customer service situations as well as transport and telecommunication. Queueing theory is directly applicable to intelligent transportation systems, call centers, PABXs, networks, telecommunications, server queueing, mainframe computer queueing of telecommunications terminals, advanced telecommunications systems, and traffic flow.

Notation for describing the characteristics of a queueing model was first suggested by David G. Kendall in 1953. Kendall's notation introduced an AIBIC queueing notation that can be found in all standard modern works on queueing theory, for example, Tijms.

The AIBIC notation designates a queueing system having A as interar rival time distribution, Bas service time distribution, and C as number of servers. For example, "G/D/1" would indicate a General (may be anything) arrival process, a Deterministic (constant time) service process and a single server. More details on this notation are given in the article about queueing models.

Q. 6. (b) Arrivals at a telephone both are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially, with mean 3 minutes. Find

(i) Probability that an arrival finds that 4 persons are waiting for their turn.

(ii) Average number of persons waiting and making telephone calls.

(iii) Average length of queue that is formed from time to time.

Ans. According to the given information,

$$M = 4 \text{ persons}$$

$$\mu = 10 \text{ min.}$$

$$\lambda = 3 \text{ min.}$$

$$\therefore \quad \rho = \frac{\lambda}{\mu} = \frac{3}{10} = 0.3$$

Since ρ_0 is basic to all the formulae for this model, we shall first determine its value. It is shown in table.

Calculation of ρ_0

i	$M!/(M-i)$	$(\lambda/\mu)i$	$\frac{M!}{(M-i)!}\left(\frac{\lambda}{\mu}\right)^i$
0	1	0.3	0.3
1	4	1.2	4.8
2	12	3.6	43.2
3	24	7.2	172.8
4	24	7.2	172.8
		Total	393.9

We have,

$$P_0 = \left[\sum_{i=0}^{M} \left(\frac{M!}{(M-i)!} \left(\frac{\lambda}{\mu} \right)^i \right) \right]^{-1}$$

$$= (393.9)^{-1}$$

$$= 2.54 \times 10^{-3}$$

(i) Thus, the probability that an arrival finds that 4 persons are waiting for their turn

$$= 2.54 \times 10^{-3}.$$

(ii) Avg. number of persons,

$$L_s = M - \frac{\mu}{\lambda}(1 - P_0)$$

$$= 4 - \frac{10}{3}\left(1 - 2.54 \times 10^{-3}\right)$$

$$= 0.675 \text{ persons}$$

(iii) Avg. length of queue,

$$L_q = M - \frac{\lambda + \mu}{\lambda}(1 - P_0)$$

$$= 4 - \frac{(3+10)}{3}\left(1 - 2.54 \times 10^{-3}\right)$$

$$= -0.32 \text{ persons.}$$

Q. 7. (a) What is simulation? Discuss its advantages and limitations.

Ans. Simulation is the imitation of some real thing, state of affairs, or process. The act of simulating something generally entails representing certain key characteristics or behaviours of a selected physical or abstract system.

Simulation is used in many contexts, including the modeling of natural systems or human systems in order to gain insight into their functioning. Other contexts include simulation of technology for performance optimization, safety engineering, testing, training and education. Simulation can be used to show the eventual real effects of alternative conditions and courses of action. Simulation is also used when the real system cannot be engaged. The real system may not be engaged because it may not be accessible, it may be dangerous or unacceptable to engage, or it may simply not exist.

Key issues in simulation include acquisition of valid source information about the relevant selection of key characteristics and behaviours, the use of simplifying approximations and assumptions within the simulation, and fidelity and validity of the simulation outcomes. Computer simulation - A computer simulation (or "sim") is an attempt to model a real-life or hypothetical situation on a computer so that it can be studied to see how the system works. By changing variables, predictions may be made about the behaviour of the system.

Computer simulation has become a useful part of modeling many natural systems in physics, chemistry and biology, and human systems in economics and social science (the computational sociology) as well as in engineering to gain insight into the operation of those systems. A good example of the usefulness of using computers to simulate can be found in the field of network traffic-simulation. In such simulations, the model behaviour "X-II change each simulation according to the set of initial parameters assumed for the environment.

Traditionally, the formal modeling of systems has been via a mathematical model, which attempts to find analytical solutions enabling the prediction" of the behaviour of the system from a set of parameters called initial conditions. Computer simulation is often used as an adjunct to, or substitution for, modeling systems for which simple closed form analytic solutions are not possible. There are many different types of computer simulation, the common feature they all share is the attempt to generate a sample of representative scenarios for a model in which a complete enumeration of all possible states would be prohibitive or impossible.

Q. 7. (b) Discuss the methods of Monte Carlo simulation.

Ans. Monte Carlo methods (or Monte Carlo experiments) are a class of computational algorithms that rely on repeated random sampling to compute their results. Monte Carlo methods are often used in simulating physical and mathematical systems. Because of their reliance on repeated computation of random or pseudo-random numbers, these methods are most suited to calculation by a computer and tend to be used when it is unfeasible or impossible to compute an exact result with a deterministic algorithm.

Monte Carlo simulation methods are especialy useful in studying systems with a large number of coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structure if (see cellular Potts model). More broadly, Monte Carlo methods are useful for modeling phenomena with significant uncertainty in inputs, such as the calculation of risk in business. These methods are also widely used in mathematics: a classic use is for the evaluation of definite integrals, particularly multidimensional integrals with complicated boundary conditions. It is a widely sucessful method in risk analysis when compared with alternative methods or human intuition. When Monte Carlo simulations have been applied in space exploration and oil exploration, actual observations of failures, cost overruns and schedule overruns are routinely better predicted by the simulations than by human intuition or alternative "soft" methods.

The term "Monte Carlo method" was coined in the 1940s by physicists working on nuclear weapon projects in the Los Alamos National Laboratory.

There is no single Monte Carlo method; instead, the term describes a large and widely-used class of approaches. However, these approaches tend to follow a particular pattern :

Define a domain of possible inputs.

Generate inputs randomly from the domain using a certain specified probability distribution. Perform a deterministic computation using the inputs.

Aggregate the results of the individual computations into the final result.

For example, the value of TT can be approximated using a Monte Carlo method :

Draw a square on the ground, then inscribe a circle within it. From plane geometry, the ratio of the area of an inscribed circle to that of the surrounding square is TT/4.

Uniformly scatter some objects of uniform size throughout the square. For example, grains of rice or sand.

Since the two areas are in the ratio TT/4, the objects should fall in the areas in approximately the same ratio. Thus, counting the number of objects in the circle and dividing by the total number of objects in the square will yield an approximation for TT/4.

Multiplying the result by 4 will then yield an approximation for TT itself.

Q. 8. (a) Describe the steps involved in the process of decision making.

Ans. Steps To Decision Making :

Identify a problem or opportunity

- The first step is to recognise a problem or to see oppounities that may be worthwhile.
- Will it really make a difference to our customers?
- How worthwhile will it be to solve this problem or ralise this opportunity?

2. Gather information :

- What is relevant and what is not releant to the decision?'
- What do you need to know before yo can make a decision, 61 the will help you make the right one?
- Who knows, who can help, who as the power and influence to male this happen (or to

stop it)?

3. Analyze the situation :

- What alternative courses of action may be available to you?
- What different .interpretations of the data may be possible?

4. Develop options : -

- Generate several possible options.
- Be creative and positive.
- Ask "what if' questions.
- How would you like your situation to be?

5. Evaluate Alternatives :

- What criteria should you use to evaluate?
- Evaluate for feasibility, acceptability and desirability.
- Which alternative wit best achieve your objectives?

6. Select a preferred alternative :

- Explore the provisional preferred alernative for future possible adverse consequences.
- What problems might it create?
- What are the risks of making this decison?

7. Act on the decision :

- Put a plan in place to implement the decision.
- Ha... you allocated resources to implement.
- Is the dec... n accepted and supported by colleagues?

Q. 8. (b) Explain Simo... odel.

Ans. The nature of human pro... m solving methods has been studied by psychologists over the past hundred years. There are sveral methods o. studying proble. solving, including; introspection, behaviorism, ...ulation, computer moding and experiment.

Beginning with the early experimental work of the Gestaltists in Germany (e.g. Duncker, 1935), and continuing through the 1960s and early 1970s, research on problem solving typically conducted relatively simple, laboratory tasks (e.g. Duncker's "X-ray" problem; Ewert & Lambert's 1932 "disk" problem, later known as Tower of Hanoi) that appeared novel to participants (e.g. Mayer, 1992). Various reasons account for the choice of simple novel tasks: they had clearly defined optimal solutions, they were solvable within a relatively short time frame, researchers could trace participants' problem-solving steps, and so on. The researchers made the underlying assumption, of course, that simple tasks such as the Tower of Hanoi captured the main properties of "real world" problems, and that the cognitive processes underlying participants' attempts to solve simple problems were representative of the processes engaged in when solving "real world" problems. Thus researchers used simple problems for reasons of convenience, and thought generalizations to more complex problems would become possible. Perhaps the best-known and most impressive example of this line of research remains the work by Allen Newell and Herbert Simon.

Simple laboratory-based tasks can be useful in explicating the steps of logic and reasoning that underlie problem solving; however, they omit the complexity and emotional valence of "real-world" problems. In clinical psychology, researchers have focused on the role of emotions in problem solving (D'Zurilia & Goldfried, 1971; D'Zurilia & Nezu, 1982), demonstrating that poor emotional control can disrupt focus on the target task and impede problem resolution (Rath, Langenbahn, Simon, Sherr, & Diller, 2004). In this conceptualization, human problem solving Gonsists of two related processes : problem orientation, the motivational/attitudinal/affective approach to problematic situations and problem-solving skills, the actual cognitive-behavioral steps, which, if successfully implemented, lead-to effective problem resolution. Working with individuals with frontal lobe injuries) leuropsychologists have discovered that deficits in emotional control and reasoning can be remediated, improving the capacity of injured persons to resolve everyday problems successfully (Rath, Simon, Langenbahn, Sherr, & Diller, 2003).