INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20^{th} May 2010

Subject ST6 — Finance and Investment B

Time allowed: Three hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. ***** *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor*

4. *The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

5. *Attempt all questions, beginning your answer to each question on a separate sheet.*

6. *Mark allocations are shown in brackets.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q 1) **(a)** Alpha Limited has issued a Rs. 100 crores issue of floating-rate bonds on which it Pays an **effective** interest rate of 0.5% over the MIBOR rate annually. The bonds are selling at par value. **The firm is worried that rates might be about to rise**, and it would like to lock in a fixed interest rate on its borrowings. The firm sees that dealers in the swap market are offering swaps of MIBOR for 8%. **The swaps available from the dealer have the same term as the floating-rate bonds.** What interest rate swap will convert the firm's interest obligation into one resembling a synthetic fixed rate loan? What interest rate will it pay on that synthetic fixed-rate loan? (2)

(b) At the present time, one can enter 3-year swaps that exchange MIBOR for 6%. An off-market swap would then be defined as a swap of MIBOR for a fixed rate other than 6%. For example, a firm with 8% coupon debt outstanding might like to convert to synthetic floating-rate debt by entering a swap in which it pays MIBOR and receives a fixed rate of 8%. What up-front payment will be required to induce a counterparty to take the other side of the swap? Assume notional principal is Rs. 100 crores. (2)

(c) If the yield curve is flat, prove that the duration of a coupon bond is

$$\frac{1+y}{y} - \frac{(1+y)+T(c-y)}{c[(1+y)^T - 1]+y}.$$

Where c is the coupon rate per annum with annual compounding (coupons are paid annually), y is the bond's yield per annum with annual compounding, and T is the number of years after which the bond will expire. (4)

(d) Pension funds pay life time annuities to recipients. If a firm will remain in business Indefinitely, the pension obligation will resemble a perpetuity. Suppose, therefore, that you are managing a pension fund with obligations to make perpetual payments of Rs. 120 crores per year to beneficiaries. The **annual effective** yield to maturity on all bonds is 12%.

(i) You decide to use 6-year maturity bonds with coupon rate of 10% (paid annually) and 21-year maturity bond with coupon rate 5% (paid annually) to immunize your obligation. How much each of these coupon bonds (in market value) will you want to hold to both fully fund and immunize your obligation?

(ii) What will be the par value of your holding in the two coupon bonds? (5)

[13]

Q 2) **(a)** Suppose that a bank has a total of Rs. 4000 crores of a **bond portfolio**. The 1-year probability of default averages 2% and the recovery rate averages 50%. The copula correlation parameter is 0.25. **Using Gaussian copula model**, estimate the 99% 1-year credit VaR. (3)

(b) Consider a portfolio with a delta of 700, a gamma of -4100 and a kappa (vega) of - 2100. Two traded options are available on the portfolio. The first traded option is available with a delta of 0.60, a gamma of 1.20 and a kappa (vega) of 0.60. The second traded option is available with a delta of 0.30, a gamma of 0.80 and a kappa (vega) of 0.30. What is the position in the two traded options and in the underlying asset would make the portfolio delta, gamma, and kappa (vega) neutral? (4)

[7]

Q 3) It is November 30 2008. The cheapest to deliver bond in a March 2010 Treasury bond futures contract **(which expires on March 31 2010)** is a 10% (per annum with semi-annual compounding) coupon bond and the delivery is expected to be made on March 31, 2010. Coupon payments on the bond are made on January 1 and July 1 each year. The term structure is flat, and the **risk-free rate of interest** with continuous compounding is 8% per annum. The conversion factor for the bond is 1.3200. The current quoted bond price is Rs. 120 (face value of the bond is Rs. 100). Calculate the futures price for the contract.

[7]

Q 4) **(a)** Demonstrate that an at-the-money call option on a given stock **must be worth more than** an at-the money put option on that stock with the same time to maturity. **Assume that there are no dividends and that the risk-free rate is greater than zero.** (2)

(b) The agricultural price support system guarantees farmers a minimum price for their output. Describe the program provisions as an option. What is the asset? The exercise price? (2)

(c) An executive compensation scheme might provide a manager a bonus of Rs. 500 for every rupee by which the company's stock price exceeds some cutoff level. In what way is this arrangement equivalent to issuing the manager call options on the firm's stock? (1)

(d) The delta of an at-the-money money call option on Infosys stock is 0.5. The delta of an at-the-money put option on Infosys stock is -0.7. What is the delta of an at-the money straddle position on Infosys stock? (1)

(e) Would you expect a one rupee increase in a **vanilla** call option's exercise price to lead to a decrease in the option value of more or less than one rupee? (2)

(f) If the stock price falls and call price increases, then what has happened to the call option's implied volatility? (1)

(g) What is the difference in cash flow between short-selling an asset and entering a short futures position? (1)

(h) Are the following **statements** true or false?

(i) All else equal, the futures price on high beta stock should be higher than the futures price on low-beta stock.

(ii) The beta of a short position in the NSE Nifty futures contract is negative. (2)

[12]

Q 5) You hold a Rs.100 crores stock portfolio with a beta of 1.10. You believe that the **effective** risk - adjusted abnormal return on the portfolio (the alpha, that is, the return in excess of CAPM return) over the next six months is 3%. The NSE Nifty index currently is at 5000 and the **effective** risk free rate is 3% per six month. Ignore dividends.

(a) What will be the futures price on the 6-month maturity NSE Nifty futures contract? (1)

(b) How many Nifty futures contract are needed to hedge the stock portfolio? One Nifty futures contract is on Rs. 50 times the value of the index. (1)

(c) What will be the profit on that futures position in six months as a function of the value of the NSE Nifty index on the maturity date? (1)

(d) If the alpha of the stock is 3%, show that the expected rate of return on the portfolio as a function of the market return is $r_p = .06 + 1.10(r_M - .03)$. (2)

(e) Calculate the expected value of hedged stock-plus-futures portfolio in six months as a function of the value of the index. (3)

(f) What is the expected rate of return on the hedged portfolio over the next six months? (2)

(g) What is the beta of the hedged portfolio? What is the alpha of the hedged portfolio? (1)

[11]

Q 6) You are attempting to formulate an investment strategy. On the one hand, you think there is great upward potential in the stock market and would like to participate in the upward move if it materializes. However, you are not able to afford substantial stock market losses and so can not run the risk of a stock market collapse, which you also think is a possibility. Your investment adviser suggests a protective put position: Buy both shares in a market index stock fund and put options on those shares with 2-month maturity and exercise price of Rs. 4800. The stock index fund is currently selling for Rs. 5000. However, your uncle suggests you instead buy a 2-month call option on the index fund with exercise price Rs. 4900 and buy 2-month T-bills with face value Rs. 4900.

(a) On the same graph, draw the payoff to each of these strategies as the function of stock fund values in two months. (3)

(b) Which portfolio must require a greater initial outlay to establish? (1)

(c) Suppose the market price of the securities are as follows:

Stock Fund	Rs. 5000
T-bill (Face value Rs. 4900)	Rs. 4850
Call (exercise price Rs. 4900)	Rs. 300
Put (exercise price Rs. 4800)	Rs. 100

Make a table of the profits realized for each portfolio for the following values of the stock price in 2 months: S_T = Rs. 4500, **Rs. 4800**, Rs. 4900, Rs. 5000, Rs. 5100, Rs. 5400.Graph the profits to each portfolio as a function of S_T on a single graph. (3)

(d) Which strategy is riskier? Which should have higher beta? (1)

(e) Explain why the data for securities given in part (c) does not violate the put-call parity relationship. (1)

[9]

Q 7) A standard Heath-Jarrow-Morton (HJM) forward interest rate model (without Ω), under the risk neutral martingale measure Q, is of the form

$$df(t,T) = m(t,T)dt + s(t,T)dz(t)$$

where z is assumed to be a Q-wiener process.

(a) Given the above forward rate dynamics, derive the dynamics of the zero coupon bond price, P(t,T). (5)

(b) Derive the HJM drift condition (under Q). (2)

[7]

Q 8) Consider a model for two countries. We have a domestic market (India) and a foreign market (US). The domestic and foreign interest rates, r_d and r_f, are assumed to be given real numbers. The domestic and foreign savings accounts satisfy

$$B_t^{\ d} = e^{r_d t}, \quad B_t^{\ f} = e^{r_f t}$$

Where B^d and B^f are denominated in units of domestic and foreign currency respectively. The exchange rate process E, which is used to convert foreign payoffs into domestic currency (the rupee/$ rate) is modeled by the following stochastic differential equation under the objective measure P

$$dE = \mu_E E dt + \sigma_E E dW$$

Where μ_E and σ_E are assumed to be constant and W denotes a P-Wiener process.

Assume that you have entered a financial contract which stipulated that you are to pay a certain amount of money in US dollars at a future date. You are considering to buy a pay later option on US dollar to reduce your exposure to foreign exchange risk The buyer of a pay later option has the obligation to exercise the option when it is in the money and to pay the premium. This means that as soon as the difference between the price of the underlying and the exercise price is positive exercise takes place, regardless of how big the difference is, i.e. the amount by which the option is in the money. The payoff at the exercise date T of a pay later option with exercise price K written on US dollar is thus given by $X = g(E_T)$, where the function g is defined by

$$g(z) = \begin{cases} z - K - P, & if\ z > K \\ 0 & otherwise \end{cases}$$

The P denotes the premium which the holder of the option must pay upon exercise. The premium P is determined when the contract is initiated (at time t = 0) in such a way that the current price of the contract is zero, i.e. $\Pi(0, X) = 0$.

Your task is to compute the current premium P (in domestic currency) for the pay later option described above.

[12]

Q 9) Consider a standard Black-Scholes market, i.e the market consisting of a risk-free asset, B, with P-dynamics given by

$$\begin{cases} dB_t = rB_t dt \\ B_0 = 1 \end{cases}$$

and a **non-dividend** paying stock S, with P-dynamics given by

$$\begin{cases} dS_t = \mu S_t dt + \sigma S_t dW_t \\ S_0 = s_0 \end{cases}$$

Where W denotes P-wiener process and r, μ and σ are assumed to be constants.

(a) Determine the arbitrage free price of an option at time t which at time T pays either K or the value of stock at time T, whichever is higher. (3)

(b) Now consider an option which has the same payoff function as the option in (a), except for that K is replaced by S_{T_0}, where T_0 is a fixed time such that $T_0 < T$. Determine the arbitrage free price of this option for $t \in [0, T_0]$. (6)

[9]

Q 10) A stock price is currently Rs.1000. Over each of the next two 1-year period it is expected to go up by 50% or down by 50%. The risk-free interest rate is zero. Consider a binary asset-or-nothing option with expiry in two years and payoff

$$Z = \begin{cases} S_2, & if\ S_2 > 1500 \\ 0 & otherwise \end{cases}$$

Where S_2 denote the stock price at time t = 2.

(a) What is the current value of this option? (2)

(b) Find the replicating portfolio for the option in (a) and verify that the portfolio is self-financing. (4)

[6]

Q 11) Consider an arbitrage free interest rate model under a martingale measure Q, and denote the instantaneous forward rates by f(t,T). The interest rate model is said to exhibit parallel shifts if the forward rate process has the following structure

$$f(t,T) = X_t + U(T-t)$$

Where U is a deterministic function of time to maturity (T-t) and X is a random process. We normalize so that U(0) = 0.

(a) Show that the process **X** is in fact the short rate process i.e. show that $X_t = r_t$. (1)

(b) In addition to the parallel shifts assumption in the interest rate model, also assume that r satisfies the SDE

$$dr_t = \alpha(t, r_t)dt + \sigma(t, r_t)dW_t$$

Where W is a Q-Wiener process. **Using HJM drift condition**, your task is to find out what can be said about α and σ under these assumptions. (6)

[7]

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