ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS

SEMESTER - 4

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The generating function for the numeric function

$$\left(1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots\ldots\right) \text{ is}$$

a) $\log(1+x)$ b) $\frac{1}{x}\log(1+x)$

c) e^x d) $\frac{1}{x}\log(1-x)$.

ii) If a network contains 6 vertices, then the number of cuts in the network is

a) 14 b) 15

c) 16 d) 32.

iii) The hamming distance between 0011011 and 0111001 is

a) 2 b) 3

c) 4 d) 0.

iv) The minimum number of edges in a connected graph having 21 vertices is

a) 18 b) 20

c) 10 d) 11.

v) The minimum number of pendant vertices in a tree with five vertices is

a) 1 b) 2

c) 3 d) 4.

vi) If S and T are two subgroups of a group G, then which of the following is a subgroup ?

a) $S \cup T$ b) $S \cap T$

c) $S - T$ d) $G - S$.

vii) If R is a ring without zero divisors, then $x \cdot y = 0$ implies

a) $x = 0$ or $y = 0$ b) $x = 0$ and $y = 0$

c) $x = 0,\ y \neq 0$ d) $x \neq 0,\ y = 0$.

viii) The solution of recurrence relation

$$a_{n+1} - 2a_n = 5,$$

$n \geq 0,\ a_0 = 1$ is

a) $6 \cdot 2^n - 5$ b) $5 - 6 \cdot 2^n$

c) $2^{n+1} - 1$ d) none of these.

ix) Which of the following sets is closed under multiplication ?

a) $\{1, -1, 0, 2\}$ b) $\{1, i\}$

c) $\{1, \omega, \omega^2\}$ d) $\{\omega, 1\}$.

x) In a Boolean Algebra $x + (y \cdot z') =$

a) $x + z$ b) xy

c) $x + y$ d) $x + y + z$.

xi) The generating function corresponding to the sequence 1, 1, 0, 1, 1, 1, ... is

a) $\frac{1}{1+x} - x^2$ b) $\frac{1}{1+x^2}$

c) $\frac{1}{1+x} + x^2$ d) $\frac{1}{1-x^2} - x^2$.

xii) The maximum degree of any vertex in a simple graph with 10 vertices is

a) 5 b) 9

c) 10 d) 20.

xiii) Let S be a finite set of n distinct elements. Then the number of bijective mapping from S to S is

a) n^2 b) $n!$

c) $\frac{n!}{2}$ d) 2^n.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following questions. $3 \times 5 = 15$

2. Show that the group $(Z_6, +)$ is cyclic. Find all the generators of the group $(Z_6 = \{[0], [1], [2], [3], [4], [5]\})$.

3. If G is a finite group and H is a subgroup of G, then prove that $O(H)$ is a divisor of $O(G)$.

4. Prove that the set of all even integers form a commutative ring.

5. Show that all roots of the equation $x^4 = 1$ form an Abelian group under multiplication.

6. Using generating functions solve the recurrence relation with initial conditions :

$a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions. 3 × 15 = 45

7\. a) Let $G = \{ (a, b) : a \neq 0,\ b \in R \}$ and $*$ be a binary composition defined on G by $(a, b) * (c, d) = (ac,\ bc + d)$.

b) Let G be a group, if $a, b \in G$ such that $a^4 = e$, then identity element of G and $ab = ba^2$. Prove that $a = e$.

c) Slow that the set of matrices $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$ is a subring of the ring of matrices.

5 + 5 + 5

8\. a) Using generating function solve the recurrence relation

$$a_n - 7a_{n-1} + 10a_{n-2} = 0$$

for $n > 1$ and $a_0 = 3,\ a_1 = 3$.

b) Solve the recurrence relation $a_n = 8a_{n-1} + 10^{n-1}$ for $n \geq 1$ and $a_0 = 1$.

8 + 7

9\. a) Convert $(x + y)(y + z)(x' + z)(x' + y')$ into conjunctive normal form $x, y, z \in$ Boolean Algebra B.

b) Construct the truth table of the Boolean function

$$f(x, y, z) = (yz + xz')(xy' + z)'.$$

5 + 10

10\. a) If A, B and C are three sets, prove analytically that

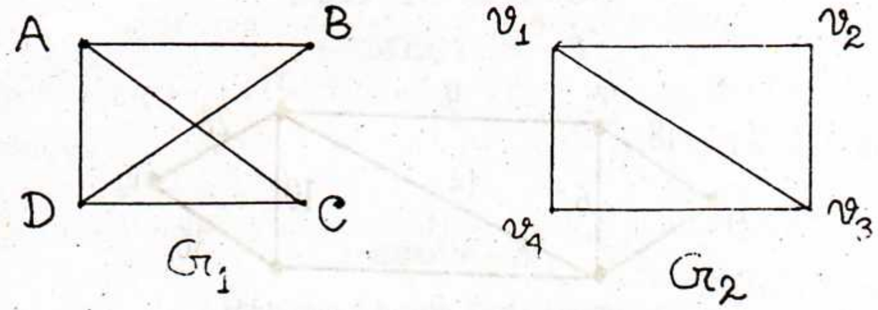
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

b) Show that the intersection of two equivalence relations is also an equivalence relation.

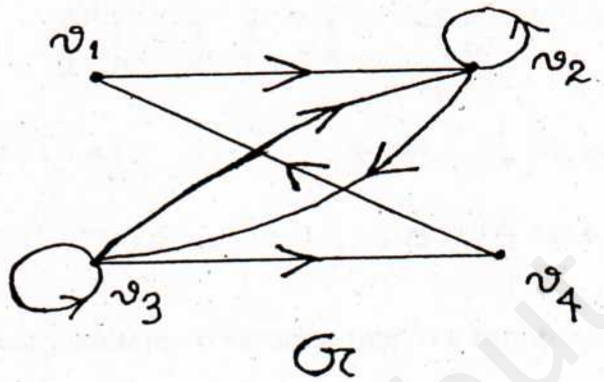
c) Prove that the order of each subgroup of a finite group is a divisor of the order of the group.

3 + 4 + 8

11. a) Examine whether the following two graphs are isomorphic :

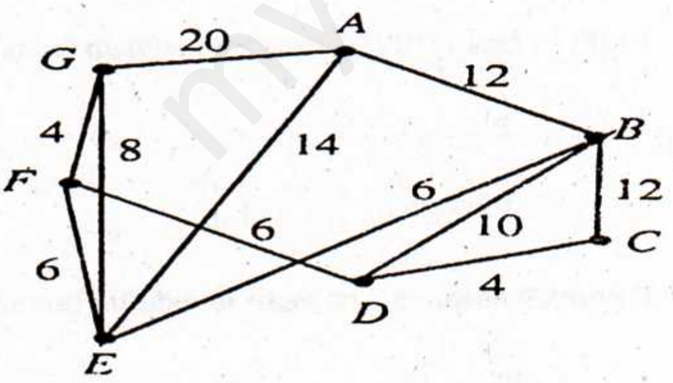


b) Find the adjacency matrix of the following digraph G :

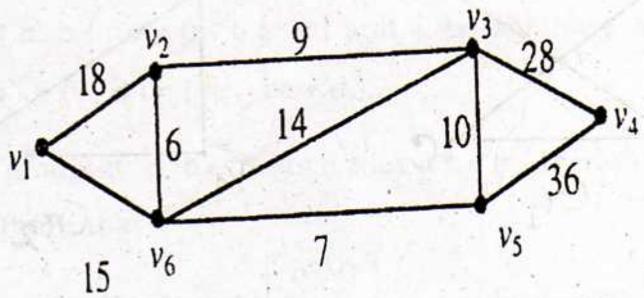


10 + 5

12. a) Find by Prim's algorithm a minimal spanning tree from the following graph :



b) Applying Dijkstra's Algorithm find the shortest path from the vertex v_1 to v_4 in the following simple graph :



8 + 7

END